Integrated Diagnosis and Prognosis of Uncertain Systems: A Bond Graph Approach

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ABSTRACT

Bond Graph (BG) methodology is used to model the dynamic uncertain systems. Uncertainty is considered on the system parameters in form of intervals. The uncertain parameters are allowed to deviate within their prescribed interval limits. Single fault hypothesis is considered in this work such that the parameter undergoing degradation is known *a priori*. A new method for generation of interval valued thresholds is briefly described in the framework of BG models in Linear Fractional Transformation form. The diagnostic module is formed using such thresholds which detect the beginning of degradation of a parameter in the real system. The new concept of Interval Extension of Analytical Redundancy Relations (IE-ARRs) is introduced which consider the parametric uncertainties and the evolution of degrading parameter in real time. Then, the *Centre and Range* method for fitting linear regression models to interval symbolic data is adapted to fit piece wise linear models to the interval valued times series data of IE-ARRs. Further, the new concept of generation of failure thresholds from a nominal system model is introduced and developed. Finally, the fitted linear model is used to estimate the remaining useful life of the parameter under degradation. Simulations are carried out on an example DC motor model. Linear and non-linear parametric degradations are considered. Results are presented in form of simulations.

1. INTRODUCTION

Health monitoring of systems is essential and significantly necessary in ensuring the correct operation of complex engineering systems.

The integral task of system health monitoring includes both the diagnostics and prognostics. Diagnostics involves detection of fault and its subsequent isolation whereas prognostics deal with the prediction of the remaining useful life of the different components or subsystems of the system.

1.1 Diagnosis of Uncertain Systems: Bond Graph and Interval Approaches

Bond Graph (BG) approach is a powerful tool for dynamical modeling and has established its efficiency for real applications. Further, because of its causal and structural properties, BG has been extensively used for Fault Detection and Isolation (FDI). A large body of research exists in the area of model based diagnosis in the framework of BG based approaches for modeling multi energetic dynamic systems. Various efficient algorithms have been implemented in dedicated software due to its graphical aspect which renders a clear insight into the physics of the system (Ould Bouamama, Staroswiecki, & Samantaray, 2006).

Recently, successful robust diagnostic methods have been developed using BG models in Linear Fractional Transformation (LFT) (Djeziri, Merzouki, & Ould Bouamama, 2007). The LFT representation of a global model can be derived from a BG model, by replacing each uncertain element by its LFT BG model. This form had been initially introduced in (Kam & Dauphin-Tanguy, 2005)for modelling and further for robust fault diagnosis (Djeziri, Merzouki, & Ould Bouamama, 2006). There in, procedures to generate robust Analytical Redundant Relations (ARRs) from a bond graph LFT model in derivative causality has been well developed .When used for FDI purpose, absolute values have been considered on the parameter uncertainties in the previous approaches. Adaptive thresholds that are robust to parameter uncertainties are generated, inside

which the behavior of the system can be considered as healthy. For diagnosis of uncertain systems, bounding approaches have been developed where the parametric uncertainty is considered in the form of interval models. Early work on treatment of uncertain parameters as intervals and subsequent usage for diagnosis is found in works of Adrot (2000). The approach, called bounded approach, represented these uncertainties by a set of possible values for which only their bounds were known. Ragot, Alhaj Dibo and Maquin (2003), proposed an interval technique for the detection and the isolation of sensor faults in the case of a static linear model .The similar case is treated for dynamic systems by (Ragot & Maquin, 2003).They treated the problem of data validation in the case of certain systems with uncertain measurements through interval approach. In the works of (Fagarasan, Ploix, & Gentil, 2004) interval calculation laws are used to generate the exact estimated output, bounds of the estimates are computed using traditional numerical integration techniques from the uncertain parameter interval vertices, assuming that monotonic property holds. Thus, the envelopes generated, are primarily by the estimation of state or parameter.

1.2 Fault Prognosis using Time Series Data

In past one decade, there has been an exceeding surge in research for the development of fault prognostic methods. Prognosis methods can be developed in three categorized approaches namely: data-driven, physics based and hybrid approaches. Data-driven approaches mainly use information from previous collected data (training data) to identify the characteristic of currently measured damage state and to predict the future trend. Physics-based approaches assume that a physical model describing the behavior of damage is available, and combine the physical model with measured data to identify model parameters and to predict the future behavior (Yang, 2002), (James & Hyungdae, 2005) and (Ming, 2012). Hybrid approaches combine the above-mentioned two methods to improve the prediction performance (Mohanty, Teale, Chattopadhyay, Peralta, & Willhauck, 2007).The data driven methods have been well developed from the point of view of time series prediction techniques. Method for predicting future conditions of machine operation, based on the time series prediction technique, associated with a classification tree and regression is proposed in (Trana, Yanga, Oha, & Tanb, 2008). (Wu, Hu, and Zhang(2007), proposed an extension of the basic Autoregressive Integrated Moving Average (ARIMA) approach, using bootstrap forecasting for machine life prognosis. Greitzer and Pawlowski (2002),propose a method of fault prognosis, based on a regression function, whose number of used points varies so that the prognosis remains consistent with the recent measures.

1.2.1 Prediction Using Interval valued data

Prediction techniques using interval data in symbolic form have been approached and developed by the communities of artificial intelligence, multivariate analysis and pattern recognition. They have been successful in dealing with prediction problem when the considered data is in interval form (Billard & Diday,2003). Such data arises in many situations such as recorded data for financial forecasting, daily interval temperatures at meteorological stations, daily interval stock prices etc. From the point of view of health monitoring of uncertain systems, such data are interesting and exploitable when the uncertain parameters are treated as intervals.

Linear regression models for predicting interval data was first approached in (Billard & Diday,2000), where the *Centre* method of fitting a linear regression model to symbolic interval data sets from the Symbolic Data Analysis(SDA) perspective is presented. It consists of fitting a linear regression model to the mid-points of the interval values assumed by the interval variables in the learning set and this model is applied to the lower and upper bounds of the interval values of the independent interval variables to predict the lower and upper bounds of the dependent variable, respectively. *Minmax* method (Billard & Diday, 2002), assumes independence between the values of lower and upper bounds of the dependent data intervals which are then estimated by different vectors of parameters. However, both of these methods consider information carried by mid-points only. As such, they fail to capture the influence of interval range on the estimation of parameters. This in turn, affects the prediction ability.

The *Centre and Range* approach to fitting a linear regression model to symbolic interval data was proposed in (Lima Neto & De Carvalho, 2008). There in, the problem was investigated as an optimization problem, which sought to minimize a predefined criterion. The approach considered the minimization of the sum of the mid-point square error plus the sum of the range square error, and the reconstruction of the interval bounds based upon the mid-point and range estimates. The lower and upper bounds of the interval values of an interval valued variable, linearly related to a set of independent interval-valued variables were predicted for independent data sets. It is shown that including information given by both center and the range of an interval data improves the model prediction performance very considerably.

1.3 Assumptions, Proposed Approach and Organization of the Work

In this work, BG methodology is used to model the dynamic uncertain systems. Uncertainty is considered only on the system parameters in form of intervals. The uncertain parameters are allowed to deviate within their prescribed interval limits. Single fault hypothesis is followed such that the parameter undergoing degradation is known *a priori*. In section 2, the new method of generation of interval valued thresholds proposed in (Jha, Dauphin-Tanguy, & Ould Bouamama, 2014) is briefly described in the framework of

BG-LFT models. The diagnostic module is formed using such thresholds which detect the beginning of degradation of a parameter in the real system. In section 3, the new concept of Interval Extension of ARRs (IE-ARRs) is introduced which considers the parametric uncertainties and the evolution of degrading parameter in real time. Then, the *Centre and Range* method (Lima Neto et al, 2008) for fitting linear regression models to interval symbolic data is adapted to fit piece wise linear models to the interval valued times series data of IE-ARRs. The procedure is explained in the subsequent subsection 3.2. Further, the new concept of generation of failure thresholds from a nominal system model is introduced and explained in subsection 3.3. Finally, the fitted linear model is used to estimate the remaining useful life of the parameter under degradation. In section 4, the developed method is validated using a pedagogical DC motor example. Linear and non-linear parametric degradation of physical components are considered. Results are presented in form of simulations. Finally conclusions are drawn in section 5.

2. ROBUST DIAGNOSIS THROUGH BG-LFT MODELS

Diagnosis based on BG-LFT models is considered in this section. Recently the authors have proposed a novel way of generating thresholds over ARR where the uncertainties are modeled as intervals (Jha, Dauphin-Tanguy, & Ould Bouamama, 2014). The novelty there comes in the treatment of uncertain part in form of intervals and using the obtained *Interval Extension Functions* (IEF) for generation of robust optimized thresholds which are adaptive and non-symmetrical in general.

2.1 Generation of Interval valued robust thresholds

A system parameter θ_i with deviations as $\Delta\theta_{i,l}$ and $\Delta\theta_{i,u}$ in the negative and positive side respectively over its nominal value $\theta_{i,n}$ is represented in Eq.(1). For the parameter θ_i, the *Interval Uncertainty* denoted as $[\partial\theta_i]$ in Eq.(2) is obtained by bounding the uncertainties $\delta\theta_i$ over its nominal value $\theta_{i,n}$. For example, for an uncertain resistance parameter R with nominal value of 10 Ohm bounded in the interval as [8 Ohm, 13 Ohm], the nominal parameter is denoted as $R_n = 10$ Ohm, uncertainty interval is $[\delta R] = [-2,3], \Delta R_l = 2, \Delta R_u = 3$. Then,

$$[R] = [R_n - \Delta R_l, R_n + \Delta R_u] = [10 - 2, 10 + 3] = [8,13]$$

.

$$[\theta_i] = \left[\theta_{i,n} - \Delta\theta_{i,l}, \theta_{i,n} + \Delta\theta_{i,u}\right] \tag{1}$$

$$[\partial\theta_i] = [-\Delta\theta_{i,l}, \Delta\theta_{i,u}] \tag{2}$$

$$0 \le \Delta\theta_{i,l}, 0 \le \Delta\theta_{i,u}$$

In general, in the framework of BG-LFT modeling, where $\theta_i \in \{R, C, I, GY, TF, RS\}$, a residual R is derived from LFT-BG with preferred derivative causality, so that the knowledge of initial conditions is not necessary for real time evaluation. Residual R is composed of two completely separated parts: a certain residual r and the uncertain part b as shown in (4),(5),(6) and (7) where TF_n and GY_n are respectively the nominal values of TF and GY moduli. R_n; C_n; I_n and RS_n are the nominal values of physical BG elements R, C, I and RS. SSe and SSf are the signal sources (measurement signals from real system) and δ_R, δ_I, δ_C, δ_{RS}, δ_{TF}, δ_{GY} are values of multiplicative uncertainty. Natural interval extension function *IEF* (Moore, 1996), B of the uncertain part b is formed by replacing each parameter multiplicative uncertainty with its prescribed interval uncertainty as in Eq.(8). The IEF, $B([\partial\theta_{1,n}], [\partial\theta_{2,n}], \ldots[\partial\theta_{q,n}], SSe, SSf)$ where q is the number of uncertain elements considered, agrees with the uncertain ARR function $b(\partial\theta_{1,n}, \ldots\partial\theta_{q,n}SSe, SSf)$ such that Eq.(8) is satisfied for each of the degenerate intervals $[\delta\theta_{1,n}, \delta\theta_{1,n}], [\delta\theta_{2,n}, \delta\theta_{2,n}], \ldots[\delta\theta_{q,n}, \delta\theta_{q,n}]$. Through *Extended Fundamental Theorem of Interval Analysis* (Moore, 1996), Eq.(9) is satisfied for every interval set of *Interval Uncertainty*.

$$R = \Phi \begin{cases} \sum Se, \sum Sf, SSe, SSf, R_n, C_n, I_n, TF_n, \\ GY_n, RS_n, \sum w_i \end{cases} \tag{3}$$

$$R = r + b \tag{4}$$

$$r = \Phi \begin{cases} Se, Sf, SSe, SSf, R_n, C_n, I_n, TF_n, \\ GY_n, RS_n \end{cases} \tag{5}$$

$$b = \sum w_i = \Psi\{\partial\theta_{1,n}, \partial\theta_{2,n}, \ldots\partial\theta_{q,n}, SSe, SSf\} \tag{6}$$

$$w = \Theta \begin{cases} Se, Sf, SSe, SSf, R_n, C_n, I_n, TF_n, \\ GY_n, RS_n, \delta_R, \delta_I, \delta_C, \delta_{TF}, \delta_{GY}, \delta_{RS} \end{cases} \tag{7}$$

$$b(\partial\theta_{1,n}, \partial\theta_{2,n}, \partial\theta_{3,n}\ldots\ldots\partial\theta_{q,n}, SSe, SSf) \tag{8}$$
$$\in B([\partial\theta_{1,n}], [\partial\theta_{2,n}], [\partial\theta_{3,n}]\ldots\ldots[\partial\theta_{q,n}], SSe, SSf)$$

When the system shows nominal behavior, an envelope around the residual R may be defined by the range of the function B. Under non-faulty conditions, the nominal residual r is around zero(theoretically). From Eqs.(5,6,7,8,9) the residual R can be written as in Eq.(10) and from Eq.(9), it is bounded by the interval valued thresholds B as shown in Eq.(11). Note that in this work, signals from dualised sensor effort sources and flow sources SSe, SSf respectively are not considered in the interval form following the hypothesis that sensor measurements are not considered faulty.

$$b(\partial\theta_{1,n}, \partial\theta_{2,n}, \ldots\ldots\partial\theta_{q,n}, SSe, SSf) \qquad (9)$$

$$\subseteq B([\partial\theta_{1,n}], [\partial\theta_{2,n}], \ldots\ldots[\partial\theta_{q,n}], SSe, SSf)$$

$$R = r + b(\partial\theta_{1,n}, \partial\theta_{2,n}, \ldots\ldots\partial\theta_{q,n}, SSe, SSf) = 0 \qquad (10)$$

$$r = 0, R = b(\partial\theta_{1,n}, \partial\theta_{2,n}, \ldots\ldots\partial\theta_{q,n}, SSe, SSf)$$

$$R \subseteq B([\partial\theta_{1,n}], [\partial\theta_{2,n}], \ldots\ldots[\partial\theta_{q,n}], SSe, SSf) \qquad (11)$$

Finding the most optimum thresholds narrows down to the problem of finding the exact range of IEFs (Armengol, 1999). The main limitation is that IEFs do not have some of the properties of real number arithmetic, for instance, the distributive property. This means that the exact range of a function is not always computable. However, the computed range is never an under-bounded one. The exact range is obtained if there are no multi-incident interval variables in $b(\partial\theta_{1,n}, \partial\theta_{2,n}, \ldots\ldots\partial\theta_{q,n}, SSe, SSf)$. The determination of the exact range of a function is a problem when there are multi-incident variables because each incidence is considered as an independent variable. In this case, this problem is similar to a global optimization one (Hansen, 1992).

3. FAULT PROGNOSIS THROUGH LINEAR REGRESSION OF INTERVAL EXTENSION ARRs (IE-ARRs)

Consider the scenario when a set of faulty parameters of the system undergo degradation and rest of the uncertain parameters deviate within their prescribed limits. In such cases, point data valued ARRs are not capable of capturing the sufficient information provided by such deviating uncertain parameters. To deal with such cases, *Interval Extension* ARRs (IE-ARRs) are proposed in this work which captures system information in form of interval data where the uncertain parameters are modelled in form of intervals.

3.1. Interval Extension ARRs (IE-ARRs)

Interval-valued functions are obtained by selecting a real-valued function f and computing the range of values $f(x)$ takes as x varies through some interval X. By definition (Moore, 1996), the result is equal to the set image $f(X)$.

Interval extensions of ARRs can be obtained by bounding each uncertain parameter involved in the ARR, within its prescribed interval limit. This is done by considering the uncertainties on the negative and positive sides $\Delta\theta_{i,l}$ and $\Delta\theta_{i,u}$ respectively, over the nominal value $\theta_{i,n}$ of the ith uncertain parameter θ_i, to obtain the interval form $[\theta_i]$ as in Eq. (2). In Eq. (12) consider a_i as any ARR with m independent parameters such that q ($q \leq m$), of them are

uncertain, $U = [u_1, u_2 \ldots]^T$ is the input vector, $\theta = [\theta_1, \ldots\theta_q \ldots\theta_m]^T$ is the nominal parameter vector and $Y = [y_1, y_2, \ldots]^T$ is the output vector. The corresponding *Interval Extension* (*IE*), IEa_i is obtained by bounding each uncertain parameter within their interval limits as shown in Eq.(12).In the BG framework, consider r in Eq.(5), which represents the point valued ARR with uncertain parameters with their nominal values $R_n, C_n, I_n, TF_n, GY_n, RS_n$.

$$a_i = f(U, \theta, Y) = f(U, \dot{U}, \ldots\ldots((\theta_1, \theta_2, \ldots\theta_q).$$
$$\ldots\theta_{m-1}, \theta_m)^T, Y, \dot{Y}\ldots) = 0$$
$$IEa_i = F(U, \theta, [\theta], Y) = F(U, \dot{U}, \ldots \qquad (12)$$
$$(([\theta_1], [\theta_2], \ldots[\theta_q])\ldots\theta_{m-1}, \theta_m)^T, Y, \dot{Y})$$

The *IE*, R is formed by considering each uncertain parameter as intervals Eq. (13). Note that the dualised signal sources (sensor measurements) are not considered faulty or uncertain. Also, it must be noted that *IE* of ARRs consider the parameter with uncertainties in the interval form $[\theta_i]$, whereas for the generation of interval valued thresholds in Eq. (9), only the parametric uncertainties are considered in the interval form $[\partial\theta_i]$.

$$R_i(t) = \Phi_i \begin{cases} Se, Sf, SSe, SSf, [R], [C], [I], [TF], \\ [GY], [RS] \end{cases} \qquad (13)$$

3.2 Fitting a Linear Regression Model to Time Series Interval Valued Data

One way to represent this type of data is through the mid-point and range of interval (Lima Neto & De Carvalho, 2008).When such data are collected in chronological sequence, the time series of interval valued data is obtained. At each instant of time, $t=1,2,3,\ldots n$, where n is the number of intervals observed in the time series, $x_{l,t}$ and $x_{u,t}$ with $x_{l,t} \leq x_{u,t}$, are the upper and lower bounds of the interval respectively. The method employed here uses two time series: the interval mid-point series x^c; and the half range interval series x^r. Considering the interval time series in Eq.(14), mid-point and half-range time series can be represented as in Eq.(15).

$$[x_{L,1}, x_{U,1}], [x_{L,2}, x_{U,2}], \ldots\ldots, [x_{L,n}, x_{U,n}]. \qquad (14)$$

$$x_t^C = \frac{x_{L,1} + x_{U,1}}{2}, x_t^R = \frac{x_{L,1} - x_{U,1}}{2} \qquad (15)$$
$$(t = 1, 2, \ldots, n)$$

In this work, the centre and range method (Lima et al., 2008) is adapted to fit a linear regression model to interval valued time series data.

Let $E = \{e_1, e_2 \ldots\ldots e_k\}$ be the set of time indexed data described by interval valued dependent variable Y and independent time variable T such that for each $e_i \in E (i = 1..k)$, $Y_i = [y_{l,i}, y_{u,i}] \in \zeta = \{[a,b] : a, b \in R, a \le b\}$ and $T_i = [t_{l,i}, t_{u,i}] \in \zeta$. Parameter vector β, is estimated using the information contained in the mid-points and ranges of the intervals.

Let Y_i^c and T_i^c respectively, assume the value of the mid-point of the interval valued variables Y_i and T_i. Also let Y_i^c and T_i^r assume the value of the half range of interval valued variables Y_i and T_i.

Then, each e_i is represented as interval quantitative feature $w_i = (t_i^c, y_i^c)$ and $r_i = (t_i^r, y_i^r)$ where,

$$
\begin{aligned}
t_i^c &= (t_{l,i} + t_{u,i})/2, \\
t_i^r &= (t_{u,i} - t_{l,i})/2, \\
y_i^c &= (y_{l,i} + y_{u,i})/2, \\
y_i^r &= (y_{u,i} - y_{l,i})/2,
\end{aligned}
\tag{16}
$$

are the observed values of T^c, T^r, Y^c and Y^r respectively.

Consider the dependent variables Y^c and Y^r related to the independent time variable T^c and T^r according to the following linear regression relationship,

$$
y_i^c = \beta_0^c + \beta_1^c t_i^c + \varepsilon_i^c
\tag{17}
$$

$$
y_i^r = \beta_0^r + \beta_1^r t_i^r + \varepsilon_i^r
\tag{18}
$$

The sum of squares of deviations is given in Eq. (19). It represents the sum of the mid-point square error plus the sum of the range square error, considering independent vectors of parameters to predict the mid-point and the range of the intervals.

$$
S = \sum_{i=1}^{k} ((\varepsilon_i^c)^2 + (\varepsilon_i^r)^2) = \sum_{i=1}^{k} (y_i^c - \beta_0^c - \beta_1^c t_i^c)^2 +
$$

$$
\sum_{i=1}^{k} (y_i^r - \beta_0^r - \beta_1^r t_i^r)^2
\tag{19}
$$

Values of $\beta_0^c, \beta_1^c, \beta_0^r$ and β_1^r that minimize S are found by differentiating Eq. (19) with respect to the parameters and setting the result equal to zero as in Eq. (20). It gives set of equations as shown in Eq. (21).The estimated parameter set $\hat{\beta}$ can be obtained by solving Eq.(21), as in Eq. (22). Note that imprecision on the independent time variable has been accommodated by modeling it in form of intervals.

This way, imprecision arising due to sensor/measurement (acquisition) delay can be taken into account. In cases where the time variable is not treated as an imprecise quantity, the upper and lower bound remain the same resulting in the interval centre being equal to the time value at that instant as $t_i^c = t_i$ and the time interval range equal to zero. It is a special case when the *Centre-Range* method reduces to the *Centre* method (Lima et al).

3.3 Remaining Useful Life Estimation

Beginning of degradation is indicated by the diagnostic module when the point valued ARRs go outside the interval valued thresholds, developed in section 2. Once, degradation is indicated, IE-ARRs are taken into account. With single degrading parameter, the IE-ARR evolves into time as the degradation proceeds.

$$
\frac{\partial S}{\partial \beta_0^c} = 0, \frac{\partial S}{\partial \beta_1^c} = 0, \frac{\partial S}{\partial \beta_0^r} = 0, \frac{\partial S}{\partial \beta_1^{\backslash r}} = 0
\tag{20}
$$

$$
\hat{\beta}_0^c k + \hat{\beta}_1^c \sum_{i=1}^{k} t_i^c = \sum_{i=1}^{k} y_i^c
$$

$$
\hat{\beta}_0^c \sum_{i=1}^{k} t_i^c + \hat{\beta}_1^c \sum_{i=1}^{k} (t_i^c)^2 = \sum_{i=1}^{k} y_i^c t_i^c
$$

$$
\hat{\beta}_0^r k + \hat{\beta}_1^r \sum_{i=1}^{k} t_i^r = \sum_{i=1}^{k} y_i^r
\tag{21}
$$

$$
\hat{\beta}_0^r \sum_{i=1}^{k} t_i^r + \hat{\beta}_1^r \sum_{i=1}^{k} (t_i^r)^2 = \sum_{i=1}^{k} y_i^r t_i^r
$$

$$
\hat{\beta} = (\hat{\beta}_0^c, \hat{\beta}_1^c, \hat{\beta}_0^r, \hat{\beta}_1^r)^T = (A)^{-1} d
\tag{22}
$$

3.3.1 Parametric Failure Threshold

For prediction of RUL of the degrading parameter, the value of the IE-ARR at the parametric failure state must be known. This is not known beforehand from the real system. It can however be provided by the system model. Let us denote the degrading parameter candidate as $\theta_{\deg}$. Its value at failure must be fixed. This can be fixed based upon system performance, stability or user defined conditions/thresholds. This value can be bounded in interval form as per the user/system dependant conditions. Let us denote such a value as $\theta_{\deg,fail}$. Then the deviation that the parameter must go in order to reach the failure state is $\Delta\theta_{\deg,fail} = \theta_{\deg,fail} - \theta_{\deg,n}$. Thus, it provides the value of parametric failure deviation $\Delta\theta_{\deg,fail}$.

Consider the interval thresholds in Eq. (9) generated in section 2, which form the envelop around the residual under nominal system condition. When the same expression is

considered with the value of failure deviation $\Delta\theta_{deg,fail}$, parametric failure thresholds are obtained as Eq. (23), where the parametric uncertainty-interval form is considered for all the uncertain parameters sensitive to the corresponding residual. Also, unlike the diagnostic thresholds where sensor measurements from real system (*SSe, SSf*) are used, B_{fail} considers the corresponding outputs from nominal system model which has all the respective parameters in nominal state. Due to the considered parametric uncertainty of each uncertain parameters, upper and lower bounds of B_{fail} are generated as Eq.(24).

$$B_{fail} = \Psi(\Delta\theta_{deg,fail},([\partial\theta_{1,n}],[\partial\theta_{2,n}],...$$
$$....[\partial\theta_{q,n}]).De_{model},Df_{model})$$
(23)

$$B_{fail} \in [B_{fail,l},B_{fail,u}]$$
(24)

3.3.2 RUL Estimation

The degradation information provided in form of interval valued data from the IE-ARRs is used to fit a linear regression model in a sliding window framework. Let the time window length be k. The interval time series data of degradation be obtained as $E = \{e_j,e_{j+1},e_{j+2}....e_{j+k}\}$, where for each time indexed $e_i (j \leq i \leq j+k)$, $Y_i = [B_{fail,i,l},B_{fail,i,u}]$ and $T_i = [t_{i,l},t_{i,u}]$. T^c,T^r and Y^c,Y^r are to be obtained using Eq. (16). The parameter vector $\hat{\beta}$ is estimated using Eq. (22). Once $\hat{\beta}$ is obtained, the degradation can be approximated by the piece wise linear model of degradation for the k time instants in the present j^{th} time window. The regression model is fitted with parameter failure value to assess the RUL in j^{th} time window as,

$$t_{fail}^c(j) = (B_{fail}^c - \hat{\beta}_0^c)/\hat{\beta}_1^c$$
$$t_{fail}^r(j) = (B_{fail}^r - \hat{\beta}_0^r)/\hat{\beta}_1^r$$
$$B_{fail}^c(j) = (B_{fail,l} + B_{fail,u})/2,$$
$$B_{fail}^r(j) = (B_{fail,u} - B_{fail,l})/2$$
(25)

$$t_{fail} = [t_{fail}^c - t_{fail}^r, t_{fail}^c + t_{fail}^r]$$
(26)

The time window is shifted to $t=j+1$ for next k time instants and a similar routine is followed.

Thus, the value of the RUL can be obtained in the bounded form based on the piece wise linear approximation of degradation in sliding time window framework. The routine is repeated to obtain the RUL in the next time window. It

should be noted that the RUL estimated, corresponds to the linear approximation of degradation. As such, in cases of gradual linear degradation an approximate constant value of RUL is obtained in interval form. However, in cases of non-linear or accelerated degradation, a distribution of RUL will be obtained. Analysis of such a distribution form has not been done here. The choice of window length is important in determining the correct linear approximation of degradation as in, a large window width is better in cases of gradual-linear degradation. This aspect has not been analyzed in this work and forms the future perspective.

4. SIMULATIONS AND RESULTS:

The proposed methodology is applied over a DC-motor model. Fig.1 shows the model schema and Fig.2 its associated BG in integral causality. The integral causal model is used for simulation purpose. The model parameters are taken as: Ra = 2.4 Ω, the resistance of stator; La = 0.84 H , the inductance of the stator; ke= 0.14 N-m/A, the motor constant; J_m = 0.08 kg m^2, the moment of inertia of rotor; f_m = 0.01 Ns/ m , coefficient of friction of motor shaft, with the inputs $Ua(t)$ being the input voltage of 220 V in magnitude and $\tau(t)$ being the load torque of 5 N m in magnitude. The observed outputs are: i_m(t) current of inductor, and ω_m(t) being the angular velocity of the motor shaft (rad/s).

Considered model has uncertain parameters as La, ke, f, Jm and Ra. Single fault hypothesis is followed with the assumption that sensors/measurements are not faulty. Parametric degradation of the electrical resistance Ra is considered and simulated under various cases of degradation. Simulations have been carried out on SIMULINK® which is integrated with MATLAB® .Interval computations have been carried out through INTLAB, (Rump, 1998) a toolbox designed for MATLAB environment. It allows the more traditional infimum-supremum as well as the midpoint-radius representations of intervals.
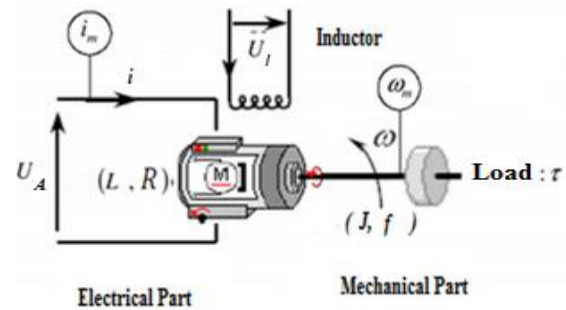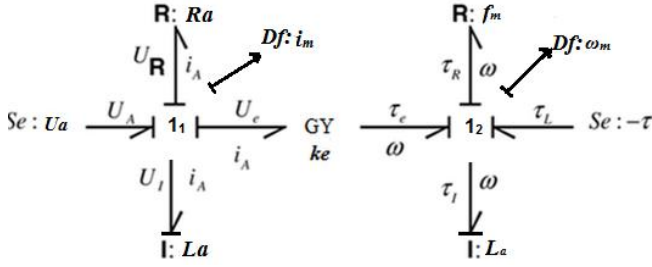

Figure1. Schema of DC motor

Figure 2. Integral causal BG model of DC motor

Consider the BG-LFT model in preferred derivative causality of DC-motor in Fig.3. The fictive inputs $w_i, i \in (R_a, L_a, k_e, J_m, f_m)$ are related to fictive outputs $z_i, i \in (R_a, ... f_m)$ as follows.

$$w_{R_a} = -\partial_{R_a}.z_{R_a} ; z_{R_a} = R_a.\{i_m\}$$
$$w_{L_a} = -\partial_{La}.z_{L_a} ; z_{L_a} = L_a.(d\{i_m\}/dt)$$
$$w_{1,ke} = -\partial_{k_e}.z_{k_e} ; z_{k_e} = k_e.\{w_m\} \qquad (27)$$
$$w_{f_m} = -\partial_{fm}.z_{fm} ; z_{fm} = f_m.\{w_m\}$$
$$w_{J_m} = -\partial_{Jm}.z_{Jm} ; z_{Jm} = J_m.(d\{w_m\}/dt)$$
$$w_{2,k_e} = \partial_{k_e}.z_{2,k_e} ; z_{2,k_e} = k_e.\{i_m\}$$

where $\partial_{Ra_a}, \partial_{L_a}, \partial_{k_e}, \partial_{f_m}, \partial_{J_m}$ are the multiplicative uncertainties on the respective parameters.
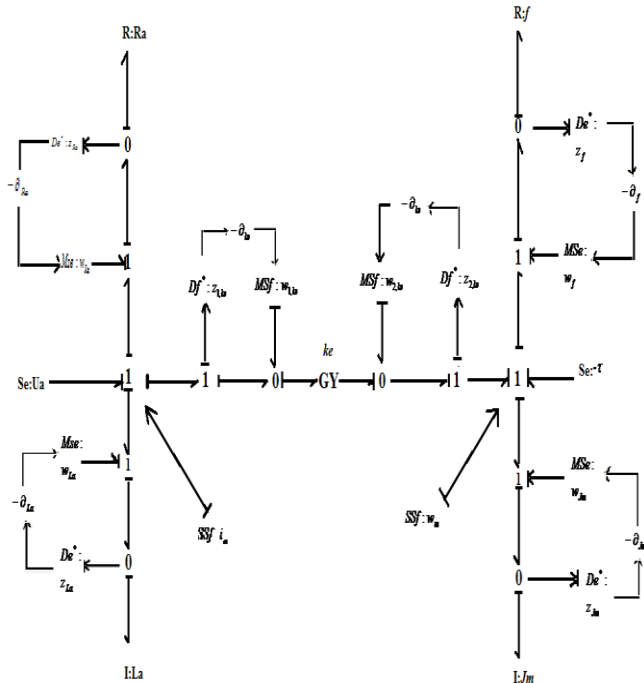
Figure 3: BG-LFT model of DC Motor

The ARR relations are derived from the model as,

$$R_1 = \underbrace{U_a - L_{a_n}.(d\{i_m\}/dt) - R_{a_n}.\{i_m\} - k_{e,n}\{w_m\}}_{r_1} +$$
$$\underbrace{w_{R_a} + w_{L_a} + w_{1,k_e}}_{b_1} \qquad (28)$$
$$b_1 = \Delta R_a.\{i_m\} + \Delta L_a.\{i_m\}| + \Delta k_e.\{w_m\}$$
$$R_2 = \underbrace{-\tau - f_{m,n}.\{w_m\} - J_{m,n}.(d\{w_m\}/dt) + k_{e.,n}\{i_m\}}_{r_2} +$$
$$\underbrace{w_{fm} + w_{Jm} - w_{2,ke}}_{b_2} \qquad (29)$$
$$b_2 = \Delta f_m.\{w_m\} + \Delta J_m.\{\dot{w}_m\} - \Delta k_e.\{i_m\}$$

where b_1 and b_2 represent the uncertain part of each residual R_1 and R_2 with $\Delta\theta_i$ denoting the additive uncertainty on parameter θ_i. Then, each additive uncertainty is bounded in interval form to form the interval valued thresholds B_1 and B_2 respectively as Eq. (30) and Eq. (31).

Since R_a is sensitive to R_1 only, R_2 is not considered for subsequent analysis. L_a and k_e are considered to deviate within their interval limits but do not undergo any kind of degradation:

$$L_a \in [L_{a_n} - (L_{a_n} * 0.1), L_{a_n} + (L_{a_n} * 0.5)]$$
$$k_e \in [k_{e_n} - (k_{e_n} * 0.1), k_{e_n} + (k_{e_n} * 0.2)].$$

Allowed deviation on R_a is such that, $R_a \in [R_{a_n} - (R_{a_n} * 0.2), R_{a_n} + (R_{a_n} * 0.1)]$.

$$B_1 = [-\Delta R_{al}, \Delta R_{au}].\{i_m\} + [-\Delta L_{al}, \Delta L_{au}].\{\frac{d}{dt}i_m\} \qquad (30)$$
$$+ [-\Delta k_{e_l}, \Delta k_{e_u}]\{w_m\} \qquad (31)$$
$$B_2 = [-\Delta f_{ml}, \Delta f_{mu}].\{w_m\} + [-\Delta J_{mi}, \Delta J_{mu}].\{\dot{w}_m\}$$
$$- [-\Delta k_e, \Delta k_{e_u}]\{i_m\}$$

4.1 Case I: No Degradation.

All the three parameters R_a, L_a and k_e which are sensitive to R_1 deviate within their interval limits. Fig.5 shows the interval thresholds generated from Eq. (30) such that $B_1 = [B_{1,l}, B_{1,u}]$

where the considered allowed interval limits are:
$\Delta R_{al} = R_{an} * 0.2, \Delta R_{au} = R_{an} * 0.1, \Delta L_{al} = L_{an} * 0.1,$
$\Delta L_{au} = L_{an} * 0.5, \Delta k_{el} = k_{en} * 0.1, \Delta k_{eu} = k_{en} * 0.2$

Fig. 5 shows the simulated residual R_1 which is generated from the real system with uncertain parameters deviating inside their prescribed interval limits. It is under the thresholds indicating no fault or degradation. The residual is different from zero indicating that parameters deviate within

7

prescribed limits. Note that for the purpose of illustration, there is no noise considered in the simulations, assuming that sensor measurements are present with negligible noise.
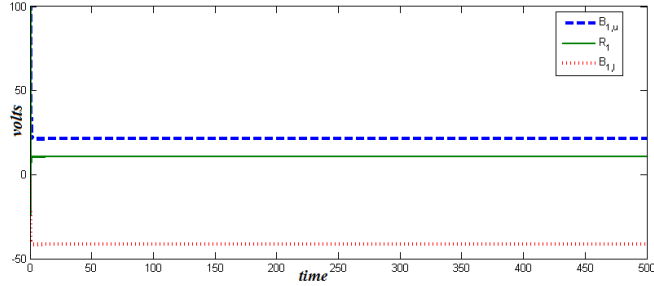

Figure 5. Residual r_1 under nominal conditions

4.2 Case II: Gradual and Linear Degradation in Winding Resistance R_a

A degradation of the form $R_a(t) = R_{an}(1+\alpha t)$ is considered in the real system model, where $\alpha = 2.5e^{-4}$. Fig. 6 shows the degradation profile. The diagnostic threshold should be crossed at t=400s when $R_a = R_{an} + \Delta R_{a_u}$. The failure value of R_a is set to be $R_{a,fail} = 3\Omega$ so that $\Delta R_{a\,fail} = 0.6\Omega$. Failure value is expected to be reached at t=1000s. Failure thresholds which consider model inputs can be formed following Eq. (23) as in Eq. (32), where the measurement inputs are from the nominal system model.

$$B_{Ra,fail} = \Delta Ra_{fail}.\{i_{m,model}\} + [-\Delta La_l, \Delta La_u].\{\frac{d}{dt}i_{m,model}\}$$
$$+ [-\Delta ke_l, \Delta ke_u]\{w_{m,model}\} \quad (32)$$

Fault detection: Detection of the degradation on R_a is done by the diagnostic thresholds B_1 as shown in Fig.7. As expected, the thresholds are crossed by the residual at t=400s indicating the beginning of degradation. Failure thresholds $B_{Ra,fail}$ are formed from the inputs of a nominal system model.

Fault prognosis: As soon as the degradation is detected, the prognostic module is triggered on. The *Interval Extension* of r_1, denoted by IEr_1 is considered from there-on i.e. after t=400s as,

$$IEr_1 = U_a - [L_{an}. - \Delta L_{al}, L_{an}. + \Delta L_{au}](d\{i_m\}/dt) \quad (33)$$
$$- R_{an}.\{i_m\} - [ke_n. - \Delta ke_l, ke_n. + \Delta ke_u]\{w_m\}$$

Fig.7 shows the evolution of r_1 and IEr_1 as the degradation proceeds in time. Failure threshold $B_{Ra,fail} \in [B_{Ra,fail,l}, B_{Ra,fail,u}]$ considered for the estimation of RUL is also shown in the same figure. Fig. 8 shows the data of Fig. 7 between time 420s and 530s presenting the various intervals for better clarity.

Linear regression model is then fitted to the interval data of IEr_1 in a sliding window of length k=5. Fig. 9 shows the obtained RUL in the interval form. As expected, the RUL is bounded around 1000s in interval form. Thus, for linear-gradual degradations, this approach is efficient in estimating the RUL .
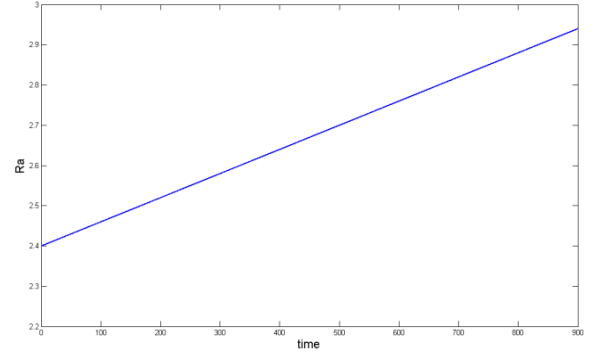

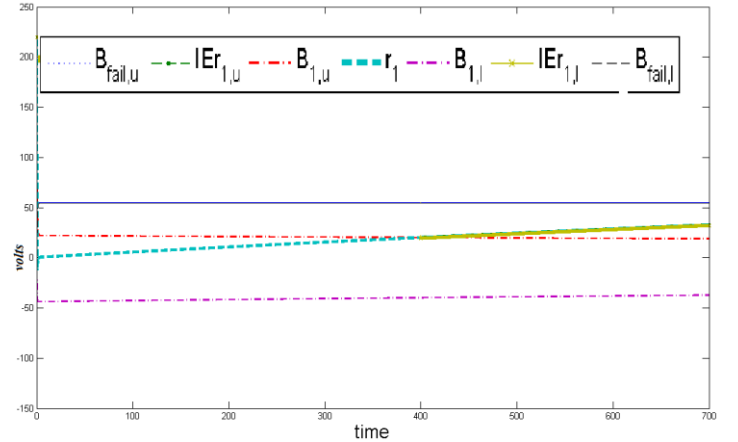Figure 6. Linear Degradation induced in R_a
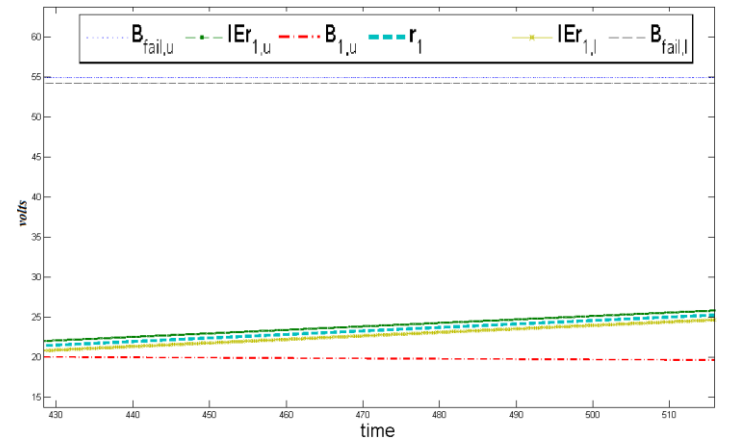

Figure 7. Detection of Degradation at t=400s.

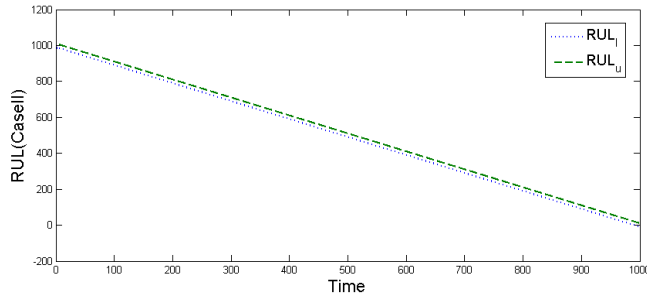Figure 8: IEr_1 profile zoomed

Figure.9 RUL estimation for Case:II

4.3 Case III: Gradual and Non-Linear Degradation of Ra

A non-linear, gradual degradation of the form $R_a = R_{an}e^{0.001t}$ is considered on *Ra*. The diagnostic threshold should be crossed at t=95s when $R_a = R_{an} + R_{an}*0.1$(so that its maximum limit for allowed deviation is reached).The failure state value is prefixed as $R_{a,fail} = 3.0\Omega$, the expected RUL is 223s.

Fault detection and Prognosis: Fig. 10 shows the simulation of the residual which crosses the thresholds at t=95s, indicating the beginning of degradation. Once degradation is detected, *IEr₁* is considered upon which the linear regression model is fit in sliding window of length $k = 5*st$ where *st* is the sample time, taken as 0.01 s here. Fig.11 shows the estimated bounded RUL .It is noticed that the RUL evolves in time starting from 250s. It is estimated by approximation of the non-linear degradation through a linear fit model. At each instant, the obtained RUL depends upon the linear approximation of nature of degradation in that time window. The linear approximation is helpful in prediction with sufficient accuracy.
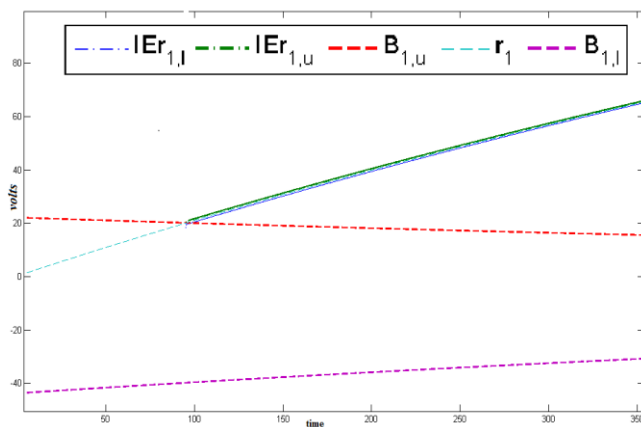

Figure 10. Detection of Degradation and *IEr₁* profile
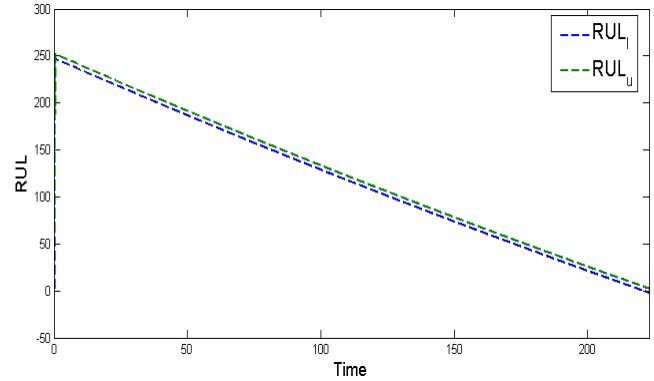

Figure 11. RUL estimation (Case: III)

5. CONCLUSION

The proposed interval valued thresholds are successful in detecting the beginning of parametric degradation in linear cases and gradual non-linear cases. The diagnostic module formed by interval valued thresholds; is derived from LFT model in derivative causality which detects the beginning of degradation. This in turn, enables the prognostic procedure where in, Interval Extensions of ARRs are used to carry the parametric degradation information in form of interval valued data time- series. Such IE-ARRs consider parametric uncertainty intervals of non-degrading uncertain parameters allowing them to deviate within their prescribed limits. For gradual, linear parametric degradation, the *Centre and Range* method can accurately predict the RUL as taking into account the imprecision brought in by the deviating uncertain parameters. For gradual, non-linear degradation, this method predicts the RUL by approximating the degradation as a linear model in sliding time window framework, with sufficient accuracy. This work does not consider noise brought in by sensor measurements or any external disturbances. Also, it lacks in being robust to outliers while approximating the linear model of degradation. Thus, further development is motivated . The proposed method needs to be developed to deal with non-linear cases, accurately. It should be noted that this methodology is developed in the BG framework of modeling, as it enables a simplified and holistic approach towards multi energetic uncertain dynamic systems.

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