Use of the Teager Kaiser Energy Operator to estimate machine speed

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ABSTRACT

For diagnostics of variable speed machines such as wind turbines, it is essential to have a measure of instantaneous speed, preferably extracted from the response signal to avoid the need for a tacho signal. The Teager Kaiser Energy Operator (TKEO) was first proposed by Teager, and refined by Kaiser. It is analogous to the total (kinetic and potential) energy of a simple oscillator. A discrete version of the TKEO can be estimated from three adjacent samples of a signal, so it is very efficient to calculate from a sampled time record in real time. It can be used to obtain estimates of amplitude and frequency modulation. A previous paper showed it can also be calculated simply as the squared envelope of the derivative of a signal, using Hilbert transform techniques via the frequency domain. The differentiation can be very efficiently performed by jω operations in the frequency spectrum, at the same time as the bandpass filtration, which can be achieved with ideal, zero phase shift filters, using FFT techniques. It cannot then be done in real-time, but this is rarely a problem in machine condition monitoring where information is typically being sought days, weeks or months in advance. The TKEO equals the product of the squared envelope of the signal and the square of the instantaneous frequency, so the latter can be obtained by dividing the squared envelope of the derivative by the squared envelope of the signal. In this paper it is shown that this can be applied to the determination of the instantaneous speed of a machine, as long as a harmonic of one of the shaft speeds is isolated in the frequency domain from interference by adjacent components. This approach is applied to a couple of practical cases, and in particular compared with the results from another more complicated approach on a wind turbine signal, based on phase demodulation of the same carrier, followed by phase unwrapping and differentiation. Virtually the same results were obtained. In both cases the differentiation gave some high frequency noise, but this could easily be smoothed using various techniques.

1. BACKGROUND

The Teager Kaiser energy operator was first proposed by Teager as a means of tracking the "energy" in speech signals, and then formalized by Kaiser (1990), who attributed it to Teager in (Kaiser, 1990) as a "private communication". It was said to combine the total energy, both kinetic and potential, in an oscillating signal, where energy is continuously transformed between the two types. Kaiser (1990) draws an analogy with the energy in a vibrating mass/spring system, where the kinetic energy is proportional to the square of the velocity of the mass, and the potential energy is proportional to the square of the extension of the spring, in other words the displacement of the mass if the other end is fixed. In any such oscillating system, the two parameters, in this case velocity and displacement, are in quadrature, and in fact Hilbert transforms of each other when scaled to be the square root of the energy. With no gain or loss of energy to/from the system, the sum of squares of the instantaneous values of these components is constant, and in fact equal to the squared envelope of either. With slow input or decay of energy to/from the system the total energy will vary accordingly, as will the (identical) squared envelope of the two quadrature components.

2. FORMULATIONS AND EQUATIONS

The TKEO is defined in both continuous and discretised forms, as given in Equations (1) and (2), respectively.

$$\Psi_c\left(x(t)\right) = \left[\dot{x}(t)\right]^2 - x(t)\ddot{x}(t) \tag{1}$$

$$\Psi_d\left(x(n)\right) = \left[x(n)\right]^2 - x(n+1)x(n-1) \tag{2}$$

Kaiser (1990) showed that Eq. (2) gives an error < 11% if the frequency of oscillation < $f_s/8$, where f_s is the sampling frequency. He also showed that for the example of the mass/spring oscillator, whose displacement is given by

$$x(t) = A(t)\sin\omega(t)\,t \tag{3}$$

if the rate of variation of the amplitude $A(t)$ and frequency $\omega(t)$ is slow, then:

$$\dot{x}(t) \approx \omega(t)A(t)\cos\omega t \tag{4}$$

and $$\ddot{x}(t) \approx -[\omega(t)]^2 A(t)\sin\omega t \quad \text{so that:} \tag{5}$$

$$\begin{aligned}\Psi_c[x(t)] &= [\dot{x}(t)]^2 - x(t)\ddot{x}(t) \\ &\approx [\omega(t)]^2[A(t)]^2(\cos^2\omega t + \sin^2\omega t) \\ &= [\omega(t)]^2[A(t)]^2\end{aligned} \tag{6}$$

Since the Hilbert transform of $\dot{x}(t)$ is $\omega(t)A(t)\sin\omega t$, the squared envelope of $\dot{x}(t)$ is equal to the sum of squares of the velocity and its Hilbert transform (the parameter whose square gives the instantaneous PE), which is seen to be the same as the TKEO.

Thus, if $\mathrm{Envsq}(x(t))$ is the squared envelope of $x(t)$, ie

$$\mathrm{Envsq}(x(t)) = x^2(t) + \hat{x}^2(t) \tag{7}$$

where $\hat{x}(t)$ is the Hilbert transform (HT) of $x(t)$, then

$$\Psi_c(x(t)) = \mathrm{Envsq}(\dot{x}(t)) \tag{8}$$

and since $$\begin{aligned}\Psi_c(x(t)) &= [\omega(t)]^2[A(t)]^2 \\ &= [\omega(t)]^2\,\mathrm{Envsq}(x(t))\end{aligned} \tag{9}$$

then $$[\omega(t)]^2 = \frac{\mathrm{Envsq}(\dot{x}(t))}{\mathrm{Envsq}(x(t))} \tag{10}$$

This is a new result, first published in Randall (2016), and provides an alternative method for performing amplitude and frequency demodulation using the TKEO.

In Maragos, Kaiser and Quatieri (1993), a set of equations were derived to relate amplitude modulation (AM) and frequency modulation (FM) signals to the TKEO as well as quantifying the errors involved for the continuous and discretised versions. They were based on the TKEO of the original signal and its derivative, as follows:

$$\omega(t) \approx \sqrt{\frac{\Psi[\dot{x}(t)]}{\Psi[x(t)]}} \tag{11}$$

$$A(t) \approx \frac{\Psi[x(t)]}{\sqrt{\Psi[\dot{x}(t)]}} \tag{12}$$

There is a subtle difference in principle between eqs. (10) and (11), but the primary difference in practice is that Eqs. (11) and (12) are intended to be evaluated in the time domain, including taking the derivative of the TKEO. Maragos, Kaiser, et al. (1993) explain that for accuracy it is necessary to use a higher order difference equation for the differentiation rather than a simple forward or backward difference.

Eqs. (7, 8, 10) are very conveniently evaluated using HT procedures via the frequency domain. In particular because the TKEO is supposed to be based on a mono-component carrier, this can be isolated in the frequency domain by a zero phase shift ideal filter, simply by retaining only the corresponding lines in the FFT spectrum. The generation of an analytic signal, whose real and imaginary parts are related by a Hilbert transform, occurs simply by inverse Fourier transformation of such a (1-sided) frequency band, and exact zero phase shift differentiation can be carried out at the same time by multiplying the (complex) spectral values in the band by $j\omega$. The noise problems that otherwise can be associated with numerical differentiation are minimised by the frequency range limitation of the band.

In other words the $\mathrm{Envsq}(x(t))$ of Eq. (10) is produced by inverse transforming the selected band directly, and taking the squared amplitude of the result, while the $\mathrm{Envsq}(\dot{x}(t))$ is obtained the same way after multiplying the selected band by $j\omega$.

The only disadvantage of the HT approach via the frequency domain is due to the wraparound effects of the FFT processing. All time and frequency functions in the FFT are considered to be one period of a periodic function, so the start and finish have to join into a loop. This does make a number of samples at each end invalid, but it is generally not a problem to transform a slightly longer record and discard the ends

The purpose of this paper is to show how this can be used to estimate the instantaneous rotational speed of a machine, provided a low harmonic of the shaft (preferably the lowest), or other speed related component, is not masked by overlapping extraneous components. This is virtually assured if a tacho signal is available, but most often a low harmonic of the machine vibration can be used as well, as in the following examples.

3. APPLICATIONS

3.1. Single stage gearbox

The first example is based on data from a gearbox test rig with two parallel shafts, driven by a 4-pole induction motor with variable frequency drive (VFD). A photograph of the rig is reproduced in Fig. 1. The torque load of the pump varies with the speed, and it was found desirable to arrange the 25-tooth and 46-tooth gears as a speedup ratio to maximise the load. A 2-per-rev tacho signal was obtained from the higher speed output shaft, and this could be used to check the speed estimates presented here from the lower speed shaft.

A series of vibration signals were taken from the accelerometer mounted on the gearbox casing. Each record started with a 30s section at constant speed, followed by a longer section where the speed was varied manually using the control buttons for the VFD. Four recordings were made for speed variations around nominal 22 Hz (Signals 1-1 to 1-4), and two for variations around 15 Hz (Signals 2-1 and 2-2). The variable speed part varied by about ±15% around the nominal speed.

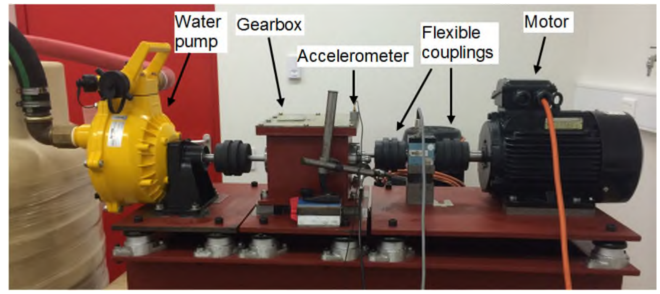

Figure 1. Layout of gearbox test rig

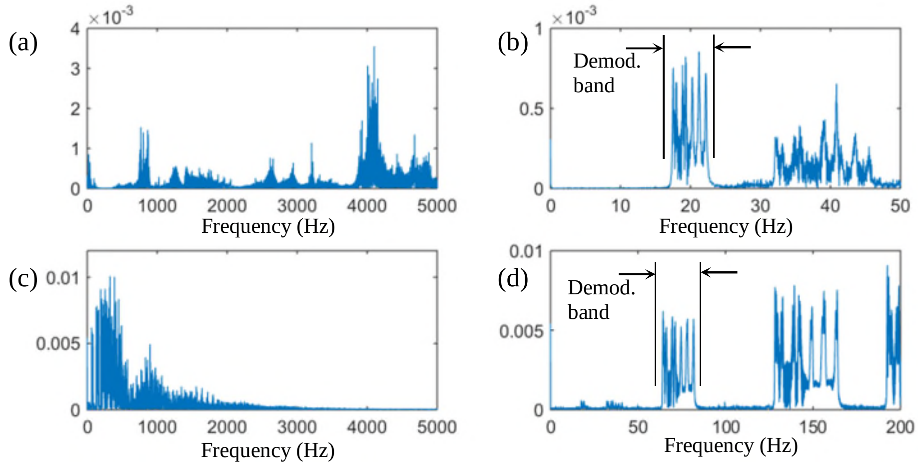

Figure 2. Spectra of vibration response and tacho signals, analysed for instantaneous speed estimation (a) Acceleration spectrum 0-5 kHz (b) Acceleration 0-50 Hz (c) Tacho 0-5kHz (d) Tacho 0-200 Hz

Figure 2 shows spectra for the variable speed part of the typical signal 1-1 from 5s after the start of speed variation to just before the end. Fig. 2(a) and (b) show the acceleration spectrum, and Fig. 2(c) and (d) show the spectrum of the tacho signal from the output shaft, with frequency 2×46 / 25 times the input shaft speed. Even though the overall spectra are quite different, the lowest harmonic of each can be seen in the zoomed spectra of Fig. 2 (b, d), with the input shaft speed (ISS) in 2(b) centred on about 20 Hz, and the second harmonic of the output shaft speed (OSS) in 2(d) centred on about 75 Hz. Each of these lowest harmonics is completely separated from adjacent components and noise, allowing for the choice of an uncontaminated demodulation band as indicated in the figure. As might be expected, the second harmonic of the tacho signal (4^{th} harmonic of OSS) in Fig. 2(d) is twice as broad as the first, but the second harmonic of the ISS in Fig. 2(b) is already overlapping with the first harmonic of OSS, and therefore not separable.

Both the indicated demodulation bands were frequency demodulated using Eq. (10) and the results (scaled for the tacho) are shown in Figure 3. Both curve estimates were smoothed as discussed below, but each had about the same amount of noise before smoothing. Figure 3 is mainly to show that equivalent results can be obtained from both response signal and tacho, and all later results are from the vibration response signals.

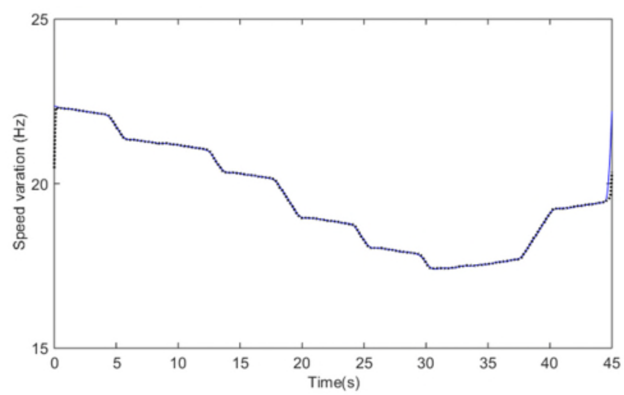

Figure 3. Comparison of speed estimates from acceleration (blue, solid) and tacho (black, dotted) adjusted for ratio

A slight difference can be seen between the two curves in Fig. 3, and that is that the end effects from wraparound error are slightly bigger for the response signal than the tacho, but this is purely because the demodulated band around 20 Hz, has almost four times fewer lines than that around 75 Hz for the tacho, and the number of time samples in the record is proportional to the demodulated band. The wraparound error is about the same in terms of number of samples. The difference between the two estimates in Fig. 3 is very small. Compared with a mean value of 19.5 Hz, the maximum absolute difference between the two curves (eliminating the section at each end with wraparound error) is 0.023 Hz (0.12%) and the standard deviation is 0.0064 Hz (0.03%).

As mentioned above, the direct application of Eq. (10) gives some additive noise in the result, which can be attributed to the differentiation, but it is usually simple to smooth it by some sort of low-pass filtration. As mentioned below for another example, two methods that were previously used for the smoothing were band reduction in the FFT spectrum, and polynomial curve fitting, but it was realised that the best method in this situation is zero phase shift FIR filtering, as it does not suffer from the effects of the step between the two ends of the record (as does the frequency domain method) and it can follow more rapid changes than the polynomial method (which goes unstable for too high order).

Figure 4 shows the results of filtering with a rectangular moving average filter, using the Matlab® function FILTFILT to avoid phase shift, as suggested in Brandt (2011). The smoothing filter in this case comprised 100 samples.

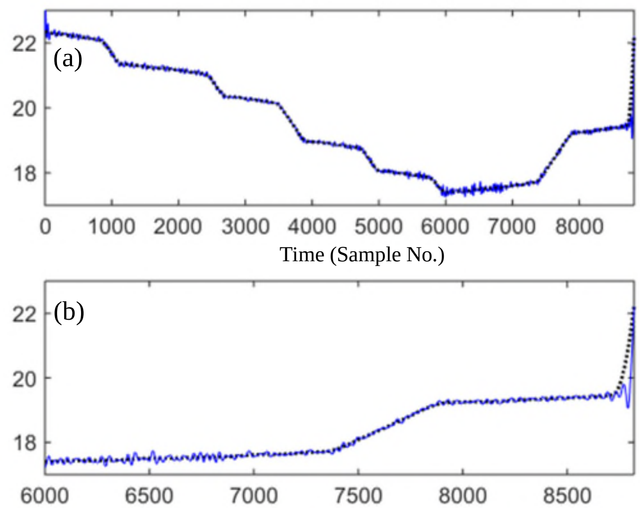

Figure 4. Original (blue, solid) vs smoothed (black, dotted) speed curves. (a) Full record (b) zoomed section showing end effects

The blue solid curve is the original noisy data, and the black dotted curve is the smoothed result. The smoothed curve is the same as the blue curve in Fig. 3, but the x-axis has been scaled in samples. The zoomed curve in Fig. 4(b) shows the noise in more detail, and also the end effects at the end of the record. It is seen that the end effect of the frequency domain HT calculations (blue solid curve) is about the same length as that due to the smoothing filter (black dotted curve) and is of the order of the filter length, viz. 100 samples.

Figure 5 shows the smoothed speed profiles of all six signals obtained by this process.

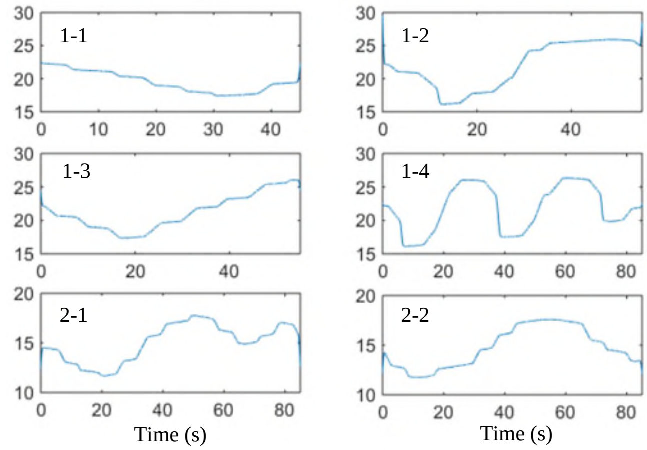

Figure 5. Smoothed speed profiles for the six signals

All signals were order tracked, phase demodulating the same demodulation band to provide a phase vs time map for the time to rotation angle conversion. In Randall and Smith (2016) they were used to demonstrate a new method for suppressing the effects of speed variation on gear diagnostics.

3.2. Wind turbine gearbox

In Randall, Coats and Smith (2015), an alternative method was published to extract the instantaneous speed of a machine from the vibration response signal. It was based on our group's submission to a contest run in conjunction with the CMMNO conference in Lyon, France, in December 2014. The contest involved making "the most relevant diagnosis of a wind turbine operating under non-stationary conditions" with particular emphasis on two points:

1) Diagnosis of a bearing fault on an unspecified shaft

2) Determination of the instantaneous speed of the input shaft over the length of the recording

Only part 2) is discussed here. All information had to be extracted from a single recording of an acceleration signal of length 550s, where the output shaft speed (generator) varied between about 20 and 30 Hz.

The sponsors of the contest were the French wind turbine manufacturer Maïa Eolis, who provided the signals for analysis, and the French company Oros, who provided the prize, a vibration analysis system.

The layout of the turbine gearbox was given by the contest organisers in the following Figure 6, where the input was to the planet carrier at the bottom left, and output to the generator at top right. A table was provided giving the numbers of teeth on all gears, so that the relative speeds of all shafts could be estimated.

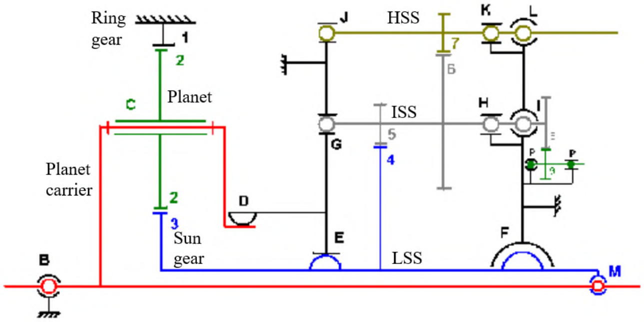

Figure 6. Schematic layout of wind turbine gearbox.
LSS = low speed shaft; ISS = intermediate speed shaft;
HSS = high speed shaft.
Gears have numbers, bearings have letters

Because the speed varied a little too much to choose a single harmonic of any shaft speed that was separated from adjacent components over the whole record length, the total record was divided into overlapping segments, in each of which the speed variation was less than about ±15%. Figure 7 shows a spectrogram of the signal over the whole length, indicating two potential carrier components to demodulate to obtain the speed information.

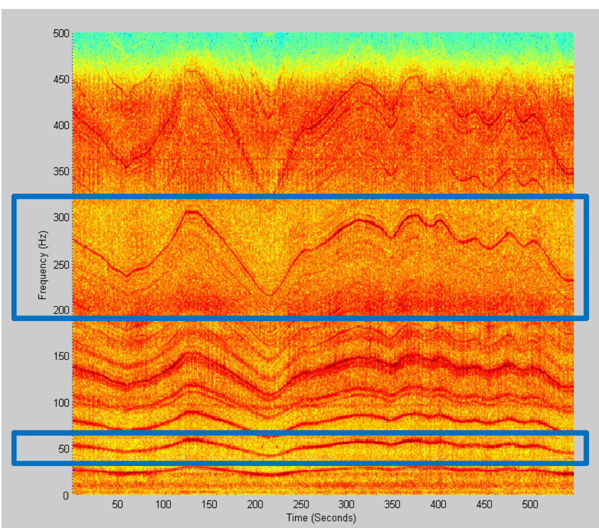

Figure 7. Spectrogram showing two separated components

The lower of these around 50 Hz was suspected to be the second harmonic of the planet gearmesh frequency (PGM), and the higher around 250 Hz was suspected to be the second harmonic of the intermediate gearmesh frequency (IGM). Detailed analysis after order tracking confirmed this.

The method used for the contest consisted in phase demodulating the selected band (this also being used for the order tracking) then differentiating the phase of each section to get angular velocity. This was because the initial phase of each section was somewhat arbitrary, but the frequency must be continuous at the junction points between windows. The demodulated phase is modulo 2π, so must be unwrapped to a continuous function of time. Figure 8 shows the estimated frequency for the eight segments into which

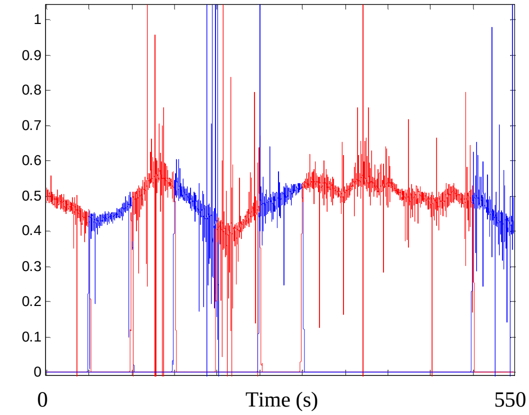

Figure 8. Initial frequency estimates for the eight segments (y-axis arbitrarily scaled)

the record was divided, most being 50s long with one long section from 300-500s. The overlapping sections were blended by cosine tapering over 2.5s on either side of each break point. It is seen that the differentiation introduced noise, which had to be smoothed. For the contest, two methods of smoothing were tried, the first being lowpass filtration in the frequency domain, and the other polynomial curvefitting. The frequency domain LP filtering had the disadvantage of end effects, which would reduce the valid length of record. The polynomial curve-fitting was chosen for the contest because it did not have these end effects, but was limited to rather smooth curves. The shorter sections could be fitted with 5th order polynomials, which was no problem, but the long section from 300-500s required a 20th order polynomial, which was on the border of becoming unstable. Figure 9 shows this section and the curve-fitted polynomial (in red).

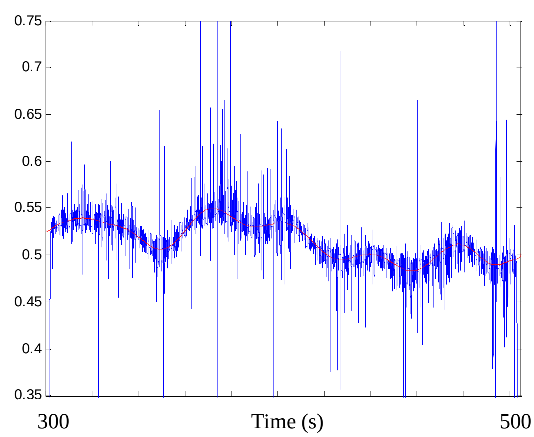

Figure 9. Long segment and curve-fitted polynomial

The polynomial sections were blended (by cosine tapering of the differences in the overlap regions) and used for the final result. This had to be converted (order tracked) to be in terms of rotation angle rather than time, and this was in fact done by integrating the instantaneous frequency to get the phase/time map for the conversion.

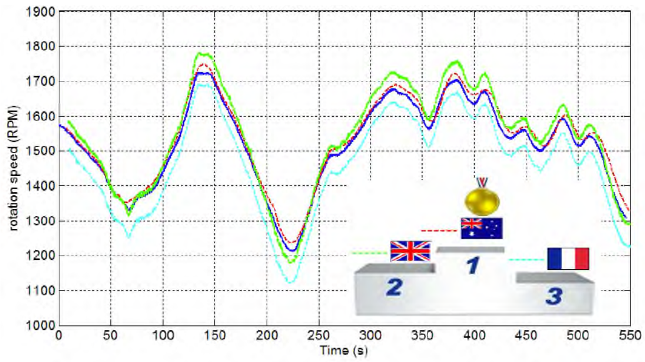

Figure 10. Comparison of the result from Randall, Coats et al. (2015) compared with the true result (blue) and the two nearest competitors.

Our entry won the contest, and in Figure 10 it is compared with the true result (and the two nearest competitors). This demonstrates that the basic method based on phase demodulation, differentiation, and smoothing is valid, but improvements could be made. We noticed that where our estimate differed most from the true result (eg near 140s and 380s) the LP filter gave a closer result than the polynomial.

Thus in applying the method of this paper to the same data, it was realized that a better method of smoothing would be to do it in the time domain with an FIR filter (to avoid the wraparound effects of the FFT) and this should at the same time give more adaptability to follow sudden changes in the curve than the polynomial method. This has already been demonstrated in Figs. 4 and 5. For the current paper the section from 250-500s was treated, and the spectrum of the second harmonic of the PGM is shown in Figure 11.

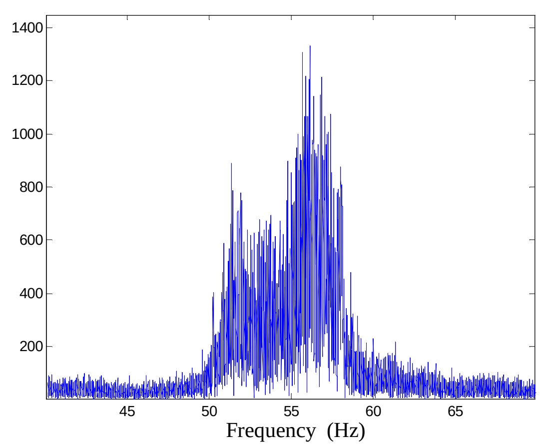

Figure 11. Spectrum of the band used for frequency demodulation by the TKEO method

It is seen that there is no significant overlap of other components, though the base noise level is perhaps a bit higher than in Fig. 2.

This band was frequency demodulated using Eq. (10), and the result is shown in Figure 12. The noise is seen to be very comparable with that in Fig. 9 using the other method.

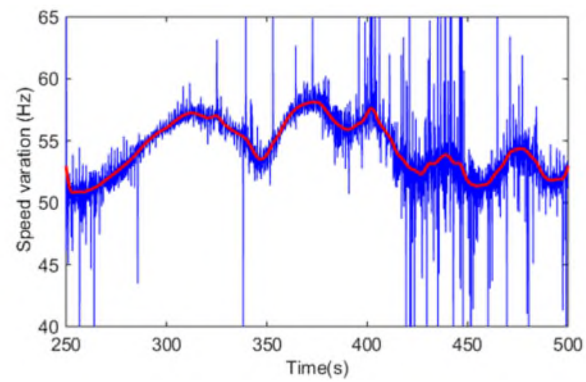

Figure 12. Speed estimate using TKEO method (blue) and smoothed version using FIR filter (red)

The result of FIR smoothing is seen in red, and can be compared with the polynomial result of Fig. 9. While quite similar, the new result has more detail, and in some small points is closer to the true result of Fig. 10 (eg, near 320s and 370s). In this case the total record length was 120,000 samples, and the rectangular smoothing window comprised 1600 samples.

4. CONCLUSION

In this paper a new method is developed for determining the instantaneous speed of a machine by frequency demodulating an isolated shaft order. It is based on the concepts of the TKEO, but evaluated using the Hilbert transform via the frequency domain. The squared instantaneous frequency is obtained by dividing the squared envelope of the derivative of the signal (the TKEO) by the squared envelope of the signal. The differentiation is simply achieved by multiplication by $j\omega$ in the frequency domain of the same section of spectrum used to obtain the squared envelope.

This is much simpler to carry out than an alternative method based on phase demodulation of the same band, followed by phase unwrapping and differentiation to angular velocity. The noise in the result is about the same for both methods, and can be smoothed by a number of methods. The best appears to be one introduced for the purpose in this paper; a zero phase shift FIR filtration whose smoothing window length can be chosen to suit the data.

The only disadvantage of the method is that it cannot be carried out in real-time, as can the traditional TKEO procedures used for speech analysis, but in general this is not a problem with machine diagnostics, and on the other hand there are considerable benefits associated with the zero phase shift processing and ideal filtration enabled by the frequency domain (FFT) Hilbert transform manipulations.

The method is validated using data from two different applications. One was a parallel shaft single stage gearbox, where identical results were obtained by analysis of an acceleration signal and a tachometer signal for each of a number of different speed profiles varying by up to ±15% around two different mean speeds. The other was a wind turbine gearbox, where the speed was extracted from the isolated second harmonic of the planetary gearmesh frequency, which also varied by about ±15%. It had previously been demonstrated that larger speed variations can be treated by dividing the signal up into overlapping sections in each of which the speed variation is limited, and then blending the individual sections together afterwards, so it was not thought necessary to repeat this.

The new method, including smoothing, is shown to be slightly better than the previous one, and is certainly easier to carry out.

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BIOGRAPHIES

Robert B. Randall has a Bachelor of Technology degree in Mechanical Engineering (1961) from the University of Adelaide, Australia, and a Bachelor of Arts degree in Pure Mathematics, Applied Mathematics and Swedish (1971) from the University of Melbourne, Australia. He is currently a visiting Emeritus Professor in the School of Mechanical and Manufacturing Engineering at the University of New South Wales (UNSW), Sydney, Australia, which he joined as a Senior Lecturer in 1988. Prior to that, he worked for the Danish company Brüel & Kjær for 17 years, after ten years' experience in the chemical and rubber industries in Australia, Canada and Sweden. He has published two books, numerous book chapters and more than 260 papers in the fields of vibration analysis and machine diagnostics, and has successfully supervised seventeen PhD projects in those areas.

Wade A. Smith has a Bachelor of Engineering (Mechanical) Diploma in Engineering Practice (2004), and a Doctor of Philosophy in Engineering (2009) from the University of Technology, Sydney, Australia. After completing his PhD in vehicle suspension and ride dynamics he worked for Australia's National Measurement Institute, before joining the University of New South Wales (Sydney, Australia) as a Research Associate in 2011. His research interests include vibration-based diagnostics of rotating machinery, and operational modal analysis. He is a member of the International Society for Condition Monitoring.