# On the use of particle filters for prognostics in industrial applications

Hai Canh VU [1], Phuc DO[2], Mayank Shekhar JHA [3], Didier THEILLIOL [4], and Flavien PEYSSON[5]

[1,2,3,4] *Laboratoire CRAN, University of Lorraine, 54506 Vandoeuvre Cedex, France*
*hai-canh.vu@univ-lorraine.fr*

[5] *PREDICT Company, 19, avenue de la Fort de Haye, CS 10508, 54519 Vandoeuvre Cedex, France*
*flavien.peysson@predict.fr*

**ABSTRACT**

Prognostics is an engineering discipline aiming at predicting the Remaining Useful Life (RUL) of an industrial system or item. Accuracy and confident prediction of the RUL are very meaningful and important for anticipating failure, controlling system operational efficiency as well as optimizing maintenance operations. Given the important role of the prognostics or RUL prediction, a number of prognostics approaches has been proposed and successfully applied in various industry. Among these approaches, particle filters (PF) are more and more studied and employed thank to their powerful performance and their flexibility in predicting the RUL of systems non-linear and non-Gaussian. However, the prediction performance strongly depends on the application contexts and the type of particle filter utilized. The choice of particle filters is therefore a critical step in real industrial applications. The paper focuses on a comparison of the three different PF techniques (Sampling importance resampling, Auxiliary particle filter, and Regularized particle filter) to support the critical step. The performance of the three PF techniques is compared by considering different degradation models, noises level. In addition, the computing time is also analyzed through different numerical examples.

## 1. INTRODUCTION

Prognostics is an engineering discipline aiming at predicting the future health of a component/system and generate its RUL, which is defined as the estimated time that the component/system can be expected to continue to serve its intended function, with taking into account the knowledge of the component/system, the historical and the current monitoring data, and future information related to mission profile, maintenance plan, etc (Voisin, Levrat, Cocheteux, & Iung, 2010). The prognostic process is structured on three mains steps: construction and fusion health indicators, RUL definition and formalization, RUL prediction using different prognostic approaches. The process starts by constructing component/system health indicators from the monitoring data. The health indicators represent the component/system health state defined as the rate and the extent of degradation from the expected normal operation condition (degradation indicator) or the degree of functional performance (performance indicator). According to the complexity of the considered system, the number of the system health indicators may be high. Thus, to reduce the dimensional space and predict the RUL effectively, the constructed health indicators are usually aggregated by using different fusion techniques to provide the most representative health indicators. The RUL is then defined and formalized based on the representative health indicators and their thresholds. Finally, prognostic approaches such as Markov models, Artificial neural networks, Kalman filter, Particle filters, etc., are applied to estimate or predict the RUL (Vogl, Weiss, & Helu, 2016).

Among these approaches, particle filters (PF) are more and more studied and implemented for the prognostics of industrial component/system thanks to their powerful performance and their flexibility in predicting the RUL of systems non-linear and non-Gaussian (An, Choi, & Kim, 2013; Zio & Peloni, 2011; Jouin, Gouriveau, Hissel, Péra, & Zerhouni, 2016). Their main idea is to use a set of samples (particles) with associated weights to represent the state density function. The state estimates are then computed based on these particle and weights. The sequential importance sampling is employed to reduce the number of particles which are required to approximate the state probability distribution. PF is therefore more efficient than the classical Monte Carlo. Given these advantages of PFs, different PF techniques have been investigated and proposed to improve the PF performance in real industrial applications. The three main PF techniques are Sampling importance resampling (SIR), Auxiliary particle filter (APF), and Regularized particle filter (RPF). Given the similar main idea, the performance of the three PF tech-

niques are not the same and strongly depends on the industrial application contexts. The choice of particle filter techniques is therefore a critical step in real industrial applications. Unfortunately, very few papers in the literature focus on the comparison of these PF techniques from a practical point of view. This paper then focuses on a comparison of the three different PF techniques (SIR, APF, and RPF) to support the critical step. The performance of the three PF techniques is compared by considering two application settings: the precision of the system degradation models and noises levels. In addition, the computing time is also analyzed through different numerical examples.

The rest sections of the paper are organized as follows: Section 2 is devoted to present the main idea of particle filters; detailed descriptions of SIR, APF and RPF are found in Section 3; Section 4 focuses on explaining how the PF techniques can be used for the RUL prediction of industrial component/system and describing the selected comparison criteria; the comparison is done through different numerical examples presented in Section 5; finally, some conclusions and perspectives are made in Section 6.

## 2. FUNDAMENTAL PRINCIPLES OF PARTICLE FILTERS

Before comparison of the three different PF techniques (SIR, APF, and RPF), we will present shortly the fundamental principles of particle filters in general, and that of each PF techniques in particular. This section focuses firstly on the mathematical descriptions of the particle filter approach.

To illustrate the basic principles of state estimation and prediction using particle filters and its commonly employed variants, a dynamic system is considered with system state at time step is represented by the vector:

$$\mathbf{x}_k = f_k(\mathbf{x}_{k-1}, v_{k-1}) \tag{1}$$

$$\mathbf{y}_k = h_k\left(x_k, w_k\right) \tag{2}$$

where, $f_k$ being state transition function (possibly non-linear); $h_k$ being the observation function that describes measurements $y_k$ which are obtained sequentially at successive time steps; $v_k$ and $w_k$ being respectively the process noise sequence and measurement noise sequence of known distribution and assumed independent and identically distributed (i.i.d).

Their equivalent representation can be obtained as:

$$\mathbf{x}_k = f_k(\mathbf{x}_{k-1}, v_{k-1}) \leftrightarrow p(\mathbf{x}_k|\mathbf{x}_{k-1}) \tag{3}$$

and

$$\mathbf{y}_k = h_k\left(\mathbf{x}_k, w_k\right) \leftrightarrow p(\mathbf{y}_k|\mathbf{x}_{k-1}) \tag{4}$$

such that $p(x_k|x_{k-1})$ represents the state transition probability and $p(\mathbf{y}_k|\mathbf{x}_{k-1})$ is the likelihood function that signifies the probability of the observation of $y_k$, given the current estimate of $x_k$.

The filtering procedure targets estimation of $\mathbf{x}_k$ based upon all the available measurement sequences $\mathbf{y}_{1:k} = \{\mathbf{y}_k, k = 1, 2, ....k\}$ . In the context of Bayesian inference, the main objective is recursive calculation of state distribution, given the set of observations $\mathbf{y}_{1:k}$ up to time $t_k$, with acceptable degree of belief. This in turn, requires construction of PDF (probability distribution function) $p(\mathbf{x}_k|\mathbf{y}_{1:k})$ commonly known as filtered posterior state PDF that provides all the information about $\mathbf{x}_k$ inferred from the measurements $\mathbf{y}_{1:k}$. The initial state PDF $p(\mathbf{x}_0)$ is assumed to be known a priori and $\mathbf{y}_0$ is the set of no measurements. Theoretically, the posterior state can be estimated from the prior $p(\mathbf{x}_{k-1}|\mathbf{y}_{1\,:\,k-1})$ at time $t_{k-1}$ in a recursive way via two sequential steps: prediction and update.

Prediction step involves application of Chapman-Kolmogorov equation that leads to the prior state PDF $p(\mathbf{x}_k|\mathbf{y}_{1:\,k-1})$ at time $t_k$:

$$p(\mathbf{x}_k|\mathbf{y}_{1\,:\,k-1}) = \int p(\mathbf{x}_k|\mathbf{x}_{k-1}) p(\mathbf{x}_{k-1}|\mathbf{y}_{1\,:\,k-1}) d\mathbf{x}_{k-1} \tag{5}$$

with the assumption that system follows $1^{st}$ order Makovian dynamics. The update step involves updating the prior as the new measurement $y_k$ arrives; eventually, leading to the posterior distribution of $x_k$ as:

$$p(\mathbf{x}_k|\mathbf{y}_{1\,:\,k}) = \frac{p(\mathbf{x}_k|\mathbf{y}_{1\,:\,k-1}) p(\mathbf{y}_k|\mathbf{x}_k)}{p(\mathbf{y}_k|\mathbf{y}_{1\,:\,k-1})} \tag{6}$$

The normalizing constant (denominator) is:

$$p(\mathbf{y}_k|\mathbf{y}_{1\,:\,k-1}) = \int p(\mathbf{x}_k|\mathbf{y}_{1\,:\,k-1}) p(\mathbf{y}_k|\mathbf{x}_k) d\mathbf{x}_k \tag{7}$$

Thus, the latest measurement is incorporated into a priori state PDF $p(\mathbf{x}_k|\mathbf{y}_{1\,:\,k-1})$ leading to the posterior state estimate PDF $p(\mathbf{x}_k|\mathbf{y}_{1\,:\,k})$.

Although the optimal solution is achievable in principle using the preceding recurrence relations, in practice, due to presence of non-linear system dynamics and non-Gaussian noises, the closed form solutions are rarely obtained in closed form. This is where the Monte Carlo methods become useful and efficient for estimation and inferences. In particular, Sequential monte carlo (SMC) methods are extensively used for recursive online estimation. Particle filters is a SMC technique in which the posterior state PDF is obtained by a set of random samples or particles where each of the particles has an associated weight based upon which the state estimates are computed. In particular, Sequential importance sampling (SIS) is one of the most popular and basic PFs, also known as Bootstrap PF, condensation PF or survival of the fittest, in which posterior state PDF $p(\mathbf{x}_{0:k}|\mathbf{y}_{1:k})$ by a set of N number of weighted particles $\left\{(\mathbf{x}^i_{0:k}), \mathrm{w}^i_k\right\}_{i=1}^N$ with $\{\mathbf{x}^i_{0:k}, i = 1, ...N\}$ being the set of particles representing the state value with respective weights as $\{w^i_k, i = 1, ...N\}$. Moreover, $\mathbf{x}_{0\,:\,k} =$

$\{\mathbf{x}_j, j = 0, ...., k\}$ is the set of all states up to time k. The weights are the approximations of the relative posterior probabilities of the particles and normalized such that $\sum_i {w^i}_k = 1$.

Then, posterior PDF is approximated as

$$p(\mathbf{x}_{0:k}|\mathbf{y}_{1\,:\,k}) \approx \sum_{i=1}^{N} \mathrm{w}_k^i.\delta(\mathbf{x}_{0:k} - \mathbf{x}_{0:k}^i) \tag{8}$$

where $\delta$ denotes the Dirac delta function, leading to discrete weighted approximation to the true posterior state distribution $p(\mathbf{x}_{0:k}|\mathbf{y}_{1\,:\,k})$. As $N$ tends to large numbers, the Monte Carlo approximation becomes an equivalent representation to the posterior state PDF. This process of obtaining particles weights is not a straight-forward procedure for which the principle of importance sampling is exploited. This involves a proposal distribution known as importance density, which is chosen such that $p(x) \propto q(x)$ and $q(x)$ is a PDF from which samples can be drawn easily. For example, if a set of samples $x^i \sim q(x)$, $i = 1, \ldots, N$ is generated from the proposal distribution $q(x)$, then the weighted approximation of the density $p(x)$ is given as :

$$p(x) \approx \sum_{i=1}^{N} \mathrm{w}^i.\delta(x - x^i) \tag{9}$$

with normalized weights as: $\mathrm{w}^i \approx \frac{p(x^i)}{q(\mathrm{x}^i)}$.

This paper does not dwell into the details of importance distributions and their properties, which are well documented in the literature available. In SIS PF, the importance density is set equal to the PDF of system state i.e.

$$q(\mathbf{x}_{0\,:\,k}|\mathbf{x}_{0\,:\,k-1}) = p(\mathbf{x}_k|\mathbf{x}_{k-1}) = f_k(\mathbf{x}_k|\mathbf{x}_{k-1}) \tag{10}$$

This enables generation of new particles from the previous set of particles by simulating the state transition function $f_k(\mathbf{x}_k|\mathbf{x}_{k-1})$. Additionally, thanks to Markovian dynamics, storage of only filtered estimate $p(\mathbf{x}_k|\mathbf{y}_{1:k})$is required at each step for a sequential online implementation. In other words, only $x_k^i$ and $y_{1:k}$ need to be stored and the previous state path up to $x_{0:k-1}^i$ can be neglected. As a consequence, weights are updated as $\mathrm{w}_k^i \propto \mathrm{w}_{k-1}^i \cdot p(\mathbf{y}_k|\mathbf{x}_k^i)$ leading to an approximation of posterior filtered PDF $p(\mathbf{x}_k|\mathbf{y}_{1:k})$ as

$$p(\mathbf{x}_k|\mathbf{y}_{1\,:\,k}) \approx \sum_{i=1}^{N} \mathrm{w}_k^i.\delta(\mathbf{x}_{0:k} - \mathbf{x}_{0:k}^i) \tag{11}$$

This simplified algorithm can be used for recursive estimation of state as the observations arrive sequentially. The likelihood functions of the new observations $p(\mathbf{y}_k|\mathbf{x}_k^i)$, result in evaluation of weights of particles constituting the next state estimate.

## 3. DIFFERENT PARTICLE FILTER TECHNIQUES

Given the above general principle, the implementation of PF faces many difficulties and its performance is quite limited in real applications. To this end, different PF techniques have been developed to attack specific PF weaknesses and to improve the PF's performance. In that which follows, we will explain in more details three major PF problems and their corresponding solutions.

### 3.1. Sampling Importance Resampling (SIR)

A usual problem with recursive SIS is that previous steps lead to an undesirable situation where the importance weights become increasingly skewed (Arulampalam, Maskell, Gordon, & Clapp, 2002). After few iterations, only few particles tend to have non-negligible weights. This leads to the problem of particle degeneracy where all but one particle must be discarded. To avoid this situation, new set of particles are resampled from the approximate posterior distribution obtained previously in the update stage, constructed upon the weighted particles. This step is known as resampling step and is well established in the literature. In general, the particles are navigated in the region of high likelihood by accepting high weighted particles and abandoning low weighted particles. The new particles are again normalized. There are various techniques developed for efficient resampling, few of them being multinomial resampling, residual resampling, systematic resampling methods. A standard SIS when accompanied by a resampling step gets referred-to as Sampling-Importance resampling (SIR). In this paper, systematic resampling scheme which is based upon an ordered technique, is applied to implement SIR.

### 3.2. Auxiliary particle filter (APF)

The SIR exhibits two basic weaknesses arising out of empirical approximation of the filtering posterior: poor outlier performance and poor posterior tail performance. In order to mitigate the aforementioned issues, auxiliary particle filter that employs sampling-importance-resampling (APF) is used. The basic approach of APF remains in mimicking the operation of the minimum variance importance distribution by introducing an auxiliary variable, $K$, that represents weight used for empirical prediction distribution estimate (Pitt & Shephard, 1999; Douc, Moulines, & Olsson, 2009). The main idea is to perform resampling at time $k-1$ using the available measurement at time $k$ before the particles $\left\{\mathbf{x}^i\right\}_{i=1}^{N}$ are propagated to time $k$ through the transition and likelihood distributions. In this way, particles that are likely to survive (largest weights) at the next time step $k$ are favored at time step $k-1$. The APF is a two-stage process such that: (1) particles with large predictive likelihoods at time-step $(k-1)$ are propagated; and (2) the resulting particles are then re-weighted and drawn from the resulting posterior. Under the assumption that

the joint posterior at time index $k-1$ is well approximated by particle presentation, the joint importance proposal for new particles is obtained as

$$q(\mathbf{x}_{0:k}) = \overbrace{q(\mathbf{x}_{0:k-1}|\mathbf{y}_{0:k})}^{past} \text{ x } \overbrace{q(\mathbf{x}_k|\mathbf{x}_{k-1}, \mathbf{y}_k)}^{new} \tag{12}$$

The empirical distribution for $p(\mathbf{x}_{0:k-1}|\mathbf{y}_{1:k})$ is expressed as:

$$q(\mathbf{x}_{0:k-1}|\mathbf{y}_{1:k}) \approx \sum_{i=1}^{N} K_{k-1}^i \, \delta(\mathbf{x}_{0:k-1} - \mathbf{x}_{0:k-1}^i) \tag{13}$$

where, $\sum_{i=1}^{N} K_{k-1}^i = 1$ and $K_{k-1}^i > 0$. In fact, the $i^{th}$ weight for each particle is based upon pre-selected particles that are good fit to the new data. One of the commonly preferred choice for these weights is to choose a point estimate of the state as its mean:

$$\widehat{m}_k^i = \int \mathbf{x}_k \; p(\mathbf{x}_k|\mathbf{x}_{k-1}^i) d\mathbf{x}_k \tag{14}$$

followed by computation of the weighting function as the likelihood evaluated at this point as

$$K_{k-1}^i = \; w_{k-1}^i \, p(\mathbf{y}_k|\widehat{m}_k^i) \tag{15}$$

which follows from the marginal $p(\mathbf{x}_{0:k-1}|\mathbf{y}_{1:k})$ which is a smoothing function. Using this proposal, the generalized weight

$$Waux_k^i = \frac{p(\mathbf{x}_{0:k}|\mathbf{y}_{1:k})}{q(\mathbf{x}_{0:k})} \tag{16}$$

can be obtained from the ratio of the posterior to the joint proposal giving:

$$Waux_k^i = \frac{p(\mathbf{x}_{0:k}|\mathbf{y}_{1:k})}{q(\mathbf{x}_{0:k})} = \frac{w_{k-1}^i}{\kappa_{k-1}^i} \text{x} \frac{p(\mathbf{y}_k|\mathbf{x}_k^i).p(\mathbf{x}_k^i|\mathbf{x}_{k-1}^i)}{q(\mathbf{x}_k^i|\mathbf{x}_{k-1}^i, \mathbf{y}_k)} \tag{17}$$

### 3.3. Regularized particle filter (RPF)

Resampling technique is employed in SIS in order to avoid particle degeneracy which in turn leads to the formation of SIR. One of the problems arising due to resampling procedure is the loss of diversity in the particles sampled. This occurs mainly because samples are drawn from a discrete rather than a continuous distribution. If not corrected, we risk a collapse of all particles to a single location due to sever particle impoverishment, leading to a poor representation of the posterior distribution. On of the ways to tackle this is to sample the posterior $p(\mathbf{x}_k|\mathbf{y}_{1:k})$ from a continuous rather than a discrete approximation of the empirical distribution using the kernel density estimator. This is commonly known as regularization step such that diversification is achieved by a certain shaking-up of the particles. Thus, the method is called the regularized particle lter. In fact, the RPF is similar to SIR except in the resampling stage which employs a kernel density estimator (Musso, Oudjane, & Le Gland, 2001; Giremus, Tourneret, & Djuric, 2005). More specifically, samples are drawn from approximation:

$$p(\mathbf{x}_k|\mathbf{y}_{1:k}) \approx \sum_{i=1}^{N} \mathrm{w}_k^i . K_{\Delta x}(\mathbf{x}_k - \mathbf{x}_k^i) \tag{18}$$

where,

$$K_{\Delta_x} = \left(\frac{1}{\Delta_x}\right)^{N_x} K\left(\frac{\mathbf{x}}{\Delta_x}\right)$$

is the rescaled kernel; $N_x$ being the state vector dimension; $\Delta_x > 0$ being the bandwidth; $K(\mathbf{x})$ is the regularization kernel which is asymmetric probability density function such that: $K(\mathbf{x}) \geq 0$; $\int K(\mathbf{x}) \, d\mathbf{x} = 1$; $\int \mathbf{x} K(\mathbf{x}) \, d\mathbf{x} = 0$; and $\left\|\mathbf{x}^2\right\| K(\mathbf{x}) \, d\mathbf{x} < \infty$.

The regularization property of the kernel density assures for any distribution $P(\mathbf{x}) \in R^{N_x \text{x} 1}$, the regularization leads to an absolute continuous probability distribution $K_{\Delta_x}(\mathbf{x}) * P(\mathbf{x})$, with * being the convolution operator such that:

$$\frac{d}{d\mathbf{x}}[K_{\Delta_x}(\mathbf{x}) * P(\mathbf{x})] = \int K_{\Delta_x}(\mathbf{x} - \alpha) P(\alpha) d(\alpha) \tag{19}$$

Then, if is an empirical distribution, it can be approximated as:

$$\hat{P}(\mathbf{x}) \approx \sum_{i=1}^{N} \delta(\mathbf{x} - \mathbf{x}^i)$$

with $x^i$ being a sample from the posterior. This leads to the transformation as:

$$\begin{aligned} \frac{d}{d\mathbf{x}}[K_{\Delta_x}(\mathbf{x}) * \hat{P}(\mathbf{x})] &= \left(\frac{1}{\Delta_x}\right)^{N_x} \sum_{i=1}^{N} w^i K(\frac{\mathbf{x} - \mathbf{x}^i}{\Delta_x}) \\ &= \sum_{i=1}^{N} w^i K_{\Delta_x}(\mathbf{x} - \mathbf{x}^i) \end{aligned} \tag{20}$$

Here, the bandwidth and kernel are selected in such a way that the mean integrated error between the posterior and regularized distribution is minimized. Some of the commonly used kernels (with special assumptions) are: Epanechnikov, Box, Triangle, Gaussian kernels etc. Now, the approximated posterior samples are used to obtain the ensemble mean and covariance at each step by the ensemble approach followed by factorization of covariance using the Cholesky decomposition. This gives matrix square roots $L$ used in the whitening transformations and leads to new scaled kernel

$$K_{\Delta_x}(\mathbf{x}) = \frac{1}{|L^{1/2}|(\Delta_x)^{N_x}} K\left(\frac{L^{1/2}\mathbf{x}}{\Delta_x}\right) \tag{21}$$

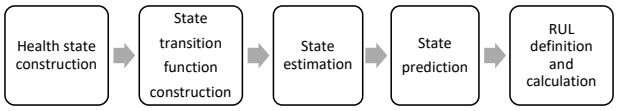


Figure 1. Particle filter based prognostics

The old particles are then shaken-up at each time instant as:

$$\tilde{x}_k^i = x_k^i + \Delta_x L_k^{1/2}.\varepsilon_k^i$$

where $\varepsilon_k^i$ are drawn from the new scaled kernel above.

## 4. PARTICLE FILTERS FOR PROGNOSTICS IN INDUSTRY: COMPARISON CRITERIA

### 4.1. Particle filter based prognostics

Prognostic aims at predicting the future health of industrial items (components, units, subsystems) and generate their RUL, which is defined as the estimated time that the items can be expected to continue to serve their intended function, with taking into account the knowledge of the item, the historical and the current monitoring data, and future information related to mission profile, maintenance plan, etc (Sun, Zeng, Kang, & Pecht, 2012).

Particle filter based prognostic process can be divided into five mains steps (See Fig. 1). The process starts by constructing the item health state ($x$), which can be composed of one or several representative health indicators such as crack size of mechanical parts, motor temperature and vibration, battery state-of-charge, etc. The state transition function $x_k = f(x_{k-1}, v_{k-1})$ is then constructed based on the knowledge about the system function/dysfunction and available data. This step is very critical since the precision of state transition models has significant impacts on the prognostics performance (this point will be investigated later in the next section). The availability of the state transition function is also an important factor that limits the application of PF techniques in real applications. That is the reason why PF is usually applied to the problems such as crack propagation, battery state of charge in which the state transition function/degradation model is available.

In the third step, the PF techniques are then applied to estimate the probability distribution of the real state from $t = 1$ to the current time $t = k$ based on the historical data. It should be noted that in many cases the state transition function is designed in the parametric form which contains a set of unknown parameters. The PF techniques are then used not only for estimation of the item state, but also the unknown parameters. The simple solution to do this is to redefine the augmented state vector as $X_k = [x_k \quad para_k]$, where $para_k$ is the estimated values of the unknown parameters at time $t = k$. A set of initial values of unknown parameters are then randomly generate at the beginning according to initial probability distributions of unknown parameters ($p_0(para)$). The PF techniques are applied to estimate the probability distributions until time $t = k$ ($para_k$). At the end of this step, the set of particles representing the distributions of unknown parameters and industrial item state are obtained.

Based on the obtained particles at time $t = k$, the prediction is carried out. It can be done for the next K-steps ahead (K-steps ahead prediction) or until the instant at which the item state reaches its critical threshold (to determine the item RUL). In the prediction step, the particles propagate in the same way as in the estimation step, except that their weights are not updated. The weights of the particles are not updated under the assumption that process noise remains same throughout the long term prediction regime. Let $L$ denote a vector recorded the instants at which a particle reaches the critical threshold. The predicted mean value of RUL can be calculated as $\overline{RUL} = \frac{1}{N}\sum_{i=1}^{N}(L_i - k)$.

### 4.2. Comparison criteria

Three PF techniques are applied for the RUL prediction of a case study. Different scenarios are created to compare the three techniques by setting different accuracy levels of the state transition function/degradation model and the process noise level. In addition, the impacts of the length of prediction horizon and the calculation time are also investigated.

These three metrics have been chosen based upon authors experience . Apart from being intuitive, these metrics also present a reasonable way of evaluating the pragmatic (implementation) aspects of PF for their industrial utility. As the PF enabled approach is often model based, accuracy of the employed degradation model that directly affects the efficiency of the employed PF method, remains of paramount significance for determining the overall useful/accuracy of the prognostics. Thus, model accuracy is chosen as a metric for the comparative study. On the other hand, it is an established fact that accompanying noise levels (state and parameter) have correlation with state estimation accuracy, hence affecting the overall prognostic result; thus, making it a suitable choice as one of the metrics used in this paper. Finally, PF has often eluded industrial application mainly owing to its computational complexity, manifesting in form of computational time. Thus, computation time is chosen as the third and final evaluating metric for the comparative study.

**Computing time.** With the same prediction horizon, the computing time of the three techniques are also investigated. Due to additional algorithmic steps involved in APF as well as RPF as compared against SIR, additional computational time is expected for the former.

**Accuracy of the degradation model.** As we mentioned before, the accuracy of the degradation model is very important

and has significant impacts on the performance of PF techniques. However, model accuracy does not impact the three PF methods in exactly the same manner. If the degradation model is available in parametric form with unknown parameters. For SIR and APF techniques, only the weights of unknown parameters of the degradation model are changed over time. The initial values remain unchanged due to the fact that the samples are drawn from discrete distribution. The initialization of parameters values is then very important. Otherwise, by utilizing RPF, new values of parameters can be discovered over time, since the samples are drawn from continuous distribution. Therefore, from theoretical point of view, RPF is more suitable for the estimation of unknown parameters than the SIR and APF. In this paper, we also investigate the cases in which the degradation model is unknown completely in advance. Some basic and most often employed degradation models such as linear, polynomial and exponential models are then used to test the performance of the three PF techniques.

**Noise level.** The second important criterion is the level of noises. If it is high, then a point estimate such as mean $\widehat{m}_k^i$ does not well represent the transition probability $p(\mathbf{x}_k|\mathbf{x}_{k-1}^i)$, thereby leading to a worse performance of APF technique when compare to that of SIR or RPF ones. However, in presence of small noises, the point estimate employed characterizes the transition probability well leading to better robustness to outliers and a better estimation/prediction performance than the SIR and RPF.

## 5. NUMERICAL EXAMPLES

### 5.1. Degradation model and simulated data.

Consider a mechanical component subjected to fatigue. The crack size is chosen as the unique indicator representing the component's health state (YANG, YUAN, QIU, ZHANG, & LING, 2012; Liu, Jia, He, & Sun, 2017). Its transition function can be developed based on the Paris model (Paris & Erdogan, 1963) as following

$$x_k = x_{k-1} + C_k \cdot (\Delta\sigma \cdot \sqrt{\pi_{k-1}})^{m_k} \cdot \Delta N \qquad (22)$$

where, $C_k$ and $m_k$ are unknown parameters of the degradation model; $\Delta\sigma = 78$ (MPa) is the stress range; and $\Delta N = 1$ is the number of cycles. The initial crack size at time $t = 0$ is $x_0 = 0.01$ (m). The component is considered to be failed if the crack size reaches its threshold which is fixed at $x_{max} = 0.0463$ (m).

Given that the crack usually occurs inside the mechanical component, the direct measure of the crack size is normally impossible. The indirect inspections are then applied. In this paper, we consider that the observations follow a lognormal distribution with mean value $x_k$, and standard deviation

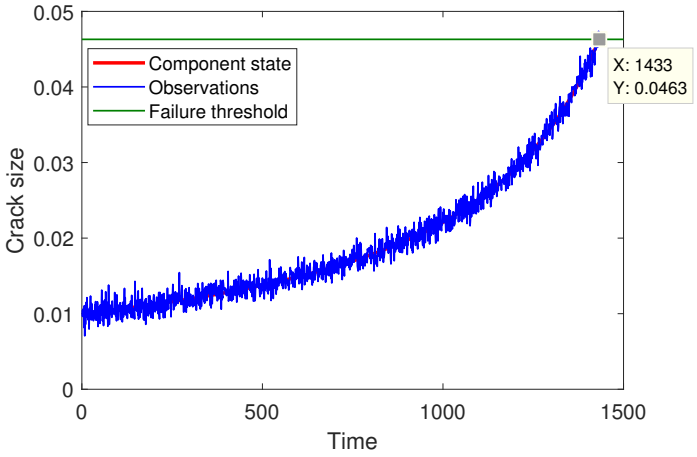


Figure 2. Simulated data from 0 to 1433

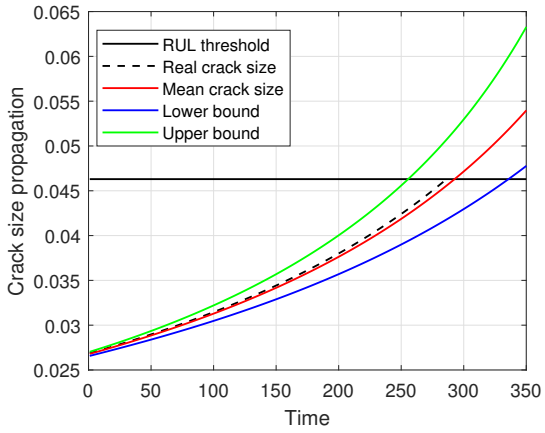


Figure 3. Crack size prediction

$\omega = 0.001$.

$$f_{y_k}(x) = \frac{1}{x} \cdot \frac{1}{p_2\sqrt{2\pi}} \exp\Big[-\frac{(\ln x - p_1)^2}{2p_2^2}\Big] \qquad (23)$$

where,

$$p_1 = \ln\Big[\frac{x_k}{\sqrt{1+\big(\frac{\omega}{x_k}\big)^2}}\Big] \text{ and } p_2 = \ln\Big[1+\big(\frac{\omega}{x_k}\big)^2\Big] \qquad (24)$$

To compare the three PF techniques, the data are generated from the above degradation and observation functions. The unknown parameters of the degradation model (state transition function) are set as $m_{true} = 4.0$ and $\ln(C_{true}) = -22.6204$. The simulated data from $k = 0$ to 1433 is presented in Fig. 2 Three PF techniques are then applied to estimate the unknown parameters and predict the component's RUL. The initial values of the unknown parameters are generated as $x_0 \sim \mathcal{N}(0.01, 0.005^2)$; $m_0 \sim \mathcal{N}(4, 0.5^2)$; and $C_0 \sim \mathcal{N}(-22, 2.00^2)$.

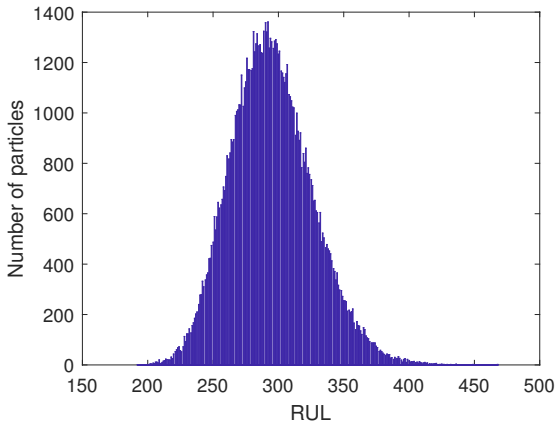


Figure 4. RUL prediction based on SIR at time $k = 1146$

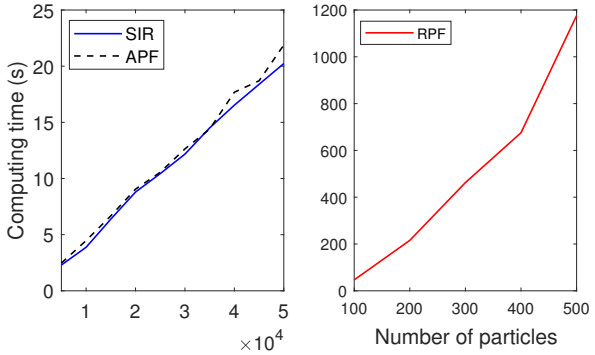


Figure 5. Computing time with different numbers of particles

Figures 3 and 4 present the predicted values of the component's crack size and component's RUL at time $t = 1146$ ($80\%$ component life-cycle). The real value of the component's RUL at the time $t$ is equals to 287. The mean predicted value of RUL is 293. The error is around $2.1\%$. The lower bound and upper bound are respectively 256 and 335.

### 5.2. Comparison between SIR and APF and RPF

**Computing time.** In this study, the training (parameters estimation) is done from $k = 1$ to 430 ($30\%$ component life-cycle). The three PF techniques are implemented with different number of particles. Fig. 5 shows that the computing times of SIR and APF are quite the same and linear. Otherwise, the computing time of RPF is very high when compared to that of SIR and APF. Indeed, the computing times of SIR and APF are only 2.29 and 2.47 seconds when the number of particles is 5000, otherwise RPF takes around 46.75 seconds for only 100 particles. In addition, the computing time of RPF is growing exponentially over the number of particles. With a high computing time, the RPF faces many challenges in the applications where the computing resources are limited or the online updating is required, e.g., cyberphysical systems.

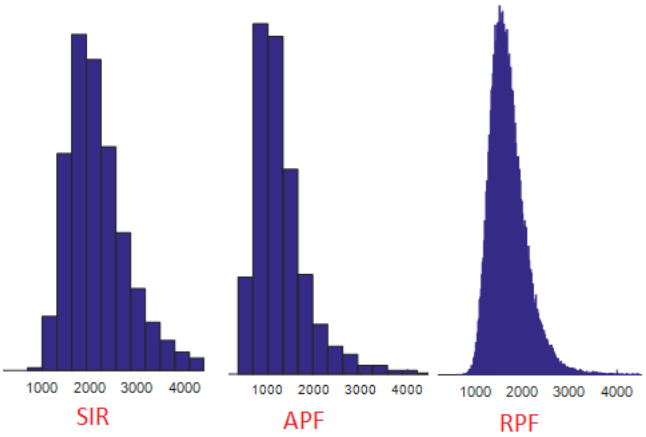


Figure 6. RUL histogram with the exponential model

Table 1. Predicted RUL by using the exponential model

| PF techniques | Mean predicted RUL | Error (%) |
|---|---|---|
| ISR | 2062 | 105.58% |
| APF | 1640 | 63.51% |
| RPF | 1137 | 13.35% |

**Accuracy of the degradation model.** In this study, we consider that the state transition function is not available. To predict the component's RUL, approximate models containing linear, polynomial and exponential models, are constructed from the given data by using of different regression methods. Finally, exponential model is selected since it fits the crack data in the best manner with least regression error. The setting of the exponential model as following

$$x_k = x_{k-1} + a \cdot e^{b \cdot k} \cdot \left[(1 - \frac{1}{e^b})\right] \tag{25}$$

where, $a \sim \mathcal{U}(0.0096, 0.0100)$ and $b \sim \mathcal{U}(0.00059, 0.00073)$.

The three PF techniques are then applied to predict the component's RUL at time $t = 430$. The obtained results are shown in Figure 6 and Table 1. From the obtained results, we can observe that the performance of PF techniques is decreased significantly as the degradation model is not an exact one. In addition, RPF is the best among the three techniques since it is suitable for the problems where the parameters are unknown. This in turn is largely due to the fact that constituting particles are "jittered" or shaken up at each step which enables them to estimate the true values. It also should be noted that given the stable performance, the RPF takes a lot of computing resources.

**Process noise level.** In this study, the noise level in the observation function is increased by setting $\omega = 0.01$. Otherwise, the true degradation model is used instead of the exponential one. The three PF techniques are then applied to predict the component's RUL. The obtained results are reported

Table 2. Predicted RUL vs a high level of the measure noise

| PF techniques | Mean predicted RUL | Error (%) |
|---|---|---|
| ISR | 916 | 8.67% |
| APF | 1434 | 42.97% |
| RPF | 1047 | 4.38% |

in Table 2. The table shown that, if the observation and state transition functions are well defined, the noise level has only slight impacts on the performance of SIR and RPF. In addition, by resampling from the continuous distribution instead of the discrete one, the performance of RPF is always better than that of SIR as resampling from continuous distributions leads to a 'smooth' and hence better approximation of posterior distribution. Moreover, the performance of APF is weak in the case of high noise level. It is also worse than that of SIR.

## 6. CONCLUSIONS

In this study, we focus on the comparisons of three PF techniques (SIR, APF, RPF) for RUL prediction (prognostic) of industrial components/systems. The comparisons are done by considering different criteria consisting of computational time, degradation model accuracy and the noise level. The obtained results shown that the RPF is the most powerful technique. However, the application of RPF faces challenges related to the computing resources and the online updating. It is then reasonable to use the RPF in the cases where the online updating is not necessary or the computing resources are available. The performance of SIR and APF are worse than that of RPF. However, they do not require a lot of computing time, and therefore can be applied in cases of the online updating or the limited computing resources. Finally, it should be noted that the performance of APF is very sensitive to the noise level, and the APF is then recommended for the cases where the noise level is small. To complete the study, in the future work, more comparison criteria and real industrial applications will be considered.

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