Finite Element based Bayesian Particle Filtering for the estimation of crack damage evolution on metallic panels

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ABSTRACT

A lot of studies are nowadays devoted to structural health monitoring, especially inside the aeronautical environment. In particular, focusing the attention on metallic structures, fatigue cracks represent both a design and maintenance issue. The disposal of real time diagnostic technique for the assessment of structural health has led the attention also toward the prognostic assessment of the residual useful life, trying to develop robust prognostic health management systems to assist the operators in scheduling maintenance actions. The work reported inside this paper is about the development of a Bayesian particle filter to be used to refine the posterior probability density functions of both the damage condition and the residual useful life, given a prior knowledge on damage evolution is available from NASGRO material characterization. The prognostic algorithm has been applied to two cases. The former consists in an off-line application, receiving diagnostic inputs retrieved with manual structure scanning for fault identification. The latter is used on-line to filter the input coming from a real-time automatic diagnostic system. A massive usage of FEM simulations is used in order to enhance the algorithm performances.

1. INTRODUCTION

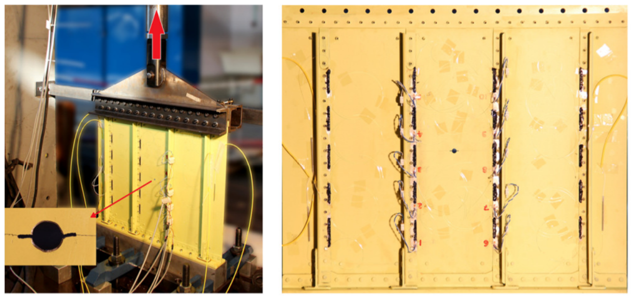
Fatigue crack nucleation and propagation is a major issue when considering aeronautical structures, both from a design (Schmidt & Schmidt-Brandecker, 2009) and maintenance points of view (Lazzeri & Mariani, 2009). From one hand, a proper design is required in order to guarantee the structure damage tolerance or the safe life, depending on the criticality of the selected component. From the other hand, a strict inspection schedule has to be programmed in order to guarantee structural health, due to the uncertainties in the design assumptions for damage nucleation and evolution (material non-uniformities, manufacturing tolerances, not easily predictable load spectrum, uncertainty in stress field knowledge in hot spots, etc.). Moreover, maintenance stops often require dismounting large portions of structure, thus reducing the availability of the aircraft and raising the operative costs.

Real time Structural Health Monitoring (SHM), as part of a complete Prognostic Health Management system (PHM), could potentially reduce the aircraft operative costs, while maintaining a high level of safety (Boller, 2001). A lot of research is thus directed to the development of systems for automatic fault detection, able to perform a continuous on-board inference on structural health. The evolution of Diagnostic Monitoring Systems (DMS) has led to the recognition that predictive prognosis is both desired and technically possible. As a matter of fact, the availability of a huge amount of data coming from DMS, once statistically treated, would allow for a stochastic estimation of the structure Residual Useful Life (RUL) as well as for the estimation of the Probability Density Function (PDF) relative to the current damage state. The approach would allow deciding in real time whether a component must be substituted or repaired, according to some predefined safety parameters.

Bayesian updating methodologies perfectly fit the PHM target (Arulampalam, Maskell, Gordon & Clapp, 2002). Their approach consists in updating the a priori information on RUL (based essentially on material characteristics) according to the actual observations (treated stochastically) taken in real time by the DMS, thus coming to the estimation of the posterior required distributions, conditional on the measures. Unfortunately, it is impossible to analytically evaluate these posterior distributions apart from the cases when the degradation process is linear and the noise is Gaussian (like happens when using Kalman Filters). Focusing on fatigue damage, being crack evolution not a linear process and all the involved uncertainties (comprehending also the measure error) not Gaussian, a numerical approach is suggested. Monte Carlo Sampling (MCS) methods are a valid tool to approximate the required posterior distributions (Cadini, Zio & Avram, 2009). Among them, Particle Filters, also known as Sequential Importance Sampling (SIS) are a MCS method taking its name from the fact that the continuous distributions of interest are approximated by a discrete set of weighted particles, each one representing a Markov Process trajectory of evolution in the state space, being its weight an index of probability of the trajectory itself (Arulampalam et al., 2002). It is however important to consider that, though as the number of samples becomes very large, the MCS characterization of the PF approaches the optimal Bayesian estimate. In addition, Sequential Importance Resampling (SIR) algorithm is a similar technique which allows for particle resampling when the initially drawn samples are not able to describe with sufficient accuracy the system dynamics. In this case, new particles are usually sampled taking into account the information about the system gained up to the resampling instant.

It is however important to consider the two main differences raising when considering real time DMS based upon a network of sensors installed over the structure with respect to classical Non Destructive Technologies (NDT) used to manually scan the structure during maintenance stops (scheduled or unscheduled). The first point is related to the target damage dimension that can be identified. NDTs can detect cracks at a very early stage of propagation, often detecting anomalies in the length order of 1mm or less. On the other hand, the on-board DMS is expected to be designed for a longer target crack length (typically an order of magnitude greater, however strictly dependent on the allowed number and position of sensors as well as on the geometry of the structure that is going to be monitored), like reported by Sbarufatti, Manes and Giglio (2011). This is however in compliance with actual specification requirements for damage tolerance (JSSG, 2006), at least for the aeronautical panel structure which is going to be tested inside this framework (Figure 1). The second point concerns the uncertainty related to the provided measure. Obviously, the variance of damage inference that can be obtained with a manual scan over the entire structure is by far more precise with respect to the PDF of the damage state estimated with a smart sensor network, due to the complicated algorithms for data fusion and damage characteristic evaluation.

The work reported inside this paper is about the development and testing of a Particle Filtering algorithm for the prognosis of aeronautical stiffened skin panels. The aim of the work is to appreciate the advantages due to the application of PF for the estimation of RUL, as a comparison with a classical methodology for the estimation of fatigue crack evolution. Moreover, this work represents the final testing of a complete PHM system that also comprehends an automatic DMS for the real time evaluation of damage. A real dynamic crack propagation test has been executed, with acquisition from a network of 20 FBG strain sensors (Figure 1), with contemporaneous manual crack length track. A detailed and validated Finite Element model of the structure under monitoring has been developed and used in a massive way inside both the DMS and the PF algorithm. PF has been applied separately to two cases. The former, namely off-line PHM, consists in providing as input for the PF the crack lengths manually recorded (with an hypothesis of the associated distribution). Concerning the second case, namely on-line PHM, as anticipated, the output of the real time DMS (processing the signal from the sensor network) is given as input to the PF algorithm. The two approaches have been compared, providing some comments on relative performances. To be noticed that the present article is focused on the prognostic part of the SHM, while the interested reader could refer to the work of Sbarufatti, Manes and Giglio (2012) for a detailed description of the DMS design and performances (taken as input for the current paper).



(a) (b)

Figure 1. (a) Test rig for dynamic crack propagation test starting from a notch artificially initiate on the aluminum panel structure. (b) Typical aeronautical stiffened skin panel structure with sensor network for diagnosis installed (20 FBG strain sensors)

In particular, a brief overview of PF theory is provided in section 2 of the present paper, followed by a description of the stochastic crack propagation model and the measurement model, respectively presented in sections 3 and 4. The PF theory has been tested for the off-line and on-line PHM, reporting results inside section 5. A conclusive section is also provided.

2. Overview of Particle Filter theory

When modeling the behavior of dynamic systems under degradation, at least two models are required (Cadini et al., 2009). Firstly, a model describing the sequential evolution of the state (or the system model) and, second, a model relating the noisy measurements to the state (or the measurement model). The former consists of a hidden Markov process describing the health state $\{\mathbf{x}_k; k = 1{:}N\}$, or the Transition Density Function (TDF) f that relates the health state at time k-1 to the condition at instant k. It consists in a Discrete time State Space (DSS) model. The latter is the equation describing the distribution of the observations $\{\mathbf{y}_k; k = 1:N\}$, or the statistical function h that relates the condition of the monitored component to its noised measure at time stamp k. In a Bayesian framework, all the relevant information about the state can thus be inferred from the posterior distribution of the state $\mathbf{x}_k$, given the history of collected measurements $\mathbf{y}_{1\ldots k}$. This is true also concerning Particle Filters, apart from the fact that the posterior distributions are estimated by means of MCS from f and h. What follow are the basic steps of the mathematical formulation of PF theory, while for a deeper description the interested reader could refer to a tutorial on particle filter theory (Arulampalam et al., 2002). The DSS and measurement models will be thoroughly defined inside the following section.

Given the stochastic damage evolution can be described through the TDF, the aim of the PF is the selection of the most probable damage state x_k at current time k (or in alternative the entire damage state history up to k), according to the noisy measurements that have been collected up to the current discrete time k. This means estimating the posterior PDF of the health state at k, like reported in Eq. (1), which is valid for the entire state sequence up to k.

$$\mathrm{p}(\mathbf{x}_{0:k}|\mathbf{y}_{0:k}) = \int \mathrm{p}(\boldsymbol{\alpha}_{0:k}|\mathbf{y}_{0:k})\delta(\boldsymbol{\alpha}_{0:k} - \mathbf{x}_{0:k})d\boldsymbol{\alpha}_{0:k} \tag{1}$$

Equation (1) indicates that the posterior PDF of the health state can be expressed as an integral inside the space of all possible damage evolutions $\boldsymbol{\alpha}_{0:k}$, where only those propagations similar to the target evolution $\mathbf{x}_{0:k}$ give contribution. According to MCS theory, the integral could be solved by sampling $\boldsymbol{\alpha}_{0:k}$ from the true posterior PDF $\mathrm{p}(\mathbf{x}_{0:k}|\mathbf{y}_{0:k})$. Unfortunately, this is not possible, being that distribution the objective of the inference. Thus, SIS-SIR technique is a well-established method to overcome this problem. The method allows generating samples from an arbitrarily chosen distribution called Importance Density Function (IDF) $q(\mathbf{x}_{0:k}|\mathbf{y}_{0:k})$, allowing to rewrite Eq. (1) in the form of Eq. (2), without applying any bias to the required $\mathrm{p}(\mathbf{x}_{0:k}|\mathbf{y}_{0:k})$.

$$\mathrm{p}(\mathbf{x}_{0:k}|\mathbf{y}_{0:k}) =$$
$$= \int \mathrm{q}(\boldsymbol{\alpha}_{0:k}|\mathbf{y}_{0:k})\frac{\mathrm{p}(\boldsymbol{\alpha}_{0:k}|\mathbf{y}_{0:k})}{\mathrm{q}(\boldsymbol{\alpha}_{0:k}|\mathbf{y}_{0:k})}\delta(\boldsymbol{\alpha}_{0:k} - \mathbf{x}_{0:k})d\boldsymbol{\alpha}_{0:k} \tag{2}$$

An estimation of Eq. (2) can be derived through MCS (based on q distribution), thus coming to Eq. (3), where $\mathbf{x}^{\mathrm{i}}_{0:k}$, $\mathrm{i} = 1,2,\ldots,\mathrm{N_s}$ is a set of N_s independent random samples (particles) drawn from $\mathrm{q}(\mathbf{x}_{0:k}|\mathbf{y}_{0:k})$ and δ is the so called Dirac delta function. Finally, $\mathrm{w}_k^{(i)}$ are the importance weights calculated as the ratio between p and q distributions, each one relative to the i^{th} particle (possible propagation history) and valid for the k^{th} discrete instant.

$$\hat{\mathrm{p}}(\mathbf{x}_{0:k}|\mathbf{y}_{0:k}) = \frac{1}{\mathrm{N_s}}\sum_{i=1}^{\mathrm{N_s}} \mathrm{w}_k^{*(i)}\delta\left(\mathbf{x}_{0:k} - \mathbf{x}_{0:k}^{(i)}\right) \tag{3}$$

Equation (3) expresses the required posterior PDF as a combination of the weights associated to each particle (or to each damage propagation sample). After some mathematical transformations available in literature (Arulampalam et al., 2002), one could express $\mathrm{w}_k^{*(i)}$ as a recursive formula dependent on the weights that have been calculated at previous discrete time k-1, as reported inside Eq. (4), where $\mathrm{w}_k^{(i)}$ are called Bayesian Importance Weights and are calculated like in Eq. (5).

$$\mathrm{w}_k^{(i)} = \mathrm{w}_{k-1}^{(i)}\frac{\mathrm{p}\left(\mathrm{y}_k|\mathrm{x}_k^{(i)}\right)\mathrm{p}\left(\mathrm{x}_k^{(i)}|\mathrm{x}_{k-1}^{(i)}\right)}{\mathrm{q}\left(\mathrm{x}_k^{(i)}|\mathbf{x}_{0:k-1}^{(i)},\mathbf{y}_{0:k}\right)} \tag{4}$$

$$\mathrm{w}_k^{(i)} = \mathrm{w}_k^{*(i)}\mathrm{p}(\mathbf{y}_{0:k}) \tag{5}$$

Inside Eq. (4), $\mathrm{p}\left(\mathrm{x}_k^{(i)}|\mathrm{x}_{k-1}^{(i)}\right)$ is the TDF (f) indicating the statistical correlation between two consecutive steps of damage evolution. Moreover, $\mathrm{p}\left(\mathrm{y}_k|\mathrm{x}_k^{(i)}\right)$ is the probability of having a certain measure at k, given a state sample is considered among the particles propagated up to k. This is available once the measurement model (h) is statistically described, like described inside section 4. Finally, $\mathrm{q}\left(\mathrm{x}_k^{(i)}|\mathbf{x}_{0:k-1}^{(i)},\mathbf{y}_{0:k}\right)$ is the IDF from which one has to sample in order to generate particles, or the random Markov Process describing the damage evolution, which can be arbitrarily selected.

The choice of IDF distribution is a crucial step for the PF algorithm design. In fact, the algorithm convergence is mathematically demonstrated to be independent from the choice of IDF given a sufficient number of samples is generated. If the allowed number of samples is limited, due to computational requirements, the algorithm performances are dependent on the choice of the importance density function. However, as a first approximation, it is often worth trying to select the IDF equal to the TDF (Bootstrap approximation (Haug, 2005)). This would allow for a strong complexity reduction of Eq. (4) as IDF and TDF will be simplified. This means generating particles according to the prior knowledge on material properties (however statistically defined), then updating weights identifying the most suitable samples according to the measure distribution and history. Nevertheless, it could happen that the real propagation that is measured behaves like an outlier with respect to the stochastic damage propagation, thus forcing almost all the particle weights to zero. When this happens, resampling of particles is required, from a different IDF, somehow taking into account the history of measurement collected up to the resampling instant.

Finally, once the health state PDF is approximated assigning an importance weight to each particle, also the distribution of the Failure Cycle (N_f) can be updated and refined, conditioned on the health state, like expressed in Eq. (6), thus allowing for the estimation of the updated RUL distribution.

$$p(N_f|\boldsymbol{y}_{0:k}) = \frac{1}{N_s}\sum_{i=1}^{N_s} w_k^{*(i)}\delta(N_f - N_{f,k}^{(i)}) \tag{6}$$

3. THE DISCRETE TIME STATE SPACE MODEL

DSS is the model describing the a priori knowledge of probabilistic damage evolutions (particles). In other words, it represents the possibilities for damage evolution (given the uncertainties in material characterization as well as the noise inevitably present inside the operating environment), from which the algorithm selects the samples that best fit with the measures. The model used inside the current framework for damage propagation is based on the NASGRO Eq. (7), though other less complicated models such as Forman law or Paris equation (Budynas & Nisbett, 2006) have been usually adopted in literature for crack propagation prognosis (Cadini et al., 2009). NASGRO law allows describing not only the stable crack propagation, but also damage initiation and the unstable crack evolution. It also takes into account the load ratio (R) of the applied spectrum, defined as the ratio between the valley and peak values of the load cycle, as well as the crack closure effect induced by plasticity near the crack tips.

$$\frac{da}{dN} = C \cdot \left[\left(\frac{1-f}{1-R}\right)\cdot \Delta K\right]^m \cdot \frac{\left(1-\frac{\Delta K_{th}}{\Delta K}\right)^p}{\left(1-\frac{\Delta K_{max}}{K_c}\right)^q} \tag{7}$$

Inside Eq. (7), a is the crack dimension and da/dN represents the crack growth rate per cycle (N). ΔK is the variation of the Stress Intensity Factor (SIF) inside one load cycle, calculated as the difference between the SIFs evaluated in correspondence of the maximum and minimum load. Moreover, ΔK_{th} is the threshold variation of SIF (crack shouldn't propagate below ΔK_{th}), K_c is the critical value of SIF (fracture toughness) and f is the crack opening function. Finally, C, m, p and q are parameters defined for material characterization. The interested reader could refer to NASGRO reference manual (2005) for a deeper insight to the parameter definition.

Equation (7) allows calculating the crack growing rate as a function of the applied load cycle, given the needed constant are defined. Some comments arise relative to the work presented hereafter. First of all, to develop a methodology as general as possible, SIFs have not been calculated with simple analytical formulas (usually valid for simple skins). A large database of FEM simulated damages has been generated, collecting SIF parameters for each case. An Artificial Neural Network has been trained in order to fit the function that relates the crack position and dimension to the SIF at crack tips. The method would allow evaluating crack propagation also for complex geometries, obviously given a validated FEM is available (the subject of current monitoring is an aluminum skin, stiffened through some riveted stringers, with crack propagating on the skin).

Moreover, Eq. (7) has been stochastically described by means of some experimental data available in literature [Giglio & Manes, 2008]. In particular, C and m parameter distributions have been derived from a crack propagation test campaign made on aluminum structures. While simulating crack propagation with Eq. (7), C and m are randomly sampled at each step of crack evolution, thus obtaining a model that relates the health state at discrete instant k-1 to the condition at k, or the Transition Density Function shown in Eq. (8). A Gaussian noise has also been introduced, like described by Cadini at al. (2009).

$$\mathrm{p}(\mathrm{x_k}|\mathrm{x_{k-1}}),\ \forall \mathrm{k} \geq 0 \tag{8}$$

Thus, the probabilistic a priori information on damage evolution is shown inside Figure 2, where the real crack propagation (over structure presented in Figure 1) is reported together with the random Markov Process evolution of the simulated damage. In particular, the initial

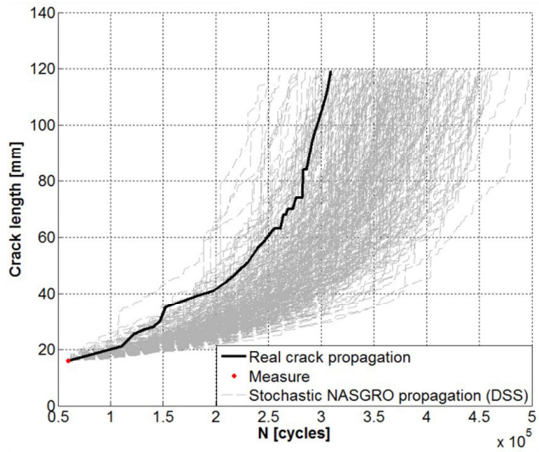


Figure 2. NASGRO DSS model for off-line PHM. Comparison of particles with real crack propagation measured during experiments. Particles have been generated starting from a 16mm measure, corresponding to the length of the artificially initiated crack.

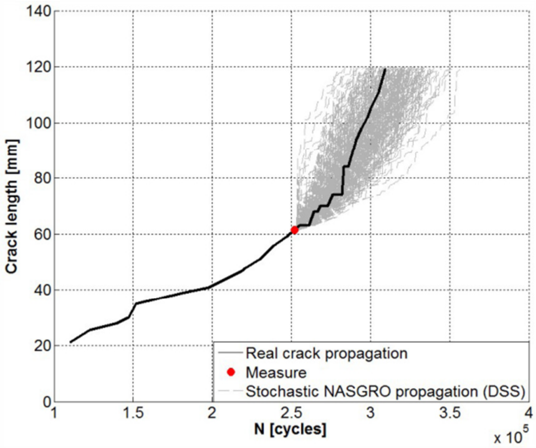


Figure 3. NASGRO DSS model for on-line PHM. Comparison of particles with real crack propagation measured during experiments. Particles have been generated starting from a 60mm measure, corresponding to the length of the crack in correspondence of the anomaly detection by the automatic diagnostic unit.

crack length has been set to 16mm, corresponding to the artificial notch introduced to fasten crack nucleation and to control crack position. As one could notice, the random simulated crack propagation covers a very wide range of possibility, including also the real case measured during test. An efficient algorithm (based on probability theory) is thus needed in order to select which are the particles that best fit the reality, given some measures (with noise and uncertainty) have been taken, thus reducing the uncertainty on the RUL estimation. The DSS model presented inside Figure 2 will be adopted when considering the application of PF to the off-line PHM system (measurements are manually collected during maintenance stops). On the other hand, Figure 3 shows the stochastic simulation of crack propagation for the on-line case (measures of crack length are estimated by a sensor network installed over the structure). Simulated crack propagation has been initiated after the anomaly detection is performed by the automatic diagnostic system (about 60mm for the sensor network Vs. damage configuration shown in Figure 1). The first thing to be noticed is the reduced dispersion of particles in Figure 3 with respect to Figure 2, being the model initiated in correspondence of a longer crack length. Moreover, the random process of simulated crack propagation appears to be centered on the real damage evolution in Figure 3, where the randomness of damage evolution from 16mm to about 60mm has not been considered.

4. THE MEASUREMENT SYSTEM

Two measurement systems have been adopted, trying to analyze the PF algorithm performances when off-line and on-line PHMs are going to be considered (Figure 4).

Off-line PHM simulates the case when the aircraft is stopped for maintenance and the structure is manually scanned by operators for crack identification. In the case a damage tolerant structure is considered, the aim is to identify if it is possible to postpone dismounting and repairing until the prognostic system declares a critical condition. In order to statistically characterize the off-line measure, it has been decided in first approximation to consider the measurement system PDF Gaussian, with mean value equal to the real crack length (measured with a caliber during the real test). Nevertheless, a standard deviation (σ_{off}) has also been selected so that the 95% confidence band is inside the ±3% range with respect to the measure.

On the other hand, the on-line PHM simulates the case when the structural health condition is automatically inferred by means of a diagnostic unit that processes data coming from a smart sensor network. The concept consists in maintaining the aircraft operative until the PHM system declares further operations unsafe, given a predefined safety parameter. The diagnostic unit used inside the current framework has been thoroughly described by Sbarufatti et al. (2012). It basically consists of two Artificial Neural Networks (ANN), trained with FEM simulations in order to understand the complex functions that relate the damage parameters (existence, position and length) to the strain field modifications due to damage. The first ANN (anomaly detection algorithm) receives strain data as input and generates an alarm when the damage index (ranging from 0 to 1) falls above 0.5. The second algorithm (damage quantification), activated in series to the anomaly detection, receives again strain data and gives crack length distribution [1] as output (a deeper explanation about diagnostic unit output is again provided by Sbarufatti et al. (2012)).

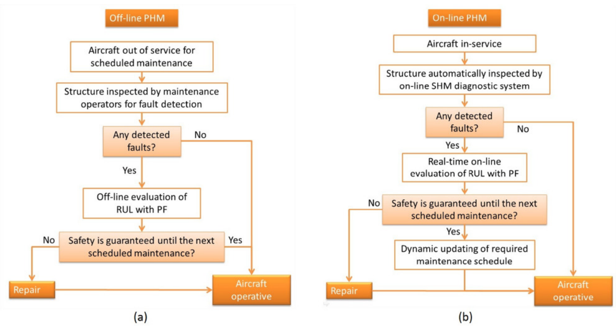


Figure 4. Comparison between (a) the Off-Line PHM procedure and (b) the On-Line PHM process. The On-Line process is based upon the diagnosis performed through an on-board SHM system that detects and characterizes structural faults.

[1] The quantification algorithm is composed by 50 ANNs, trained with randomly selected damage samples (with random position and length). Each one receives the strain pattern from the FBG acquisition system and returns an estimation of crack length.

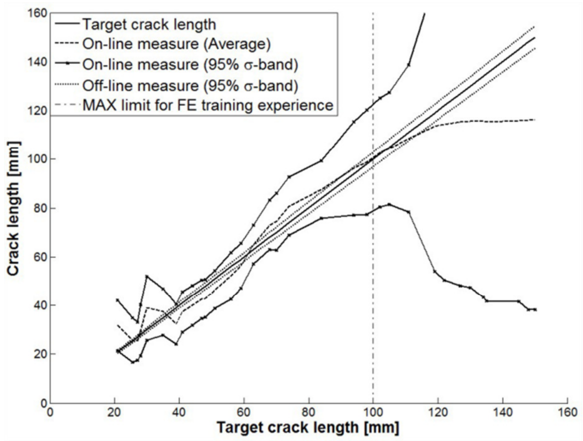


Figure 5. Measurement system uncertainties. Comparison of the on-line diagnostic system performance with respect to the off-line manual structural scan methodology. The on-line diagnostic system has been trained with FEM damage simulations, with crack length up to 100mm.

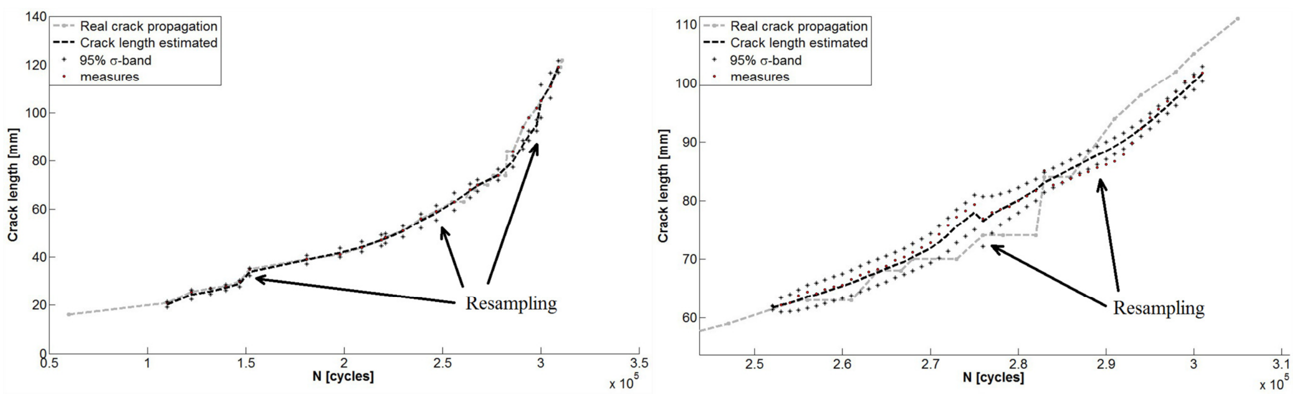
The PF algorithm is thus activated after the anomaly is detected and an estimation of the damage state distribution is provided from the diagnostic algorithm.

A comparison of the on-line vs. off-line measurement system is provided in Figure 5. It can be noticed that the ±2σ-band adopted to simulate the behavior of a generic system for manual surface scan is by far narrower with respect to the uncertainty correlated to the real-time automatic diagnostic system. For instance, considering a 70mm target crack length, the ±2σ-band ranges between 63mm and 86mm for the on-line diagnosis, while ranging between 67.5mm and 72.5mm for the off-line measure. However, it can be noticed that the average value of the quantification distribution correctly estimate the target crack length. The strong degeneracy for the σ-band of the on-line measure of longer cracks is due to the fact that the database of simulated experience used to train the ANN algorithms for diagnosis has been limited up to 100mm cracks.

5. COMPARISON OF ON-LINE VERSUS OFF-LINE RESULTS

The performances of the PF algorithm when applied to the two maintenance approaches introduced above are now deeply investigated. The main output of the PF probabilistic calculation is the estimation of the health condition of the structure, like reported inside Figure 6 relatively to both off-line and on-line PHM. In few words, the main advantage of the PF technique is that it allows to update the posterior PDF for the damage condition, taking into account the history of all the measures taken up to the k^{th} discrete time instant, as well as the analytical a priori knowledge given by the underlying model for damage evolution. This becomes particularly attractive when autonomous diagnostic systems are considered. As a matter of fact, they could provide continuous information relative to damage existence and level; nevertheless they are characterized by a robustness and precision inferior with respect to classical NDT technologies (herein simulated with off-line measures). In practice, PF could filter the most suitable states at k^{th} instant, inside the database of possible damage evolutions (particles) calculated a priori with respect to any measure. Particles relative to the off-line and on-line PHM have been shown in Figure 2 and Figure 3 respectively. Once the actual state distribution is updated and refined, the distribution of the RUL could also be updated, becoming conditional on the whole history of the monitored component, and consistent with the analytical and empirical knowledge which is inside the TDF.

The state posterior PDF estimation is shown inside Figure 6, relatively to the off-line (Figure 6(a)) and on-line (Figure 6(b)) cases. PF has been applied to a real crack propagation test, with contemporaneous manual acquisition of crack



(a) (b)

Figure 6. Filtering of the health state distribution. (a) Posterior PDF of the health state for the off-line measure and (b) Posterior PDF of the health state for the on-line structural diagnosis. The real crack propagation is shown, as well as the collected measures. The posterior 95% σ-band is also plotted, to be compared with the a priori σ-band reported inside Figure 5. The instants when the algorithm required particle resampling have also been indicated.

length measures (processed in Figure 6(a)) and automatic estimation of crack measure by means of an on-board smart sensor network based upon strain field (processed in Figure 6(b)). It is immediately clear that, while the manual structure scan would allow to detect and to measure shorter cracks (the inferior limit is imposed herein by the length of the artificial damage for crack initialization, set to 16mm), the anomaly detection threshold for the sensor network and damage configuration reported in Figure 1 is around 60mm. On the other hand, off-line measures are available at predefined scheduled intervals, while the on-line health assessment is retrieved in continuous every 1000 load cycles through the diagnostic unit developed by Sbarufatti et al. (2012). However, on-line measures are affected by a large uncertainty if compared to the off-line case, like described into Figure 5.

Concerning the off-line PHM system, the health state estimation (Figure 6(a)) appears to characterize precisely the damage evolution, being the 95% σ-band mostly centered on the real damage condition. However, it is clear from Figure 2 that the damage evolution occurred during the test is not centered with respect to the stochastic model used to define the TDF. This resulted in resampling requirement after few updating iterations, as the available particles were not enough to describe the posterior PDF of the health state (only few particles retains a weight which is significantly different from zero).

Relating to the on-line PHM system, it can be noticed that the posterior PDF of the health condition is by far narrower with respect to the output of the diagnostic algorithm shown inside Figure 5. For instance, relatively to a 70mm crack, the 95% σ-band of the quantification algorithm (Figure 5) ranges from 63mm to 86mm, while after the PF updating process it ranges from 68mm to 72.5mm (Figure 6(b)). However, the estimated σ-band sometimes doesn't comprehend the real state evolution. This is mainly due to the fact that the measures are affected by a higher error (with respect to the off-line system), which is in part confirmed by the evolution of some stochastic particles. This means that, if a lot of measures over/underestimate the real damage condition and their assumptions are also confirmed by the DSS model, the PF precision will decrease. However, under the reasonable assumption (Figure 5) that the measure PDF is centered on the target, the PF inference will converge toward the real damage evolution. In other words, PF tends to interpolate the measures, nevertheless taking into account the a priori knowledge which is inside the DSS model. Though the DSS model used for the a priori description of the damage evolution for the on-line PHM results centered on the real crack propagation (Figure 3), particle resampling was also required, due to the fact that the updating process focused on a particular set of particles.

Some specifications are required concerning the adopted resampling technique. As a matter of fact, the DSS model used to initialize the algorithm has been kept as general as possible (considering the distribution of material parameters inside the NASGRO law), in order to be representative of many experimental tests for crack propagation on the same material (aluminium). The resulting DSS spreading is high, thus provoking premature particle degeneracy and requirement for resampling. Nevertheless, if a sufficient number of iterations have been concluded, it is possible to generate new particle samples from a different importance density $q\left(x_k^{(i)} \middle| \mathbf{x}_{0:k-1}^{(i)}, \mathbf{y}_{0:k}\right)$, taking now the history of measures into account but preventing from the possibility to adopt Bootstrap approximation. Concerning the work herein reported, new particles are generated considering a TDF with deterministic material parameters (C and m are now obtained by fitting the specific measures taken relatively to the specimen under monitoring) and random white noise. From one hand, this would allow to reduce the uncertainty

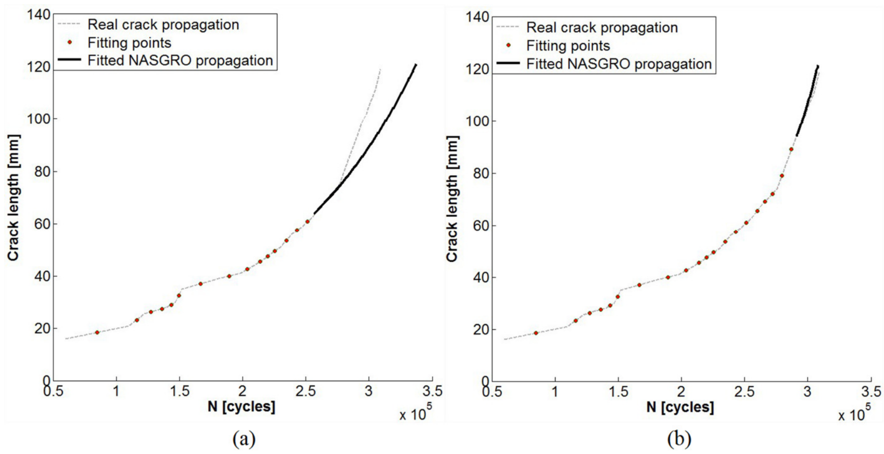
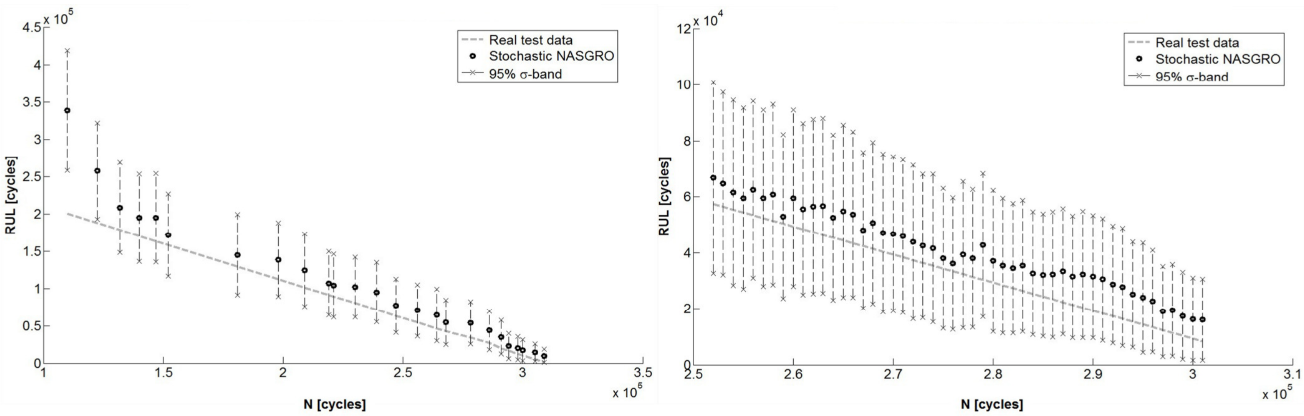


Figure 7. Effect of NASGRO parameter dynamic fitting. A sudden (unpredicted) change in the slope of the crack propagation curve cannot be described before it has happened.

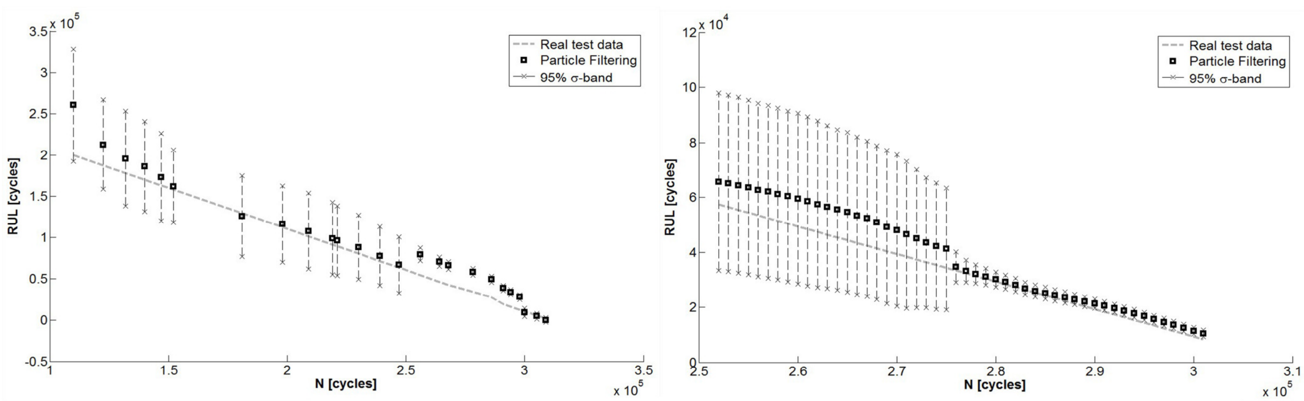
related to prognosis. From the other hand, like described into Figure 7 (where the noise has been eliminated for just description purposes), this method is less robust to unexpected changes in the system dynamics. It is clear from Figure 7 that, if C and m are considered to be deterministic, they cannot take into account for sudden changes in the curve slope (Figure 7(a)), unless a new resampling is executed fitting the propagation curve with new measures (Figure 7(b)). The effect is visible in the RUL estimation, relative to the off-line PHM case (Figure 9(a)); the error in RUL estimation with PF increases after resampling is executed at 250000 load cycles, until a new resampling is executed at about 300000 load cycles, taking into account the unexpected change in the crack evolution slope.

Once the PDF of the health state is filtered by the PF algorithm, also the RUL of the component under monitoring can be updated according to Eq. (6). In order to appreciate the advantages and drawbacks of the PF algorithm, it has been compared with a second technique. The method consists in evaluating the RUL PDF by performing a stochastic crack propagation based on the NASGRO law. In few words, given the PDFs of the material related constants are provided, 3000 crack propagations (particles) have been simulated, sampling at each step the material constants from the available distributions. Once the target crack length is identified (120mm have been selected as limit crack length, due to the limits of the FEM database), the RUL can be stochastically defined with a PDF. The same procedure is repeated each time a new estimation of the crack length is provided either from the on-line or the off-line diagnostic system. To be noticed that this method just depends on the last measure provided by diagnostic and doesn't take into account the trend of historical measures (which is, on the contrary, the advantage of PF). Each inference is thus completely uncorrelated to the previous ones. Moreover, it requires simulating many crack propagation every time a new RUL PDF is needed. Stochastic NASGRO (SN) and Particle Filter RUL evaluations are respectively reported in Figure 8 and 9, again relatively to the on-line and off-line



(a) (b)

Figure 8. 95% σ-band for RUL estimation with stochastic NASGRO law. Comparison of off-line PHM (a) versus on-line PHM (b).



(a) (b)

Figure 9. 95% σ-band for RUL estimation with Particle Filtering algorithm. Comparison of off-line PHM (a) versus on-line PHM (b).

PHM. The estimated RUL (intended hereafter as the remaining number of cycles before reaching the 120mm long crack) is reported during the component life (as a function of load cycles). The real RUL is shown as well as its estimation calculated with SN law (Figure 8) and PF (Figure 9). In particular, the expected value of the RUL PDF has been reported, as well as the 95% σ-band. The first thing to be pointed out is that SN only depends on the knowledge of material properties (and applied load); for this reason, if a discrepancy between the DSS and reality is present at the beginning, there won't be an updating process on the basis of the collected measures, thus maintaining the same error during life, as clearly appreciable from Figure 8(b). Moreover, the SN prognosis is very sensitive to the quality of the measure, being an issue especially when the on-line PHM is considered, where the inevitable fluctuations in the inference on structural condition (due to the high level of uncertainties) will be reflected in an unstable prognosis (Figure 8(b)). On the other hand, PF technique is able to filter these uncertainties (Figure 9(b)), thus estimating a RUL which is dependent on the entire trend of measures that have been collected since the anomaly is identified. The variance of the RUL PDF evaluated with the two prognosis methods appears to be of the same order, unless resampling is performed in PF algorithm. As explained above, the information retrieved from the collected measures would allow decreasing significantly the uncertainty in prognosis (as at least the uncertainty related to material properties can be by far reduced). This is well reflected in Figure 9(b) where an important reduction in the variance of PF estimation of RUL is obtained. After 275000 load cycles, only few particles remained with a non-negligible weight, thus provoking degeneracy of the algorithm. New particles have thus been generated, nevertheless without considering the material uncertainty inside the DSS (C and m parameters inside the NASGRO equation are deterministic and obtained through a non-linear fitting of the historical data available up to resampling instant). Nevertheless, the resampling technique has to be improved in order to avoid focusing in a too narrow region inside the DSS. In fact this is the reason for the deviation of the estimated RUL PDF from real RUL inside Figure 9(a), like described in Figure 7.

Finally, two comments arise while comparing off-line versus on-line PHM. Firstly, the 95% σ-band of the RUL based on the off-line measure is narrower due to the more precise measuring system. Nevertheless, the disposal of a real-time diagnostic tool would increase the availability of data relative to the health state, thus reducing the time needed to the PF algorithm to converge on the correct estimation.

6. CONCLUSIONS

A Particle Filtering (PF) Bayesian updating technique has been used inside this framework for the dynamic estimation of component Residual Useful Life. Two applications have been compared. The first one consists in applying particle filters to a Condition Based Maintenance where the structural health monitoring (SHM) has been off-line performed by maintenance operators. The second one consists in an automatic SHM performed on-board by a diagnostic unit trained with Finite Element damage simulation to recognize crack damage existence and length, based upon strain field measure. The methodology has been tested in laboratory on a specimen representative of a typical aeronautical structure, constituted by a skin, stiffened through some riveted stringers. Though the uncertainty related to the on-line structural diagnosis is by far larger than the one associated to the off-line measure, PF algorithm proved to correctly describe the posterior RUL distribution (conditional on the measures) in both cases. The additional uncertainty in the on-line measures resulted to be compensated by the availability of a continuous measure, thus allowing the algorithm to reach convergence in a relatively inferior time. PF algorithm has also been compared to a simpler technique based upon stochastic NASGRO (SN) law propagation. The advantage of PF with respect to SN is that it takes into account the whole history of measures taken on the monitored component as well as the prior knowledge coming from the propagation model. This results in a more robust and precise estimation of the health state as well as of the RUL PDF. Finally, the adoption of a robust filtering methodology that merges the information coming from a wide sensor network with the numerical or analytical knowledge about the phenomenon subject of monitoring appears to be a suitable technique for the performance increase of automatic SHM systems, thus leading toward the real on-board PHM.

NOMENCLATURE

ANN	Artificial Neural Network
DMS	Diagnostic Monitoring System
DSS	Discrete State-Space
FBG	Fiber Bragg Grating
FEM	Finite Element Model
IDF	Importance Density Function
MCS	Monte-Carlo Sampling
NDT	Non Destructive Technology
PDF	Probability Density Function
PF	Particle Filter
PHM	Prognostic Health Management
RUL	Residual Useful Life
SHM	Structural Health Monitoring
SIF	Stress Intensity Factor
SIR	Sequential Importance Resampling
SIS	Sequential Importance Sampling
SN	Stochastic NASGRO
TDF	Transition Density Function

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