A Model-Based Prognostics Framework to Predict Fatigue Damage Evolution and Reliability in Composites

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ABSTRACT

In this work, a model-based prognostics methodology is proposed to predict the remaining useful life (RUL) of composite materials under fatigue loads. To this end, degradation phenomena such as stiffness reduction and increase in matrix micro-cracks density are predicted by connecting micro-scale and macro-scale damage models in a Bayesian filtering framework. The proposed Bayesian filtering framework also allows incorporating various uncertainties in the prediction that are generally associated with material defects, sensing and monitoring noise, modeling errors, etc., to name a few. This, however, results in a explosion of search space due to high dimensionality, and hence a high computational complexity not conducive for real-time monitoring and prediction. To reduce the dimensionality of the problem without significantly compromising on prediction performance (precision and accuracy), a model tuning is first carried out by means of a Global Sensitivity Analysis. This allows identifying and subsequently down selecting the parameters for online adaptation that affect prediction performance the most. Resulting RUL estimates are then used to compute a time-variant reliability index for composite materials under fatigue stress. The approach is demonstrated on data collected from run-to-failure tension-tension fatigue experiments measuring the evolution of fatigue damage in CRFP cross-ply laminates. Micro-cracks are considered as the primary internal damage mode that are estimated from measurements obtained by active interrogation using PZT sensors. Results are presented and discussed for the prediction of growth in micro-cracks density and loss of stiffness for a given panel along with the reliability index calculation for the damaged component.

1. INTRODUCTION

Composites are high-performance materials used extensively in the construction of engineering structures, with a wide range of applications such as aeronautical, marine and mechanical structures. Most of these applications involve components subject to cyclic loads, which make them susceptible to fatigue degradation. This degradation leads to a progressive decrease of the performance reliability of the material, and ultimately, to the catastrophic failure of the structure. The prediction forward in time of such fatigue degradation and the reliability of the composite structure is of a paramount importance for safety and cost reasons, however it is still a partially understood problem.

In contrast to metals, fatigue damage in composites is governed by complex multi-scale processes driven by internal fracture mechanisms that ultimately lead to the alteration of the macro-scale mechanical properties (Reifsnider & Talug, 1980; Jamison, Schulte, Reifsnider, & Stinchcomb, 1984). The inherent complexity of this process implies uncertainty, that comes not only from the variability of loading conditions and material heterogeneity, but also from the incomplete knowledge of the underlying damage process. This uncertainty can increase dramatically when dealing with full-scale structures in real environments. Nevertheless, real time measurements of the structural performance are now available through state-of-art Structural Health Monitoring (SHM) techniques, and a large variety and amount of response data can be readily acquired, processed and further analyzed to assess various health-related properties of structures. Thus a SHM-based prognostic approach is best suited to deal with this uncertainty, and furthermore to accurately predict the service life and the time-varying reliability of the composite structure.

In the last few years, the topic of fatigue damage prognostics is slowly gaining interest. There is an increasing number of articles dealing with probability-based approaches for fatigue damage prognostics (Myötyri, Pulkkinen, & Simola, 2006; Cadini, Zio, & Avram, 2009; Guan, Jha, & Liu, 2011; Zio & Di Maio, 2012; An, Choi, & Kim, 2013; Gobbato, Kosmatka, & Conte, 2014), most of them in the context of metals. However the number of contributions for composites materials is still very limited (J. Chiachío, Chiachío, Saxena, Rus, & Goebel, 2013), precisely where the benefits of the probabilistic SHM-based prognostic approach can be fully exploited to deal with the variability and complexity of the fatigue damage accumulation process.

Damage prognostics is concerned with determining the health state of system components and predicting their RUL based on predefined thresholds, given an evolutionary damage model. As with diagnostics, prognostics methods are typically categorized as either model-based or data-driven, depending on whether the damage model is based on physical first principles, or, alternatively uses damage data to capture trends of degradation. Model-based approaches provide RUL estimates that are more accurate than data-driven approaches, when suitable models are available (M. Daigle & Goebel, 2010). Specifically, model-based approaches have the ability to adapt to different systems (specimen, materials, conditions, etc.) without much training, and furthermore, they can incorporate monitoring data in a SHM context.

This paper integrates a model-based damage prognostics problem with reliability theory in application to fatigue in composite materials, which distinguishes from the recent paper presented by the authors at PHM2013 (2013 Annual Conference of the Prognostics and Management Society) (J. Chiachío et al., 2013). In that article, a model-based prognostics framework was proposed to sequentially estimate the health state as well as the parameters of the underlying damage model, based on available SHM data. From this estimation, the RUL of the estructure was computed. A Sequential Importance Resampling algorithm (Arumlampalam, Maskell, Gordon, & Clapp, 2002) was used for the joint state-parameter sequential estimation, and an artificial dynamics approach (Liu & West, 2001; M. J. Daigle & Goebel, 2013) was adopted to improve the predictability of the algorithm.

The new contributions of this research work with respect to (J. Chiachío et al., 2013) are (i) the consideration of two different-scale damage signatures to represent the health state of the system: matrix-cracks density and longitudinal stiffness reduction, and (ii) the prediction of the time-varying reliability of the structure, as a unified health indicator of the system.

As a case study, SHM data from a tension-tension fatigue experiment in a cross-ply CFRP laminate is used. Damage data used in this example are taken from the Composite dataset, NASA Ames Prognostics Data Repository (Saxena, Goebel, Larrosa, & Chang, 2008), corresponding to laminate L1S19. More details about these tests are reported in (Saxena et al., 2011). Results shows the suitability and accuracy of the proposed approach.

The rest of the paper is organized as follows. Section 2 discusses the theory behind fatigue damage in composites and presents the proposed methodology for fatigue damage modeling. The sequential state estimation problem by means of particle filters is presented in Section 3. Section 4 formally defines the prognostics problem and describes the methodology to compute the time-varying reliability. Section 5 presents the demonstration of the approach on real data of fatigue considering a cross-ply CFRP laminate. Finally, some concluding remarks are presented in Section 6.

2. FATIGUE DAMAGE MODELING

The progression of fatigue damage in composites involves a progressive or sudden change of the macro-scale mechanical properties, such as stiffness or strength, as a consequence of different fracture modes that evolve at the micro-scale along the lifespan of the structure (Jamison et al., 1984). In this work the longitudinal stiffness loss is chosen as the macro-scale damage variable, given that, in contrast to the strength, it can be measured through non-destructive methods during operation. This is of key importance for the filtering-based prognostics approach proposed. At the micro-scale level, matrix micro-cracking (J. A. Nairn, 2000) is selected as the dominant fracture mode for the early stage of damage accumulation. Matrix cracks usually initiate from internal defects in 90° plies during first loading cycles, and grow rapidly along fibers direction spanning the entire width of the specimen (J. A. Nairn, 2000). Continued loading leads to formation of new cracks between the already formed cracks thereby progressively increasing the matrix-crack density of the ply until saturation. This saturated state, usually termed as *characteristic damage state* (CDS) (Reifsnider & Talug, 1980), is long recognized as a precursor of more severe fracture modes in adjacent plies, such as delamination and fiber breakage (Lee, Allen, & Harris, 1989; Beaumont, Dimant, & Shercliff, 2006), which may subsequently lead to the catastrophic failure of the laminate. In addition, matrix micro-cracking may itself constitute failure of the design when micro-crack induced degradation in properties exceeds the predefined threshold.

To accurately represent the relation between the internal damage and its manifestation through macro-scale properties, several families of *damage mechanics* models have been proposed in the literature (Talreja & Singh, 2012). These models, that are based on first principles of admissible ply stress fields in presence of damage, can be roughly classified into 1) computational methods, 2) semi-analytical methods and 3) analytical methods. Among them, computational and semi-analytical methods have been shown to be promising, however they are computationally prohibitive in a filtering based prognostics approach, where a large number of model evaluations is required. Therefore, we focus here on the set of analytical models, that depending on the level of assumptions, they can be classified into *shear-lag* models (Garrett & Bailey, 1977; Highsmith & Reifsnider, 1982), *variational* models (Hashin, 1985), and *crack opening displacement* based models (Gudmundson & Weilin, 1993; Lundmark & Varna, 2005).

Shear-lag models use one-dimensional approximations of the equilibrium stress field after cracking to derive expressions for stiffness properties of the cracked laminate. Their main assumption is basically that, in the position of matrix cracks, axial load is transferred to uncracked plies by the axial shear stresses at the interfaces. These models have received the most attention in the literature and, as a consequence, a vast number of modifications and extensions can be found. However, as stated by Talreja and Singh (Talreja & Singh, 2012), all the one-dimensional shear-lag models are virtually identical, except for the choice of the *shear-lag parameter*, as explained later in this section. Variational models are based on a two-dimensional approximation of the equilibrium stress field, that in contrast to shear-lag analysis, is obtained from the Principle of Minimum Complementary Energy (Reddy, 2002; Dym & Shames, 2013). Finally, COD-based models use a 3-D homogenization procedure derived from the study of the average crack-face opening displacement of a single matrix crack as a function of the applied load, that can be calculated either analytically (Gudmundson & Weilin, 1993) or numerically (Varna, Akshantala, & Talreja, 1999; Joffe, Krasnikovs, & Varna, 2001; Lundmark & Varna, 2005). The reader is referred to the recent work of Talreja and Singh (Talreja & Singh, 2012) for a detailed overview of these models.

Variational and COD models are expected to better capture the various complex damage mechanisms, since they involve a more complex damage mechanics analysis, but it might be at expense of more information extracted from the data (J. Chiachío et al., 2014). Then, if such models are utilized for future prediction, as arises in prognostics, the results are expected to significantly depend on the details of the available data. In contrast, the most simple shear-lag model provide reasonable accuracy results while it extracts less information from data. To this end, it is expected to be less sensitive to the noise on data. It is an example of the principle of Ockham's razor in the context of fatigue of materials, that has been shown to hold true for composites materials by a recent study (J. Chiachío et al., 2014).

2.1. Stiffness reduction model

Following the unifying formulation of (Joffe & Varna, 1999), the effective longitudinal Young's modulus E_x^* can be calculated in $\left[\phi_{\frac{n_\phi}{2}}/90_{n_{90}}/\phi_{\frac{n_\phi}{2}}\right]$ laminates (where $\phi \in [-90°, 90°]$) as a function of the crack-spacing in 90° layers for both, shear-lag and variational models, as follows:

$$E_x^* = \frac{E_{x,0}}{1 + a\frac{1}{2\bar{l}}R(\bar{l})} \tag{1}$$

In the last equation, $E_{x,0}$ is the longitudinal Young's modulus of the undamaged laminate, $\bar{l} = \frac{l}{t_{90}}$ is the half crack-spacing normalized with the 90° sub-laminate thickness, $R(\bar{l})$ is the average stress perturbation function, and a is a function of ply and laminate properties, defined as follows:

$$a = \frac{E_2 t_{90}}{E_1 t_\phi}\left(1 - \nu_{xy}^{(\phi)} \frac{\frac{\nu_{xy}^{(\phi)} t_{90}}{E_y^{(\phi)}} + \frac{\nu_{12} t_\phi}{E_2}}{\frac{t_{90}}{E_y^{(\phi)}} + \frac{t_\phi}{E_1}}\right)\frac{1 - \nu_{12}\nu_{xy}^{(\phi)}}{1 - \nu_{12}^2\frac{E_2}{E_1}} \tag{2}$$

In the last equation, the superscript (ϕ) denotes: "property referred to the $\left[\phi_{\frac{n_\phi}{2}}\right]$-sublaminates". The reader is referred to the Nomenclature section for a description of the ply and laminate properties used in the calculations.

It should be noted that the matrix-cracks density is usually termed as $\rho = \frac{1}{2l}$, so that the normalized half crack-spacing $\bar{l}$ can be expressed as a function of ρ as $\bar{l} = \frac{1}{2\rho t_{90}}$. For shear-lag models, the function $R(\bar{l})$ takes the next expression (Joffe & Varna, 1999):

$$R(\bar{l}) = \frac{2}{\xi}\tanh(\xi\bar{l}) \tag{3}$$

where ξ is the aforementioned shear-lag parameter. Depending on the choice of ξ, different shear-lag models, that have been proposed in the literature, can be obtained. See (Talreja & Singh, 2012) for further discussion about shear-lag analysis. In this paper, the "classical" shear-lag model (Garrett & Bailey, 1977; Manders, Chou, Jones, & Rock, 1983) is adopted. For this model, ξ takes the following expression:

$$\xi = \sqrt{G_{23}\left(\frac{1}{E_2} + \frac{t_{90}}{t_\phi E_x^{(\phi)}}\right)} \tag{4}$$

2.2. Damage propagation model

Having identified the model to express the relationship between the effective Young's modulus and micro-cracks density, the next step is to address the time evolution of the micro-cracks density. To this end, the previously explained shear-lag model is used to obtain the energy released per unit crack area due to the formation of a new crack between two existing cracks, denoted here as G. This energy, known as energy release rate (ERR), can be calculated as (J. A. Nairn, 1989):

$$G = \frac{\sigma_x^2 h}{2\rho t_{90}} \left(\underbrace{\frac{1}{E_x^*(2\rho)}}_{\text{Eq. 1}} - \underbrace{\frac{1}{E_x^*(\rho)}}_{\text{Eq. 1}} \right) \tag{5}$$

where σ_x is the applied axial tension, and h and t_{90} are the laminate and 90° sublaminate half-thickness, respectively. The energy released calculated by Eq. (5) is further introduced into the modified Paris' law (J. Nairn & Hu, 1992) to obtain the evolution of matrix-cracks density as a function of fatigue cycle n, as shown below:

$$\frac{d\rho}{dn} = A(\Delta G)^\alpha \tag{6}$$

where A and α are fitting parameters, and ΔG is the increment in ERR for a specific stress amplitude, i.e., $\Delta G = G(\sigma_{x,max}) - G(\sigma_{x,min})$. Due to the complexity of the expression for ΔG, which involves the underlying micro-damage mechanics model for the computation of $E_x^*(\rho)$, a closed-form solution for Eq. (6) is hard to obtain. To overcome this drawback, the resulting differential equation can be solved by approximating the derivative using "unit-time" finite differences, considering that damage evolves cycle-to-cycle as:

$$\rho_n = \rho_{n-1} + A\left(\Delta G(\rho_{n-1})\right)^\alpha \tag{7}$$

To summarize, a shear-lag damage-mechanics model is selected to compute $E_x^*(\rho)$, i.e. the relationship between the effective longitudinal Young's modulus (macro-scale) and the matrix-cracks density (micro-scale). The evolution of matrix-cracks density is modeled using the modified Paris' law in Eq. (7), that incorporates the damage mechanics model to evaluate the increment in ERR.

3. FILTERING-BASED STATE-PARAMETER ESTIMATION

3.1. Stochastic embedding

For the purpose of filtering and prognostics, a probability-based description of the deterministic models described in Section 2 is needed. To this end, let consider a generic model defined by a deterministic relationship $\mathbf{g} = \mathbf{g}(\mathbf{u}, \boldsymbol{\theta}) : \mathbb{R}^{N_i} \times \mathbb{R}^{N_d} \to \mathbb{R}^{N_o}$, between the model input $\mathbf{u} \in \mathbb{R}^{N_i}$ and the model output $\mathbf{g} \in \mathbb{R}^{N_o}$, given a set of N_p model parameters $\boldsymbol{\theta} \in \boldsymbol{\Theta} \subset \mathbb{R}^{N_p}$. This damage model can be "embedded" stochastically (Beck, 2010) by adding a model-error term $\mathbf{v}$ that represents the difference between the actual system response $\mathbf{x}$ and the model output $\mathbf{g}$, as follows:

$$\mathbf{x} = \mathbf{g}(\mathbf{u}, \boldsymbol{\theta}) + \mathbf{v} \tag{8}$$

The probability model chosen for the error term $\mathbf{v}$ in Eq. (8) determines the probability model for the system output $\mathbf{x}$. For example, if $\mathbf{v}$ is assumed to be a zero-mean Gaussian distribution, then the system output $\mathbf{x}$ will be also distributed as a Gaussian, as shown below:

$$\mathbf{v} = \mathbf{x} - \mathbf{g}(\mathbf{u}, \boldsymbol{\theta}) \sim \mathcal{N}(0, \Sigma) \implies \mathbf{x} \sim \mathcal{N}(\mathbf{g}(\mathbf{u}, \boldsymbol{\theta}), \Sigma)$$

where $\Sigma \in \mathbb{R}^{N_o \times N_o}$ is the covariance matrix. Thus, a stochastic damage model can be defined as a function of model parameters $\boldsymbol{\theta} \in \boldsymbol{\Theta}$, as [1]

$$p(\mathbf{x}|\mathbf{u}, \boldsymbol{\theta}) = \left((2\pi)^{N_o} |\Sigma|\right)^{-\frac{1}{2}} \exp\left(-\frac{1}{2} (\mathbf{x} - \tilde{\mathbf{x}})^T \Sigma^{-1} (\mathbf{x} - \tilde{\mathbf{x}})\right) \tag{9}$$

where $\tilde{\mathbf{x}} = \mathbf{g}(\mathbf{u}, \boldsymbol{\theta})$. As discussed in Section 2, the progression of damage is studied at every cycle n by focusing on two of its manifestations: the matrix-cracks density, ρ_n, and the normalized effective stiffness, defined as $D_n = \frac{E_x^*}{E_{x,0}}$. Then, according to Eq. (8), the actual damage response can be represented by:

$$\rho_n = \underbrace{g_1(\rho_{n-1}; \mathbf{u}, \boldsymbol{\theta})}_{\text{Eq. 7}} + v_1 \tag{10a}$$

$$D_n = \underbrace{g_2(\rho_n; \mathbf{u}, \boldsymbol{\theta})}_{\text{Eq. 1}} + v_2 \tag{10b}$$

where subscripts 1 and 2 denote the corresponding damage subsystems: matrix-crack density and relative stiffness reduction, respectively.

From Eqs. (10a) and (10b), the three main elements defining the stochastic damage model in Eq. (9) are identified: (1) the actual system output $\mathbf{x}_n = [\rho_n, D_n]$, (2) the damage model $\mathbf{g} = [g_1, g_2]$, and (3) the corresponding model error vector $\mathbf{v} = [v_1, v_2]$. A key concept here is the consideration of model errors v_1 and v_2 as stochastically independent, even though the models corresponding to the damage subsystems, g_1 and g_2, are mathematically related, as shown in Section 2. This means that the covariance operator Σ is a diagonal matrix, and therefore, the stochastic damage model of the overall system can be readily expressed as a product of univariate Gaussians, as:

$$p(\mathbf{x}_n|\mathbf{u}, \boldsymbol{\theta}) = p(\rho_n|\rho_{n-1}; \mathbf{u}, \boldsymbol{\theta}) p(D_n|\rho_n; \mathbf{u}, \boldsymbol{\theta}) \tag{11}$$

where

$$p(\rho_n|\rho_{n-1}; \mathbf{u}, \boldsymbol{\theta}) = \mathcal{N}\big(g_1(\rho_{n-1}; \mathbf{u}, \boldsymbol{\theta}), \sigma_{v_1}^2\big) \tag{12a}$$

$$p(D_n|\rho_n; \mathbf{u}, \boldsymbol{\theta}) = \mathcal{N}\big(g_2(\rho_n; \mathbf{u}, \boldsymbol{\theta}), \sigma_{v_2}^2\big) \tag{12b}$$

The parameters σ_{v_1} and σ_{v_2} in Eq. 12a and 12b are the standard deviation of the error terms v_1 and v_2, respectively. Observe that the stochastic damage model provided in Eq. (11) implicitly encloses a stochastic *state transition equation*, so that Eq. (8) can also be expressed as:

$$\mathbf{x}_n = \mathbf{g}(\mathbf{x}_{n-1}; \mathbf{u}_n, \boldsymbol{\theta}_n) + \mathbf{v}_n \tag{13}$$

[1] $p(\cdot)$ is used here to express a probability density function, whereas $P(\cdot)$ is used to denote probability

where a new variable $\mathbf{z}_n = \{\mathbf{x}_n, \boldsymbol{\theta}_n\} \in \mathcal{Z} \subset \mathbb{R}^{N_o \times N_p}$ can be suited defined as the system (health) state at time or fatigue cycle n. As explained before, Eq. (13) can be expressed probabilistically as:

$$\begin{aligned} p(\mathbf{x}_n|\mathbf{x}_{n-1}\mathbf{u}_n, \boldsymbol{\theta}_n) &= \mathcal{N}(\mathbf{g}(\mathbf{x}_{n-1}; \mathbf{u}_n, \boldsymbol{\theta}_n), \Sigma_{\mathbf{v}_n}) \\ &= \mathcal{N}(g_1, \sigma^2_{v_{1_n}})\mathcal{N}(g_2, \sigma^2_{v_{2_n}}) \end{aligned} \tag{14}$$

3.2. Filtering equations

Let suppose that the actual system response $\mathbf{x}_n$ can be measured during operation and that, at a certain fatigue cycle n, the measured system response can be expressed as a function of $\mathbf{x}_n$ as:

$$\mathbf{y}_n = \mathbf{x}_n + \mathbf{w}_n \tag{15}$$

where $\mathbf{y}_n = \left[\hat{\rho}_n, \hat{D}_n\right]$ is a vector of measurements for matrix-cracks density and normalized effective stiffness, respectively, and $\mathbf{w}_n$ is a measurement error that can be defined as zero mean Gaussian process, i.e., $\mathbf{w}_n \sim \mathcal{N}(0, \Sigma_{\mathbf{w}_n})$. Then, the *measurement equation* defined in Eq. (15) can be expressed in probabilistic terms as:

$$p(\mathbf{y}_n|\mathbf{x}_n) = \mathcal{N}(\mathbf{y}_n; \mathbf{x}_n, \Sigma_{\mathbf{w}_n}) \tag{16}$$

Note that, since the measurements of each subsystem (micro-cracks and stiffness loss) are considered to be stochastically independent, the covariance matrix will be a diagonal matrix, and the measurement equation defined in Eq. (15) can be readily expressed as:

$$p(\mathbf{y}_n|\mathbf{x}_n) = p(\hat{\rho}_n|\rho_n)p(\hat{D}_n|D_n) \tag{17}$$

$$= \mathcal{N}(\hat{\rho}_n; \rho_n, \sigma_{w_{1_n}})\mathcal{N}(\hat{D}_n; D_n, \sigma_{w_{2_n}}) \tag{18}$$

Then, the focus of the filtering problem is on sequentially updating the probability density function (PDF) of the system state given a set of system measurements up to time n, $\mathbf{y}_{1:n}$, i.e., $p(\mathbf{x}_n, \boldsymbol{\theta}_n|\mathbf{y}_{1:n}) = p(\mathbf{z}_n|\mathbf{y}_{1:n})$, using the previously defined state transition equation and measurement equation. A *particle filter* (Arumlampalam et al., 2002) is used to approximate the joint state-parameter distribution by a set of discrete weighted *particles*, $\{\mathbf{z}^i_n, \omega^i_n\}^N_{i=1}$, as

$$p(\mathbf{z}_n|\mathbf{y}_{1:n}) \approx \sum_{i=1}^{N} \omega^i_n \delta(\mathbf{z}_n - \mathbf{z}^i_n) \tag{19a}$$

$$= \sum_{i=1}^{N} \omega^i_n \delta(\mathbf{x}_n - \mathbf{x}^i_n)\delta(\boldsymbol{\theta}_n - \boldsymbol{\theta}^i_n) \tag{19b}$$

where $\mathbf{y}_{1:n} = \{\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_n\}$ denotes the sequence of measurements, N denotes the number of particles, $\mathbf{z}^i_n$ denotes the estate estimate for particle i, and ω^i_n the "weight" of particle i. Particle filters are best suited to sequential estate estimation in nonlinear systems with possibly non-Gaussian noise, where optimal solutions are unavailable or intractable, as in our problem. We employ the *sampling importance resampling* (SIR) particle filter, and implement the resampling step using systematic resampling (Arumlampalam et al., 2002). In our problem, the system state is defined as $\mathbf{z}_n = \{\rho_n, D_n, \boldsymbol{\theta}_n\}$ and the measurements $\mathbf{y}_{1:n}$ are compounded by simultaneous measurements of both, micro-cracks density and normalized effective stiffness $\mathbf{y}_{1:n} = \{\hat{\boldsymbol{\rho}}_{1:n}, \hat{\boldsymbol{D}}_{1:n}\}$. Thus, Eq. (19) can be rewritten as:

$$p(\rho_n, D_n, \boldsymbol{\theta}_n|\mathbf{y}_{1:n}) \approx \sum_{i=1}^{N} \omega^i_n \delta(\rho_n - \rho^i_n)\delta(D_n - D^i_n)\delta(\boldsymbol{\theta}_n - \boldsymbol{\theta}^i_n) \tag{20}$$

As observed in Eq. (20), model parameters augment the state vector, then the particle filter is being used to perform joint state-parameter estimation. Here the parameters $\boldsymbol{\theta}_n$ evolve by some unknown random process that is independent of the state $\mathbf{x}_n$, so that the particles with parameter values closest to the true ones should be assigned higher weights, thus allowing the particle filter to converge to the true values. In this context, standard Sequential Monte Carlo (SMC) methods (Doucet, De Freitas, & Gordon, 2001) fail and it is necessary to rely on more sophisticated algorithms. Although this problem is still open in the specific literature (Liu & West, 2001; Storvik, 2002; Kantas, Doucet, Singh, & Maciejowski, 2009), here we choose the "artificial dynamics" approach (Liu & West, 2001) due to its pragmatism and simplicity, by which model parameters performs a random walk by introducing a small (and decreasing with n) artificial white noise term, as $\boldsymbol{\theta}_n = \boldsymbol{\theta}_{n-1} + \boldsymbol{\xi}_n$. Thus,

$$p(\boldsymbol{\theta}_n|\boldsymbol{\theta}_{n-1}) = \mathcal{N}(\boldsymbol{\theta}_{n-1}, \sigma_{\boldsymbol{\xi}n}) \tag{21}$$

To sequentially reduce the standard deviation of this artificial error sequence, $\sigma_{\boldsymbol{\xi}n}$, there are many alternative methods in the literature (Kantas et al., 2009). In this paper, the recent method proposed by (M. Daigle & Goebel, 2010; M. J. Daigle & Goebel, 2013) is chosen by its simplicity and efficiency.

A pseudocode for a single step of the SIR filter proposed for estimating Eq. (20) is provided in Algorithm 1.

Note that the proposed sequential state-parameter estimation approach for damage prognostics in composites involves a filtering problem defined over a multi-dimensional parameter space $\boldsymbol{\Theta} \subset \mathbb{R}^{N_p}$. It is clear that the higher N_p is, the higher the complexity and computational cost of the filtering and prognostics algorithms. To this end, GSA (Saltelli, Ratto, Tarantola, & Campolongo, 2006) is used to simplify the model parameterization by identifying the subset of most sensitive model parameters $\boldsymbol{\theta}$ among the set of mechanical and fitting parameters defining the damage models.

Through this study, the ply properties $\{E_1, E_2, t\}$ together with the Paris' law fitting parameter $\{\alpha\}$ emerged as the key parameters in terms of model output uncertainty. Then the set

Algorithm 1 Particle Filter

At $n = 0$
Generate $\{(\rho_0^i, D_0^i, \boldsymbol{\theta}_0^i)\}_{i=1}^N$, sampling from prior PDFs $\pi_{\boldsymbol{\theta}}(\cdot)$, $\pi_\rho(\cdot)$ and $\pi_D(\cdot)$, respectively.
Assign the initial weights: $\{\omega_0^i = {}^1\!/\!_N\}_{i=1}^N$
At $n \geqslant 1$
for $i = 1 \to N$ **do**
  Sample from Eq. (21): $\boldsymbol{\theta}_n^i \sim p(\cdot|\boldsymbol{\theta}_{n-1}^i)$
  Sample from Eq. (12a): $\rho_n^i \sim p(.|\rho_{n-1}^i, \boldsymbol{\theta}_n^i)$
  Sample from Eq. (12b): $D_n^i \sim p(\cdot|\rho_n^i, \boldsymbol{\theta}_n^i)$
  Update weights: $\omega_n^i \propto p(\hat{D}_n|D_n^i)p(\hat{\rho}_n|\rho_n^i)\omega_{n-1}^i$
end for
for $i = 1 \to N$ **do**
  Normalize $\omega_n^i \leftarrow {}^{\omega_n^i}\!/\!_{\sum_{i=1}^N}$
end for
$\{(\rho_n^i, D_n^i, \boldsymbol{\theta}_n^i)\}_{i=1}^N \leftarrow$ Resample $\{(\rho_n^i, D_n^i, \boldsymbol{\theta}_n^i), \omega_n^i\}_{i=1}^N$

of updatable parameters was defined by adding the standard deviation of the model error and measurement error to the last choice, i.e., $\boldsymbol{\theta} = \{\alpha, E_1, E_2, t, \sigma_v, \sigma_w\}$. The rest mechanical and geometrical parameters act as static non-updatable input parameters.

4. DAMAGE AND RELIABILITY PROGNOSTICS

4.1. Damage prognostics

As previously explained in Section 3.1, $\mathbf{z}_n \in \mathcal{Z} \subset \mathbb{R}^{N_o \times N_p}$ represents the actual health state of the structure, which may enclose different degradation modes (e.g., micro-cracks, stiffness loss, delaminations, etc). We define the *useful domain* as the non empty subset $\mathcal{U} \subset \mathcal{Z}$ of "authorized" damage states of our system. The complementary subset $\bar{\mathcal{U}} = \mathcal{Z} \setminus \mathcal{U}$ represents degradation states that do not fulfill the design requirements, even though the system could still work.

For predicting the RUL of a composite laminate, we are interested in predicting the time when the damage grows beyond the useful domain, using the most current knowledge of the system state estimated by means of the particle filter (Eq. 20). The time or fatigue cycle at which it occurs is known as the expected end of life (EOL).

To compute EOL as a probability, each particle (damage state) is propagated forward in time using the stochastic damage model as state transition equation, until the boundary of the useful domain is reached. To this end, a *threshold function* $T_{\mathcal{U}}(\mathbf{z}_n)$ can be defined such that it that maps a given point in the joint state-parameter space to the Boolean domain $\{0, 1\}$ (M. Daigle & Goebel, 2011), as follows:

$$T_{\mathcal{U}}(\mathbf{z}_n) = \begin{cases} 0 & \text{if } \mathbf{z}_n \in \mathcal{U} \\ 1 & \text{if } \mathbf{z}_n \in \bar{\mathcal{U}} \end{cases} \tag{22}$$

Thus, the EOL of a given particle i at cycle n can be defined as the time $n' \geqslant n$ such that $T_{\mathcal{U}}(\mathbf{z}_{n'}) = 1$ by first time. Mathematically:

$$EOL_n^i = \inf\{n' \in \mathbb{N} : n' \geqslant n \wedge T_{\mathcal{U}}(\mathbf{z}_{n'}^i) = 1\} \tag{23}$$

Using the updated weights at the starting time n, a probabilistic estimation of the EOL can be obtained as:

$$p(EOL_n|\mathbf{y}_{1:n}) \approx \sum_{i=1}^N \omega_n^i \delta(EOL_n - EOL_n^i) \tag{24}$$

where ω_n^i is the weight of the i^{th} particle at time or cycle n. Once EOL_n is estimated, the remaining useful life can be readily obtained as $RUL_n = EOL_n - n$. Thus,

$$p(RUL_n|\mathbf{y}_{1:n}) \approx \sum_{i=1}^N \omega_n^i \delta(RUL_n - RUL_n^i) \tag{25}$$

An algorithmic description of the proposed prognostic procedure is provided as Algorithm 2. Note that the prediction requires hypothesizing future inputs of the system $\mathbf{u}_n$ (recall Eq. (14)). For simplicity but no loss of generality, we assume in this work that no variation of inputs parameters are expected on future states.

Algorithm 2 RUL prediction

Requires: $\{(\rho_n^i, D_n^i, \boldsymbol{\theta}_n^i), \omega_n^i\}_{i=1}^N$
Output: $\{EOL_n^i, \omega_n^i\}_{i=1}^N$
for $i = 1 \to N$ **do**
  Calculate: $T_{\mathcal{U}}(\rho_n^i, D_n^i, \boldsymbol{\theta}_n^i)$
  while $T_{\mathcal{U}} = 0$ **do**
    Sample from Eq. (21): $\boldsymbol{\theta}_{n+1}^i \sim p(\cdot|\boldsymbol{\theta}_n^i)$
    Sample from Eq. (12a): $\rho_{n+1}^i \sim p(.|\rho_n^i, \boldsymbol{\theta}_{n+1}^i)$
    Sample from Eq. (12b): $D_{n+1}^i \sim p(\cdot|\rho_{n+1}^i, \boldsymbol{\theta}_{n+1}^i)$
    $(\rho_n^i, D_n^i, \boldsymbol{\theta}_n^i) \leftarrow (\rho_{n+1}^i, D_{n+1}^i, \boldsymbol{\theta}_{n+1}^i)$
    $n \leftarrow n + 1$
  end while
  $EOL_n^i \leftarrow n$
  $RUL_n^i = EOL_n^i - n$
end for

4.2. Time varying reliability estimation

In addition to know the remaining useful life of the structure, it is also of much interest to estimate and predict the probability of the system to fulfill the design requirements, using the most up-to-date information of the system at cycle n, $\mathbf{y}_{1:n}$. In mathematical terms, the performance reliability of the system at cycle n can be defined as (M. Chiachío, Chiachío, & Rus, 2012):

$$R_{n|n}(\mathbf{z}_n) = P(\mathbf{z}_n \in \mathcal{U}|\mathbf{y}_{1:n}) = \int_{\mathcal{U}} p(\mathbf{z}_n|\mathbf{y}_{1:n}) d\mathbf{z}_n \tag{26}$$

where $p(\mathbf{z}_n|\mathbf{y}_{1:n})$ is the updated PDF of the system health state at time n. Given that the event $\{\mathbf{z}_n \in \mathcal{U}\}$ is the complementary of $\{\mathbf{z}_n \in \bar{\mathcal{U}}\}$, then $P(\mathbf{z}_n \in \mathcal{U}|\mathbf{y}_{1:n}) = 1 - P(\mathbf{z}_n \in \bar{\mathcal{U}}|\mathbf{y}_{1:n})$; thus the reliability can be rewriten as:

$$R_{n|n}(\mathbf{z}_n) = 1 - \int_{\mathcal{Z}} T_{\mathcal{U}}(\mathbf{z}_n) p(\mathbf{z}_n|\mathbf{y}_{1:n}) d\mathbf{z}_n \tag{27}$$

where $T_{\mathcal{U}}$ is the threshold function previously defined in Eq. (22). Using the particle filter approximation of $p(\mathbf{z}_n|\mathbf{y}_{1:n})$ defined in Eq. (19), the last multidimensional integral can be estimated as follows:

$$R_{n|n}(\mathbf{z}_n) \approx 1 - \int_{\mathcal{Z}} T(\mathbf{z}_n) \sum_{i=1}^{N} \omega_n^i \delta(\mathbf{z}_n - \mathbf{z}_n^i) d\mathbf{z}_n \tag{28a}$$

$$= 1 - \sum_{i=1}^{N} \omega_n^i T_{\mathcal{U}}(\mathbf{z}_n^i) \tag{28b}$$

For a forward time reliability prediction at general cycle $n + \ell$, where $\ell \in \mathbb{N} > 1$, a probability-based estimation of the damage state at cycle $n + \ell$ is needed, i.e., $p(\mathbf{z}_{n+\ell}|\mathbf{y}_{1:n})$. It can be accomplished by Total Probability Theorem using the updated state of the system at cycle n, as (Doucet et al., 2001)

$$p(\mathbf{z}_{n+\ell}|\mathbf{y}_{1:n}) = \int_{\mathcal{Z}} \left[\prod_{t=n+1}^{n+\ell} p(\mathbf{z}_t|\mathbf{z}_{t-1}) \right] p(\mathbf{z}_n|\mathbf{y}_{1:n}) d\mathbf{z}_{n:n+\ell-1} \tag{29}$$

Note that last equation can be sampled by drawing one conditional sample trajectory $\mathbf{z}^j_{n+1:n+\ell} = \{\mathbf{z}^j_{n+1}, \mathbf{z}^j_{n+2}, \ldots, \mathbf{z}^j_{n+\ell}\}$ from the state transition equation, by means of conditional sampling (Doucet et al., 2001). Thus, an estimate of the ℓ-step predictive ahead PDF can be expressed as

$$p(\mathbf{z}_{n+\ell}|\mathbf{y}_{1:n}) \approx \sum_{j=1}^{N} \omega_n^j \delta(\mathbf{z}_{n+\ell} - \mathbf{z}^j_{n+\ell}) \tag{30}$$

where ω_n^j is the weigh of particles updated at time n. Finally, the reliability at cycle $n + \ell$ using the updated information at cycle n can be obtained as:

$$R_n(\mathbf{z}_{n+\ell}) = 1 - \int_{\mathcal{Z}} T(\mathbf{z}_{n+\ell}) p(\mathbf{z}_{n+\ell}|\mathbf{y}_{1:n}) d\mathbf{z}_{n+\ell} \tag{31a}$$

$$\approx 1 - \sum_{j=1}^{N} \omega^j_{n+\ell|n} T(\mathbf{z}^j_{n+\ell}) \tag{31b}$$

5. CASE STUDY

The proposed framework is exemplified using SHM data obtained from a set of carefully designed run-to-failure fatigue experiments in cross-ply graphite-epoxy laminates. Both stiffness data and NDE measurements of internal damage, such as micro-crack density and delamination area, were periodically measured during the fatigue test (Saxena et al., 2011). Torayca T700G unidirectional carbon prepreg material was used for $15.24\ cm \times 25.4\ cm$ coupons with dogbone geometry and $[0_2/90_4]_s$ stacking sequence, whose mechanical properties are listed in Table 1. A notch ($5.1\ mm \times 19.3\ mm$) was created in these coupons to induce damage modes others than matrix-cracks, such as delamination, thereby introducing additional sources of uncertainty and then demonstrating the proposed framework under more realistic conditions.

Fatigue tests were conducted under load-controlled tension-tension cyclic loading, with a maximum applied load of 31.13 KN, a frequency $f = 5$ Hz, and a stress ratio $R = 0.14$ (relation between the minimum and maximum stress for each cycle). Monitoring data were collected from a network of 12 piezoelectric (PZT) sensors using Lamb wave signals and three triaxial strain-gages. Additionally, periodic X-rays were taken to visualize and characterize subsurface damage features, in particular, the micro-cracks density. This information was then used to develop a mapping between PZT raw signals and micro-cracks density, as reported in Larrosa and Chang (Larrosa & Chang, 2012). More details about these tests are reported in the Composite dataset, NASA Ames Prognostics Data Repository (Saxena et al., 2008). Damage data used in this example correspond to laminate L1S19 in (Saxena et al., 2008).

Results for sequential state estimation for both micro-cracks density and stiffness loss are presented in Figures 1a and 1b, respectively. Every time new data arrive, the damage variables (ρ_n, D_n) together with model parameters $\boldsymbol{\theta}_n$ are updated using a SIR algorithm with N=500 particles. This information is further used to propagate the models into the future to compute the RUL, calculated as: $RUL_n = EOL_n - n$, using the methodology described in Section 4.1. For this example, the useful domain is defined as $\mathcal{U} = \{(\rho, D) \in [0, 0.42] \times [1, 0.88]\} \subset \mathbb{R}^2$. The predictions of RUL are plotted against time in Figure 1c.

Observe that the RUL prediction is appreciably inaccurate within the first stage of the fatigue process. This stage corresponds to the interval of cycles required for data to train model parameters. From this period, the prediction precision clearly improves with time. We use the two shaded cones of accuracy at 10% and 20% of true RUL, denoted as RUL* to help evaluating the prediction accuracy and precision. Notice also in Figure 1a that accuracy seems to depart from true RUL at the final stage, which indicates that the model and its variance structure do not fully capture the damage dynamics towards the end. Such behavior have been previously reported in (Saxena, Celaya, Saha, Saha, & Goebel, 2010) and may be related with the asymptotic behavior of the micro-crack evolution, which requires more efficient algorithms for prognostics in such cases.

To show the time-varying reliability prediction of the material, a multi-step forward prediction of the health state is computed every time new SHM data arrive, using the methodology described in Section 4.2. Figure 2 shows several examples of time-varying reliability predictions at different cycles. Observe in figure 2a that the prediction gradually improves as more SHM data are available. Note also that the prediction of the cycle for which reliability vanishes is consistent with the RUL estimation.

6. CONCLUSIONS

A SHM-based prognostics framework to predict the remaining useful life and reliability of composites under fatigue conditions is proposed. We consider physics-based models for damage evolution due to the benefits for estimating the RUL and reliability. Two damage variables, micro-cracks density and stiffness loss, are simultaneously considered to represent the health state of the laminate. The validity of this framework is demonstrated on SHM data collected from a tension-tension fatigue experiment using CFRP cross-ply laminate. Reliability emerges as a suitable unified system-health indicator for prognostics, as it encapsulates information of the system health state while it allows predicting the RUL of the system. More research effort is need to achieve more efficient prognostic algorithms to improve the accuracy at the final stage of the process, where damage typically reaches an asymptotic behavior, and to incorporate other damage features like delamination in the proposed model-based framework.

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NOMENCLATURE

h	Laminate half-thickness
$E_x^{(\phi)}$	Longitudinal Young's modulus
$E_y^{(\phi)}$	Transverse Young's modulus
$\nu_{xy}^{(\phi)}$	In-plane Poisson ratio
t_{90}	$[90_{n_{90}}]$-sublaminate half-thickness
t_ϕ	$[\phi_{\frac{n_\phi}{2}}]$-sublaminate thickness
t	Ply thickness
E_1	Longitudinal Young's modulus
E_2	Transverse Young's modulus
ν_{12}	In-plane Poisson ratio
G_{23}	Out-of-plane shear modulus

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Table 1. Ply properties used in the calculations.

Long. Modulus E_1 [GPa]	Trans. Modulus E_2 [GPa]	In-plane Poisson ν_{12}	Out-of-plane Poisson ν_{23}	Shear modulus G_{12} [GPa]	Out-of-plane-Shear modulus G_{23} [GPa]	Thickness t [mm]
127.55	8.41	0.309	0.49	6.2	2.82	0.152

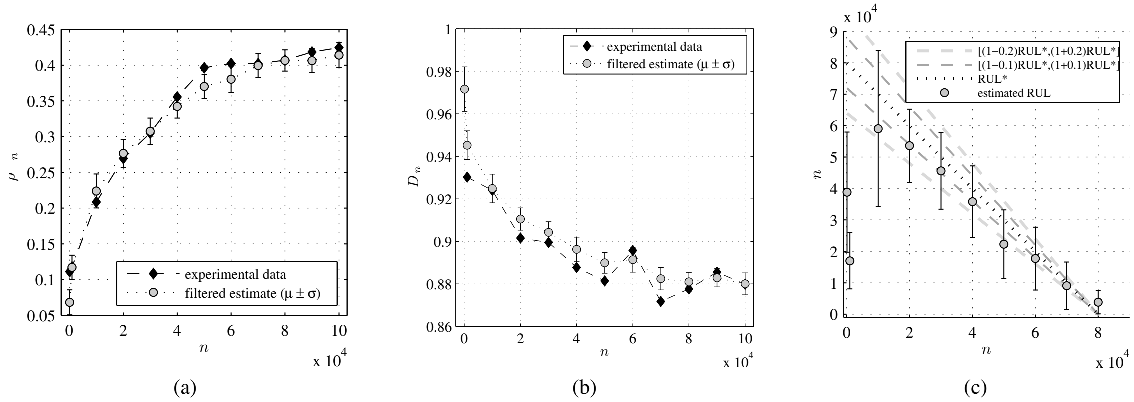

Figure 1. Results for sequential estate estimation for (a) micro-crack density, (b) normalized longitudinal Young's modulus and (c) remaining useful life. At each cycle n, the filtered estimation is calculated using the data available up to that cycle.

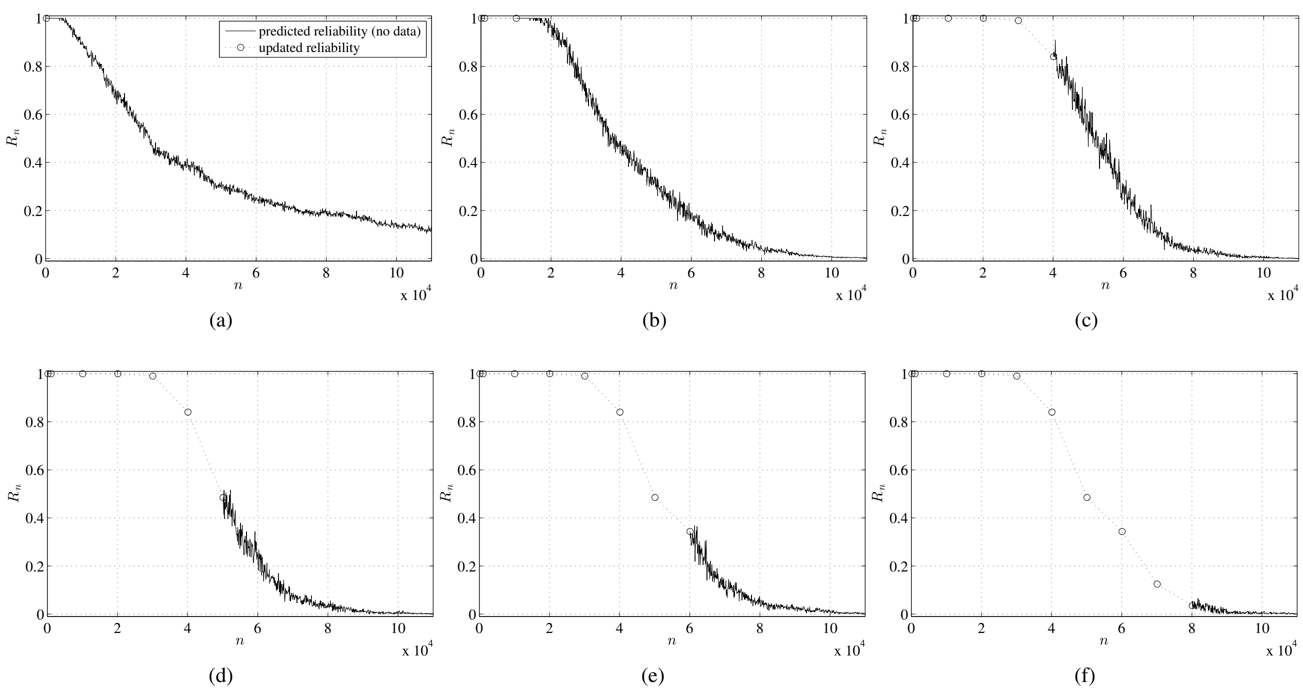

Figure 2. Time-varying reliability prediction at different cycles along the process. At each cycle n, the estimation is calculated using the data available up to that cycle.