A Bayesian assessment for railway track geometry degradation prognostics

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ABSTRACT

Advanced PHM techniques have the potential to substantially reduce railway track maintenance costs while increasing safety and availability. However, there is still a significant lack of knowledge and experience in relation to suitable PHM models and algorithms within the context of railway track geometry degradation. This paper proposes a Bayesian model class methodology for prognostics performance assessment whereby different prognostics algorithms can be rigorously assessed and ranked according to their relative probability to predict the future degradation process. The proposed framework is exemplified and tested for a case study about track degradation prognostics using published data about track settlement, taken from a simulated traffic loading experiment carried out at the Nottingham Railway Test Facility.

1. INTRODUCTION

The continuous ageing and the increasing use of the railway infrastructure calls for advanced PHM techniques to optimise the infrastructure asset management. For ballasted tracks, which represent the vast majority of the railway network worldwide, geometry degradation due to traffic loadings represents the main ageing factor requiring periodic interventions to restore the track to an acceptable geometry (Esveld, 2001; Selig & Waters, 1994). These interventions represent a very significant part of the overall infrastructure maintenance costs, and furthermore, imply temporary line closures and disruptions which reduce the effective network capacity. As a result, railway track maintenance typically needs to be planned months in advance.

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In this context of anticipated decision-making, several authors have started to look at PHM techniques for more predictive condition-based track asset management (Mishra, Odelius, Thaduri, Nissen, & Rantatalo, 2017; J. Chiachío, Chiachío, Prescott, & Andrews, 2017). However, the literature on this topic is still a very limited, and thus, there is a lack of conclusive knowledge about which of the available track degradation modelling approaches is more suitable for prognostics, and which prognostics algorithms and methods best suit this challenge. To date, railway track degradation and maintenance modelling has a strong empirical character, mostly grounded on data-based models with limited prospective capability. A review of these models can be found in Dahlberg (2001) for track settlement modelling, and more recently in Soleimanmeigouni and Ahmadi (2016) and Higgins and Liu (2017), focusing also in maintenance modelling. However, only few authors have adopted physics-based models to deal with railway track geometry degradation from first geomechanical principles. See for example Suiker and de Borst (2003); Indraratna, Thakur, Vinod, and Salim (2012).

This paper proposes a Bayesian methodology for the assessment of the prognostics performance of two contrasting families or *model classes* for railway track geometry degradation; namely, a physics-based model class, and a phenomenological data-based model class. The assessment is carried out using probabilities that measure the relative degree of belief of a candidate model class in predicting the upcoming degradation process. These probabilities are sequentially obtained using Bayes' Theorem based on the *prospective evidence* concept, which is a re-interpretation of the classical Bayesian evidence in the context of prognostics, measuring the probability of the future degradation process to be predicted by a given prognostics algorithm under a particular model class. As a case study, the proposed methodology is exemplified using experimental data taken from Aursudkij, McDowell, and Collop (2009) about permanent axial strain in a ballasted railway track, carried out at the Nottingham Railway Test Facility (Brown, Brodrick, Thom, & McDowell, 2007).

The remainder of the paper is organised as follows. Section 2 presents the fundamentals of the track geometry degradation models to be tested, and provides an overview of the methodology adopted for model-based prognostics. Section 3 presents the proposed Bayesian model class methodology for prognostics. In Section 4, the proposed Bayesian framework is applied to railway track settlement data to serve as a case study. Finally, Section 5 provides concluding remarks.

2. TRACK DEGRADATION MODELLING

2.1. Candidate model classes

In this study, two representative model classes are selected to be assessed and ranked using the proposed Bayesian assessment methodology. The first model class, denoted here by $\mathcal{M}_0$, corresponds to a physics-based elasto-plastic model originally proposed by Indraratna et al. (2012) to represent the evolution of the permanent deformation of ballast with cyclic loadings. In essence, the model predicts the cyclic accumulation of permanent deformations in the granular substructure as a function of the applied stress invariants p and q (refer to Appendix), along with some geomechanical input parameters, as[1]:

$$\frac{d\epsilon_v^p}{d\epsilon_s^p} = \frac{9(M - \eta p/p_{cs})}{9 + 3M - 2\eta M p/p_{cs}} \tag{1a}$$

$$\frac{d\epsilon_s^p}{d\eta} = \frac{2\phi\kappa\,(1 - p_{0,i}/p_{cs,i})\,(p/p_{cs})}{M^2(1+e_0)\,(2p_0/p - 1)}\frac{d\epsilon_s^p}{d\epsilon_v^p}\eta \tag{1b}$$

where ϵ_v^p and ϵ_s^p are the plastic volumetric and deviatoric strains, respectively, $\eta = q/p$ is the applied stress ratio, and ϕ is a semi-empirical factor controlling the cyclic hardening of the material, which depends on some empirical parameters θ, i.e., $\phi : \phi(\theta)$ (see Eq (21)). The rest of input parameters are defined in the Nomenclature Section and also in the Appendix.

For each loading cycle, the differential constitutive equations in Eq (1) are numerically integrated by finite differences, leading to a set of cycle-by-cycle incremental equations, as follows (J. Chiachío et al., 2017):

$$\epsilon_s^p|_n = \epsilon_s^p|_{n-1} + \Delta\epsilon_s^p|_n \tag{2a}$$

$$\epsilon_v^p|_n = \epsilon_v^p|_{n-1} + \Delta\epsilon_v^p|_n \tag{2b}$$

where $\epsilon_s^p|_n$ and $\epsilon_v^p|_n$ are the deviatoric and volumetric plastic strains at loading cycle n, respectively. The management variable of interest in this problem is the vertical permanent strain of the track after n loading cycles, $\epsilon_1^p|_n$, which can be periodically measured section by section (Selig & Waters, 1994). Using basic geomechanical derivations, $\epsilon_1^p|_n$ can be shown to be obtained as a function of the component plastic strains $\epsilon_s^p|_n$ and $\epsilon_v^p|_n$, as:

$$\epsilon_1^p|_n = \epsilon_s^p|_n + \frac{1}{3}\epsilon_v^p|_n \tag{3}$$

The other candidate model class to be tested, denoted as $\mathcal{M}_1$ in this paper, corresponds to a phenomenological logarithmic relationship between the vertical permanent strain of the track and the number of loading cycles, given by the expression:

$$\epsilon_1^p|_n = A + B\ln n \tag{4}$$

where A and B are fitting parameters. This model class has been extensively adopted by many authors for its efficiency and simplicity. See for example Alva-Hurtado and Selig (1981); Hettler (1984); Indraratna, Salim, Christie, et al. (2002). A discrete-time representation of this model can be straightforwardly obtained as

$$\epsilon_1^p|_n = \begin{cases} A & n = 1 \\ \epsilon_1^p|_{n-1} + B/n, & n > 1 \end{cases} \tag{5}$$

2.2. Model-based prognostics methodology

Let us assume that our physical system can be represented by a discrete-time state-space I/O model, as follows (Chiachío, Chiachío, Sankararaman, Saxena, & Goebel, 2015):

$$x_n = g(x_{n-1}, \theta) + v_n \tag{6a}$$

$$y_n = h(x_n, \theta) + w_n \tag{6b}$$

where x_n is the *actual state* of the system at time or load cycle n, and y_n represents a measurement about the system state at time n. The function $g(x_{n-1}, \theta) : \mathbb{R}^{n_x} \times \mathbb{R}^{n_\theta} \to \mathbb{R}^{n_x}$ is the *state transition equation*, represented by the model classes considered above (given by Eq (2) and (5) for $\mathcal{M}_0$ and $\mathcal{M}_1$, respectively), and $h(x_n, \theta) : \mathbb{R}^{n_x} \times \mathbb{R}^{n_\theta} \to \mathbb{R}^{n_y}$ is the *observation equation*, given by Eq (3) for $\mathcal{M}_0$, and by $h(x_n, \theta) = \epsilon_1^p|_n$ for $\mathcal{M}_1$. In Eq (6), $\theta \in \Theta$ is a vector of model parameters representing each model class, and v_n and w_n represent the modelling error and measurement noise, respectively. Following the Principle of Maximum Information Entropy (Jaynes, 2003), these error terms are conservatively modelled as zero mean Gaussians, i.e.:, $v \sim \mathcal{N}(0, \sigma_v)$, $w \sim \mathcal{N}(0, \sigma_w)$. If the model parameters are included within the system state as an *augmented state*, i.e., $z = (x, \theta)$, then the dynamical model defined in Eq (6) in state-space form can be probabilistically rewritten as (Chiachío et al., 2015):

$$p(z_n|z_{n-1}) = \mathcal{N}(g(z_{n-1}), \sigma_v) \tag{7a}$$

$$p(y_n|z_n) = \mathcal{N}(h(z_n), \sigma_w) \tag{7b}$$

[1]The term representing the contribution of ballast breakage in the plastic deformation of ballast proposed in Indraratna et al. (2012) is neglected here for simplicity.

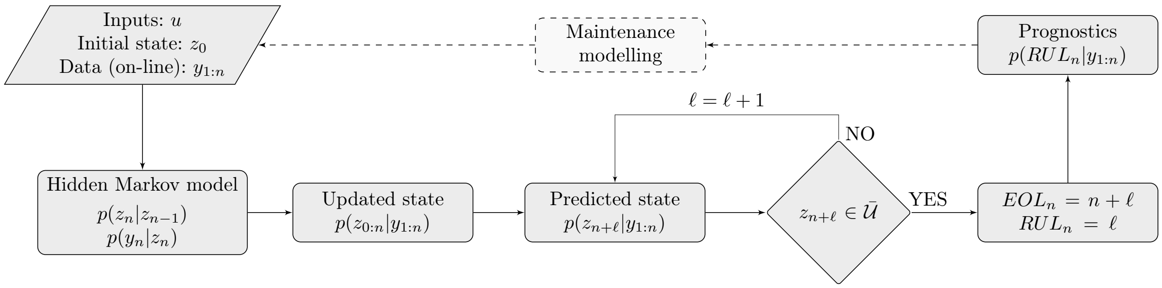

Figure 1. Schematic overview of model-based prognostics methodology

The probabilities in Eq (7) constitute the key elements in the model-based prognostics approach for track settlement (J. Chiachío et al., 2017). In particular, given a sequence of measurements up to time or cycle n, $y_{1:n} = \{y_1, y_2, \ldots, y_n\}$, where y_i denotes a track settlement measurement at the i-th loading cycle, the goal is to estimate the *updated* probability density function (PDF) of the system state at current time n. This PDF is obtained from Eqs. (7) using Bayes' Theorem, as follows:

$$p(z_{0:n}|y_{1:n}) \propto \underbrace{p(y_n|z_n)}_{\text{Eq (7b)}}\underbrace{p(z_n|z_{n-1})}_{\text{Eq (7a)}}p(z_{0:n-1}|y_{1:n-1}) \quad (8)$$

where $p(z_{0:n-1}|y_{1:n-1})$ is the system state at previous cycle $n-1$. Note that Equation (8) is analytically intractable in the general case involving both nonlinear and non-Gaussian state-space models. Sequential Monte Carlo methods are typically adopted to efficiently approximate the posterior PDF in Eq (8) using a collection of K weighted samples or *particles*, $\{z_n^{(i)}, \omega_n^{(i)}\}_{i=1}^K$, $\sum_{i=1}^K \omega_n^{(i)} = 1$, as follows (Liu & West, 2001):

$$p(z_{0:n}|y_{1:n}) \approx \sum_{i=1}^{K} \omega_n^{(i)} \delta(z_{0:n} - z_{0:n}^{(i)}) \quad (9)$$

where δ is the Dirac delta. From the estimation of the updated state of the system at current time or cycle n, a particle estimation of the ℓ-step ahead state of the system can be shown to be obtained by Total Probability Theorem as (Chiachío et al., 2015):

$$p(z_{n+\ell}|y_{1:n}) \approx \sum_{i=1}^{K} \omega_n^{(i)} \delta(z_{n+\ell} - z_{n+\ell}^{(i)}) \quad (10)$$

where $\omega_n^{(i)}$ are the particle weights corresponding to the system update at load cycle n. Having defined the boundary $\partial\bar{\mathcal{U}}$ between the *safe region* $\mathcal{U}$ of the state space and the *unsafe region* $\bar{\mathcal{U}}$ (i.e., the region where the system performance is unacceptable), a particle-filter estimation of the *End of Life* (EOL) and *Remaining Useful Life* (RUL) can be obtained based on Eq (10), as follows:

$$p(\mathrm{EOL}_n|y_{1:n}) \approx \sum_{i=1}^{K} \omega_n^{(i)} \delta(\mathrm{EOL}_n - \mathrm{EOL}_n^{(i)}) \quad (11a)$$

$$p(\mathrm{RUL}_n|y_{1:n}) \approx \sum_{i=1}^{K} \omega_n^{(i)} \delta(\mathrm{RUL}_n - \mathrm{RUL}_n^{(i)}) \quad (11b)$$

In Eqs (11a) and (11b), $\mathrm{EOL}_n^{(i)}$ and $\mathrm{RUL}_n^{(i)}$ are the particles for EOL_n and RUL_n respectively, which are obtained as

$$\mathrm{EOL}_n^{(i)} = \inf\big\{n + \ell \in \mathbb{N} : \ell \geqslant 1 \wedge \mathbb{I}_{(\bar{\mathcal{U}})}(z_{n+\ell}^{(i)}) = 1\big\} \quad (12a)$$

$$\mathrm{RUL}_n^{(i)} = \mathrm{EOL}_n^{(i)} - n \quad (12b)$$

with $\mathbb{I}_{(\bar{\mathcal{U}})}$ being an indicator function that assigns the unity if $z_{n+\ell} \in \bar{\mathcal{U}}$, and makes zero the rest. A graphical scheme summarising the model-based prognostics methodology is provided in Figure 1. The interested reader is referred to Chiachío et al. (2015); M. Chiachío, Chiachío, Shankararaman, Goebel, and Andrews (2017) for further insight about model-based prognostics.

3. BAYESIAN ASSESSMENT METHODOLOGY

3.1. Prospective evidence of candidate model class

Let suppose that measurements about system degradation are available until current time or load cycle n, denoted by $\mathcal{D}_n \triangleq y_{1:n}$, and that the system is updated until time n, using Eq (8) under model class $\mathcal{M}$. Let suppose also that data about the future performance of the system are available,[2] denoted by $\mathcal{D}_{n^+}$. Then, one might be interested in assessing the probability of $\mathcal{D}_{n^+}$ to be predicted by a particular model-based prognostics algorithm under model class $\mathcal{M}$ at current time n. This probability can be obtained by computing the *prospec-*

[2]The availability of "future data" $\mathcal{D}_{n^+}$ could be a hard assumption for purely online prognostics scenarios, but it may hold true for offline and pseudo-online prognostics analyses. For online prognostics, we may assume that $\mathcal{D}_{n^+}$ can be available through experiments run in similar conditions.

tive evidence of model class $\mathcal{M}$, as follows:

$$p(\mathcal{D}_{n^+}|\mathcal{D}_n,\mathcal{M}) = \int_\Theta p(\mathcal{D}_{n^+}|\theta_n,\mathcal{D}_n,\mathcal{M})p(\theta_n|\mathcal{D}_n,\mathcal{M})d\theta_n \tag{13}$$

where $p(\theta_n|\mathcal{D}_n,\mathcal{M})$ is the marginal posterior of the model parameters at load cycle n, which can be approximated using Eq (9). Thus, a particle approximation of the multi-dimensional integral in Eq (13) can be readily obtained as:

$$p(\mathcal{D}_{n^+}|\mathcal{D}_n,\mathcal{M}) \approx \sum_{k=1}^{K}\omega_n^{(k)} p(\mathcal{D}_{n^+}|\theta_n^{(k)},\mathcal{D}_n,\mathcal{M}) \tag{14}$$

with $p(\mathcal{D}_{n^+}|\mathcal{D}_n,\theta_n^{(k)},\mathcal{M})$ being the *prospective likelihood function* of model class $\mathcal{M}$, which measures how likely $\mathcal{D}_{n^+}$ is predicted by $\mathcal{M}$ parameterized by $\theta_n^{(k)}$. If $\mathcal{D}_{n^+}$ is given by the sequence $\mathcal{D}_{n^+} = \{y_j, y_k, \ldots, y_m\}$, where $\{j,k,\ldots,m\} \subset \mathbb{N} > n$ are the discrete times or load cycles where the upcoming data are available, then the prospective likelihood function can be obtained as:

$$p(\mathcal{D}_{n^+}|\theta_n^{(k)},\mathcal{D}_n) = \underbrace{p(y_k|y_j,\theta_n^{(k)})}_{\text{Eq (7a)}}\cdots\underbrace{p(y_m|y_n,\theta_n^{(k)})}_{\text{Eq (7a)}} \tag{15}$$

where the conditioning on $\mathcal{M}$ is dropped for the sake of simplicity.

3.2. Prospective plausibility of candidate model class

In addition to quantifying the prognostics performance for a particular model class or a particular algorithm, a question of particular interest is the assessment and rank of different model classes $\mathfrak{M} = \{\mathcal{M}_1,\ldots,\mathcal{M}_{n_M}\}$, according to their *updated* plausibility to predict the future degradation process. Thus, given an initial quantification of the relative plausibility of a candidate model class, $P(\mathcal{M}_j|\mathfrak{M})$, where $\sum_{i=1}^{n_M} P(\mathcal{M}_i|\mathfrak{M}) = 1$, the *prospective plausibility* of such model class can be obtained using Bayes' Theorem, as:

$$P(\mathcal{M}_j|\mathcal{D}_{n^+},\mathfrak{M}) = P(\mathcal{M}_j|\mathfrak{M})\frac{p(\mathcal{D}_{n^+}|\mathcal{D}_n,\mathcal{M}_j,\mathfrak{M})}{p(\mathcal{D}_{n^+}|\mathcal{D}_n,\mathfrak{M})} \tag{16}$$

where $p(\mathcal{D}_{n^+}|\mathcal{D}_n,\mathcal{M}_j,\mathfrak{M})$ is the prospective evidence of $\mathcal{M}_j$ given by Eq (13), and the term $p(\mathcal{D}_{n^+}|\mathcal{D}_n,\mathfrak{M})$ is obtained by Total Probability Theorem as

$$p(\mathcal{D}_{n^+}|\mathcal{D}_n,\mathfrak{M}) = \sum_{i=1}^{n_M} p(\mathcal{D}_{n^+}|\mathcal{D}_n,\mathcal{M}_i,\mathfrak{M}) \tag{17}$$

4. CASE STUDY

The Bayesian assessment methodology for model-based prognostics presented above is exemplified here using a case study about railway track geometry degradation. To this end, published data (Aursudkij et al., 2009) about plastic axial strain in a ballasted railway track is considered for the assessment, taken from an experiment carried out at the Nottingham Railway Test Facility (Brown et al., 2007). The dataset is represented in Figure 2 for illustration purposes, while further insight about the experimental setup is found in Aursudkij et al. (2009).

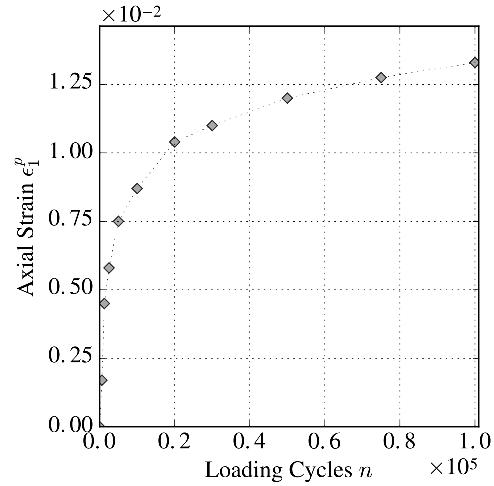

Figure 2. Track geometry degradation data taken from Aursudkij et al. (2009)

The prognostics methodology summarised in Section 2.2 is applied using the data in Figure 2 for the two model classes considered in Section 2.1. For identification purposes, the physics based model in Eq (1) is referred to as model class $\mathcal{M}_0$, while the phenomenological model proposed in Eq (3) is denoted as $\mathcal{M}_1$. A discrete-time state-space representation of both model classes is obtained and subsequently embedded within a particle-filtering algorithm using $N = 5,000$ particles, following the methodology in Section 2.2. For this analysis, both model classes are conservatively assumed to be equally plausible *a priori*, i.e., $P(\mathcal{M}_j|\mathfrak{M}) = 0.5$, $j = 0,1$. Then, as long as the process evolves and data is collected, estimations of the prospective evidence in Eq (13) are sequentially obtained, whereby the relative prospective plausibility of each model class is subsequently obtained using Eq (16). The results for the relative prospective plausibility of both model classes are shown in Figure 3 for the times (cycles) where degradation data are available. In view of these results, the prognostics algorithm using the physics-based model class $\mathcal{M}_0$ is revealed as the most plausible in predicting the future degradation of the system, and therefore the one which is more likely to provide the best prognostics estimates. This higher prospective plausibility is more accentuated at the beginning of the process due to the initial lack of data, since the physics-based model class needs less support from data to train the fitting parameters. As the process evolves towards the end, both model classes have extracted

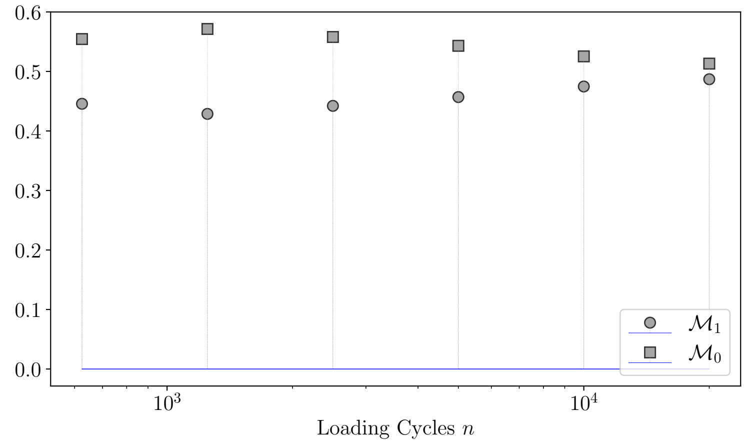

Figure 3. Relative prospective plausibility obtained for both the physics-based model class $\mathcal{M}_0$ and a phenomenological model class $\mathcal{M}_1$ for track geometry degradation, using the data in Figure 2.

enough information from the data to allow them to perform similarly in terms of prognostics, and their relative plausibilities tend to converge.

5. CONCLUSION

PHM science and technology has the potential to explore and devise optimal frameworks for railway track asset management, thus contributing to reduce maintenance cost and increase safety and availability. Notwithstanding, there is a need for more conclusive knowledge in this important area of application of PHM. A Bayesian model class methodology has been proposed in this paper to test and rank different prognostics models and algorithms for railway track degradation prognostics according to their relative probability to predict the future degradation process. The methodology is general in nature but for this paper, it has been illustrated using a case study for railway track degradation prognostics, where the performance of a physics-based model class for track degradation is compared to that obtained using a phenomenological data-based model class. According to the results for this case study, the physics-based model class provides the highest probabilities during the whole process, which suggests that among the two candidate model classes, the physics-based model class is more likely to predict the future and therefore to provide the best prognostics estimates. More research effort is needed to corroborate these results with new data and different model classes, and in general to exploit the potential of PHM in the context of optimal railway track asset management.

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NOMENCLATURE

$d\epsilon_v^p$	plastic volumetric strain increment
$d\epsilon_s^p$	plastic distortional strain increment
e	voids ratio
e_0	Initial voids ratio
p	mean stress invariant
p_0	initial mean stress
q	deviatoric stress invariant
q_{max}	in-cycle maximum deviatoric stress
q_{min}	in-cycle minimum deviatoric stress
η	stress-ratio $\eta = {}^{q}/_{p}$
M	critical stress-ratio
Γ	critical state model parameter
λ_{cs}	critical state model parameter
κ	swelling/recompression constant
σ_1	applied vertical stress
σ_3	confining stress

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BIOGRAPHIES

Juan Chiachío is a Postdoctoral Research Fellow in Infrastructure Asset Management in the Resilience Engineering Research Group at the University of Nottingham (UK). He received his PhD in Structural Engineering in 2014 (*Summa Cum Laude, International Mention*) by the University of Granada, Spain. In addition, he holds a MSc in Structural Engineering and a MSc in Civil Engineering, both by the University of Granada. His research is focused on translating reliability and prognostics methods into the life-cycle analysis of structural and infrastructural systems subjected in-service degradation. This research has led to a number of publications in highly ranked journals, a best-paper award and nominations in major conferences. Dr Chiachío has been awarded by the Spanish National Council of Education through one of the FPU annual fellowships, by the Andalusian Society of Promotion of Talent, by the Prognostics and Health Management Society with a Best Paper Award, and by the European Council of Civil Engineers. In addition, his work has attracted the interest of world-class institutions for collaborative research, like the Prognostics Center of Excellence of NASA, the California Institute of Technology, and the Hamburg University of Technology (Germany). Prior to starting his academic career in 2011, Dr Chiachío worked as a structural engineer for five years in top engineering companies in Spain.

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John Andrews is Head of the Resilience Engineering Research Group at the University of Nottingham where he holds a Royal Academy of Engineering Research Chair in Infrastructure Asset Management. Prior to this he worked for 20 years at Loughborough University where his final post was Professor of Systems Risk and Reliability. The prime focus of his research has been on methods for evaluating the system resilience, unavailability, unreliability and risk. Much of this work has concentrated on the Fault Tree analysis technique and the use of Binary Decision Diagrams (BDDs). Recently attention has turned more the degradation modelling and the effects of maintenance, inspection and renewal on asset performance. In this context the modelling he has carried out has extended the Petri net and Bayesian Network capabilities. In 2005, John founded the Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability of which was the Editor-in-chief for 10 years. He is also a member of the Editorial Boards for 6 other international journals in this field.

APPENDIX

For granular materials like ballast and suballast under three-dimensional stresses, the following relationships are used to obtain the stress invariants p and q:

$$p = \frac{1}{3}\sigma_{kk} \tag{18a}$$

$$q = \sqrt{\frac{3}{2}s_{ij}s_{ij}} \tag{18b}$$

where σ_{ij} is the stress tensor, and s_{ij} the stress deviator tensor, defined as

$$s_{ij} = \sigma_{ij} - p\delta_{ij} \tag{19}$$

with δ_{ij} the Kronecker delta function. Under the assumption of axisymmetric stress state ($\sigma_2 = \sigma_3$), the stress invariants simplify to

$$p = \frac{1}{3}(\sigma_1 + 2\sigma_3) \tag{20a}$$

$$q = \|\sigma_1 - \sigma_3\| \tag{20b}$$

In regards to the governing equations, the function ϕ in Eq (1) is a semi-empirical factor that accounts for complex phenomena observed in the yielding behaviour of granular materials under cyclic loading conditions, such as the Bauschinger effect, the effect of the stress ratio and loading history, among others (Mroz, Norris, & Zienkiewicz, 1978). In this research, the expression proposed by Indraratna et al. (2012) is adopted to account for such effects, which is given by:

$$\phi = \alpha \left(1 - \frac{\eta}{M}\frac{p}{p_{cs}}\right) \left(\frac{\langle p - p_e\rangle^2 + \langle q - q_e\rangle^2}{(\Delta p)^2 + (\Delta q)^2}\right)^{1/2} N^{\beta} \tag{21}$$

with $\Delta p, \Delta q$ being the in-cycle total stress increments, $\langle\cdot\rangle$ the Macauley brackets, and α and β empirical fitting parameters. The elastic mean stress p_e is given by the expression

$$p_e = p_{min} + \left(1 - \frac{1}{\ln(n+10)}\right)\Delta p \tag{22}$$

In Eqs. (21) and (1), p_{cs} is the value of p at the critical state, which can be obtained as (Roscoe, Schofield, & Wroth, 1958)

$$p_{cs} = \exp\left(\frac{\Gamma - e}{\lambda_{cs}}\right) \tag{23}$$

where Γ, and λ_{cs} are material parameters, which, together with M and κ (see Nomenclature section), conform the parameters of the model (Indraratna et al., 2012). The rest of elements are defined in the Nomenclature section.