Fast optimization for aircraft descent and approach trajectory

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ABSTRACT

We address the problem of online scheduling of the descent aircraft trajectory. The problem is considered in a general framework of the multiphase optimal control. First, we obtain solution of this problem using traditional approach. Next, we develop novel solution algorithm using two key components: (i) inference of the dynamical and control variables of the descending trajectory from the low dimensional flight profile and (ii) solution of the resulting low-dimensional optimization problem using efficient local search. We show that the developed algorithm is much faster than the traditional one and discuss its future application to the simultaneous optimization of the runway throughput and the descent trajectory for each aircraft in convective weather conditions.

1. INTRODUCTION

In the future air traffic management system, the trajectory becomes the fundamental element of a new set of operating procedures collectively referred to as trajectory-based operations (TBO) (Cate, 2013). The basis for TBO is that each aircraft's expected flight profile and time (or airspeed) information will be specified by a four-dimensional (4D) trajectory (K. H. Shish et al., 2015; Young et al., 2016; K. Shish et al., 2016).

One of the challenges in development of the future TBO is management of the airport congestion especially under convective weather conditions (M. Kamgarpour, W. Zhang, & C.J. Tomlin, 2011). One of the key ingredients to the solution of this problem is fast online optimization of the aircraft descent trajectory. The difficulty in solving this problem stems from the fact that there are multiple phases (configurations) that have to be flown during descent while respecting the system dynamics and satisfying a large number of linear and nonlinear constraints.

The standard approach procedure involves a continuous steady descent starting from 6,000 ft, or higher, which is followed by a steep descent to a set of cleared altitudes and a capture of the 3° glide-slope from below. The speed during final approach is based on the reference speed, V_{ref}, which is calculated on the basis of reference speed for Flaps 30 (for a specific type of the aircraft) and depends on the mass of the aircraft.

In addition, the trajectory planning operation specifies a set of transitions to given flaps and landing gear configurations with corresponding reference speed. The various phases flown by the aircraft during descent are controlled by the pilots and involve a set of nominal actions that are required to enable the designed vertical flight profile including e.g. capturing the localizer and the gliding slope.

Overall, it can be seen that the flight planning problem render itself as a complex multiphase trajectory optimization problem subject to dynamical, path, and control constraints (Betts & Cramer, 1995; C. Tomlin, Lygeros, & Sastry, 2000; C. J. Tomlin, Mitchell, Bayen, & Oishi, 2003; de Jong, 2014; de Jong et al., 2015; Park & Clarke, 2016; de Jong et al., 2017).

There are several numerical methods that can be used to address this probl em, see e.g. surveys on trajectory optimization (Betts, 1998; Rao, 2014). Among these methods the most popular ones are techniques based on so-called direct methods including direct collocation (Hargraves & Paris, 1987) and pseudospectral approach (Fahroo & Ross, 2000). In the direct methods, the stat e and/or control of the system are discretized in time and the problem is reduced to a nonlinear programming problem (NLP) (Rao, 2014).

For example, one of the best known current implementations of the optimization algorithms in an experimental Flight Management System (FMS) developed using General Pseudospectral Optimization Software (GPOPS). It requires ≥ 30 sec to solve the trajectory optimization problem from the top of descent to the runway (de Jong, 2014; de Jong et al., 2015, 2017).

We note, that future implementations of the FMS demand continuous online estimations of the time bounds for each phase of the flight and fast rescheduling of the flight plan in e.g. convective weather conditions. The constraints on the optimization time are even more strict when optimization of the arrival time for each aircraft has to be performed simultaneously with the solution of the runway scheduling problem during e.g. airport congestion. These demands call for a development of alternative fast and robust approaches to the online solution of the multiphase trajectory optimization problem.

Here we present a novel algorithm of the solution of this problem and compare its performance with the performance of a conventional technique.

The proposed technique is based on the observation that during several minutes of the flight along the final approach trajectory the aircraft has to attain several flight phases/configurations each of which has strict bounds on the distance, altitude, and speed of the aircraft. This allows one to reduce the complexity of the problem by parameterizing the aircraft altitude and velocity profiles with minimum number of parameters. These parameters – locations of the phase transition points – are the key decision variables of the reduced NLP.

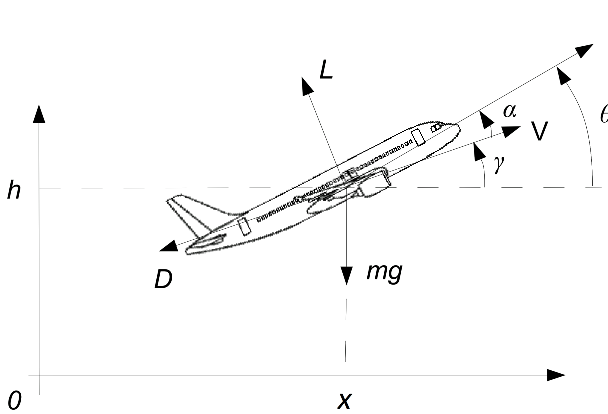

Figure 1. The aircraft forces and angles during vertical flight with nonzero climb rate.

The resulting algorithm is robust and allows fast online multiphase optimization of the vertical landing trajectory. It also paves the way to the simultaneous optimization of the landing trajectory for each aircraft and the runway throughput.

The paper is organized as follows. In the next section we provide the formulation of the multiphase vertical trajectory optimization problem. In Sec. 3 we analyze the solution of this problem using General Pseudospectral Optimization method. In Sec. 4 we describe novel algorithm and its application to the optimization of the final approach to a runway in San Francisco airport. Finally, in Conclusions we summarize the obtained results and discuss future applications of the algorithm.

2. OPTIMIZATION OF THE VERTICAL TRAJECTORY

The goal of the landing trajectory optimization is to design a trajectory that minimizes (or maximizes) some measure of the aircraft performance while satisfying a set of constraints. This problem can be conveniently formulated as an optimal control problem (Betts, 1998).

In the latter case (Betts, 1998; Zhao, 2012) we are given the initial and final states x_0, x_f of the system and initial time t_0 and the problem is to determine the final time t_f, the control input $u(t)$ and the corresponding state history $x(t)$, which minimize the cost functional $J[x(t), u(t)]$ and satisfy a set of dynamical equations set of equality and inequality constraints and set of bounds on control and dynamical variables. In the case of multiphase optimal control problem the dynamical equations, constraints, and bounds are introduced for each phase of the flight.

2.1. Model Equations

The state of the aircraft during descent in the vertical plane (see (Miquel & Suboptimal, 2015) and Figure 1) is defined in our model as

$$x = \{V, \gamma, x, h\}, \tag{1}$$

where V is the speed, γ is the flight path angle, h is the altitude, and x is the distance to the runway.

The aircraft control is represented by two virtual control inputs: thrust T and the angle of attack α.

$$u = \{\alpha, T\}. \tag{2}$$

The set of model parameters p describes the deflection of the flaps δ_{fl} and spoilers δ_{sp} and the gear settings δ_{lg}.

To simplify analysis without loss of generality we neglect the wind and the mass change due to the fuel burned. The resulting model takes the form

$$\begin{aligned} m\dot{V} &= T\cos\alpha - D - mg\,\sin\gamma, \\ mV\dot{\gamma} &= (T\sin\alpha + L) - mg\,\cos\gamma, \end{aligned} \tag{3}$$

$$\begin{aligned} \dot{x}_e &= V \cos \gamma, \\ \dot{h}_e &= V \sin \gamma, \end{aligned} \tag{4}$$

We note that the actual control variable is the pitch rate. However, due to time separation between slow and fast aircraft dynamics, the angle of attack α is considered to be virtual control input (Lombaerts, Schuet, Wheeler, Acosta, & Kaneshige, 2013; Schuet, Lombaerts, Acosta, Wheeler, & Kaneshige, 2014).

The lift L and drag D coefficients in the model are

$$\begin{aligned} D &= \frac{1}{2}\rho V^2 S \left(C_{D_0} + C_{D_\alpha}\alpha + C_{D_{\alpha^2}}\alpha^2 + \right. \\ &\qquad \left. C_{D_{\delta_{sp}}}\delta_e + C_{D_{fl}}\delta_{fl} + C_{D_{lg}}\delta_{lg}\right), \\ L &= \frac{1}{2}\rho V^2 S \left(C_{L_0} + C_{L_\alpha}\alpha + C_{L_{\delta_{sp}}}\delta_{sp} + \right. \\ &\qquad \left. C_{L_{fl}}\delta_{fl} + C_{L_{lg}}\delta_{lg}\right). \end{aligned} \tag{5}$$

where ρ is the air density, S is the net wing surface area, and C_i are the non-dimensional aerodynamic aerodynamic force coefficients.

2.2. Problem formulation

Formally, the descent trajectory optimization requires the following problem to be solved (Betts & Cramer, 1995; C. Tomlin et al., 2000; C. J. Tomlin et al., 2003; Becerra, 2010; de Jong, 2014; Patterson & Rao, 2015; de Jong et al., 2015, 2017). Find the control trajectories, $u^{(i)}(t), t \in [t_0^{(i)}, t_f^{(i)}]$, state trajectories $x^{(i)}(t), t \in [t_0^{(i)}, t_f^{(i)}]$, static parameters $p^{(i)}$, and time $t_f^{(i)}$ that minimize the following performance index (Becerra, 2010):

$$\begin{aligned} J = \sum_{i=1}^{N_p} \Bigg(& \varphi^{(i)}\left[x^{(i)}(t_f^{(i)}), p^{(i)}, t_f^{(i)}\right] + \\ & \int_{t_0^{(i)}}^{t_f^{(i)}} L^{(i)}\left[x^{(i)}(t), u^{(i)}(t), p^{(i)}, t\right] dt \Bigg) \end{aligned} \tag{6}$$

subject to the dynamical constraints:

$$\dot{x}^{(i)}(t) = f^{(i)}\left[x^{(i)}(t), u^{(i)}(t), p^{(i)}, t\right], t \in [t_0^{(i)}, t_f^{(i)}], \tag{7}$$

the path constraints

$$h_L^{(i)} \le h^{(i)}\left[x^{(i)}(t), u^{(i)}(t), p^{(i)}, t\right] \le h_U^{(i)}, \tag{8}$$

for $t \in [t_0^{(i)}, t_f^{(i)}]$, the event constraints:

$$e_L^{(i)} \le e^{(i)}\left[x_0^{(i)}, u_0^{(i)}, x_f^{(i)}, u_f^{(i)}, p^{(i)}, t_0^{(i)}, t_f^{(i)}\right] \le e_U^{(i)}, \tag{9}$$

the phase linkage constraints:

$$\begin{aligned} & \Psi_l \le \Psi[x_0^{(1)}, u_0^{(1)}, x_f^{(1)}, u_f^{(1)}, p^{(1)}, t_0^{(1)}, t_f^{(1)} \\ & \qquad \vdots \\ & x_0^{(N_p)}, u_0^{N_p)}, x_f^{N_p)}, u_f^{(N_p)}, p^{(N_p)}, t_0^{(N_p)}, t_f^{(N_p)}] \le \Psi_u \end{aligned} \tag{10}$$

where $x_{0,f}^{(i)} x^{(i)}(t_{0,f}^{(i)})$ and $u_{0,f}^{(i)} u^{(i)}(t_{0,f}^{(i)})$

the bound constraints:

$$\begin{aligned} & u_L^{(i)} \le u^{(i)}(t) \le u_U^{(i)}, \quad t \in [t_0^{(i)}, t_f^{(i)}] \\ & x_L^{(i)} \le x^{(i)}(t) \le x_U^{(i)}, \quad t \in [t_0^{(i)}, t_f^{(i)}] \\ & p_L^{(i)} \le p^{(i)} \le p_U^{(i)}, \\ & \underline{t}_0^{(i)} \le t_0^{(i)} \le \bar{t}_0^{(i)}, \\ & \underline{t}_f^{(i)} \le t_f^{(i)} \le \bar{t}_f^{(i)}, \end{aligned} \tag{11}$$

and the time constraints:

$$t_f^{(i)} - t_0^{(i)} \ge 0. \tag{12}$$

In each phase functions and variables in (6)-(11) have appropriate dimensions that may change from phase to phase.

The corresponding dynamical and control variables and the dynamical constraints (dynamical equations) are defined for our model in the previous subsection. Index $i = 1, ..., N_p$ runs through the number of phases, which are defined together with the corresponding bounds on the dynamical and control variables in the following subsections.

2.3. Flight phases

The following problem will be considered. Optimize the final approach trajectory using objective function (6) for the descending flight between CEPIN and the stabilized approach fix (500 ft) of the runway (RWY) 28R in San Francisco airport (SFO). The phases included into the analysis are shown in the Table 1.

This simplified schedule (in a sense that it does not involve pilot - air traffic control interaction) was developed by taking into account standard requirements for the aircraft descent to the given runway, see e.g. (Prats et al., 2014). In development of this schedule it was assumed that the localizer (LOC) signal extends 18 nm away from and 4500 ft above the antenna site (see e.g. FAA-S-8081-9B, June 2001). It was also assumed that the glide slope capture is initiated $2 \div 0.75$ nm away from DUMBA in a level flight. In addition, only continuous descent operations were considered, increase of the acceleration and altitude during descent were excluded from this optimization test.

A number of optimization problems can be formulated within this framework. The critical parameters of interest during airport congestions are earliest and latest time of arrival. A parameter of common interest is fuel consumption. In this work we were primarily interested in scheduling transitions times between various descent phases and estimation of the earliest and latest transition time for each phase.

Table 1. Phases included into the first test. Here ARCHI, AXMUL, CEPIN, DUMBA, GIRRR, and ZILED are the names of the waypoints near San Francisco airport (SFO).

#	Name	ALT (ft)	DST (nm)	SPD (kn)
1	initial state	10000	29	230
2	Descent to ARCHI	8000 $\leq$ h $\leq$ 10000	29 $\leq x \leq$ ARCHI	175 $\leq$ V $\leq$ 230
3	Descent to ZILED	6000 $\leq$ h $\leq$ 10000	ARCHI $\leq x \leq$ ZILED	175 $\leq$ V $\leq$ 230
4	Flaps5	5000 $\leq$ h $\leq$ 10000	ZILED $\leq x \leq$ GIRRR	175 $\leq$ V $\leq$ 230
5	LOC capture	4000 $\leq$ h $\leq$ 4500	GIRRR $\leq x \leq$ DUMBA	175 $\leq$ V $\leq$ 230
6	Flaps15	1800 $\leq$ h $\leq$ 4500	DUMBA $\leq x \leq$ CEPIN	165 $\leq$ V $\leq$ 215
7	Flaps20	1800 $\leq$ h $\leq$ 4500	CEPIN $\leq x \leq$ AXMUL-2	165 $\leq$ V $\leq$ 195
8	Gear down	1800 $\leq$ h $\leq$ 4500	AXMUL-3 $\leq x \leq$ AXMUL	165 $\leq$ V $\leq$ 195
9	GS capture	1800 $\leq$ h $\leq$ 2000	AXMUL-2 $\leq x \leq$ AXMUL+1	165 $\leq$ V $\leq$ 195
10	Flaps25	DST $\times$ $tn(\gamma \pm \delta\gamma)$	AXMUL $\leq$ x $\leq$ AXMUL+3	155 $\leq$ V $\leq$ 185
11	Flaps30	DST $\times$ $tn(\gamma \pm \delta\gamma)$	AXMUL+2 $\leq x \leq$FIX	150 $\leq$ V $\leq$ 170
12	stabilized appr fix	446	1.4	150

We will now consider solution of this problem using traditional technique and a freely available package GPOPS (Rao et al., 2010)

3. EXAMPLE OF NUMERICAL SOLUTION USING GPOPS

The GPOPS package is easy to install and to use. It was shown to perform well for multiple aerospace applications including optimization of the descent aircraft trajectory at National Aerospace Laboratory (Netherlands). We have chosen this package for the evaluation of the traditional technique of the solution of the scheduling problem. We have initially chosen a subset of phases from the Table 1 because the convergence of the algorithm was slow.

Here we provide an example of optimization for descending trajectory with the following five phases

1. flaps 15 gear up; $V_{max}^{(1)}$ = 215 kn; $V_{min}^{(1)}$ = 95.3 kn;
2. flaps 15 gear down; $V_{max}^{(2)}$ = 215 kn; $V_{min}^{(2)}$ = 95.3 kn;
3. flaps 20 gear down; $V_{max}^{(3)}$ = 195 kn; $V_{min}^{(3)}$ = 91.1 kn;
4. flaps 25 gear down; $V_{max}^{(4)}$ = 185 kn; $V_{min}^{(4)}$ = 87.5 kn;

Note, that the method allows for many different types of objective function (Rao et al., 2010). For example, the final altitude in each phase could be minimized or maximized using functions similar to (14) and (15) and substituting time t with altitude h.

Alternatively, to minimize the total mechanical energy to fly along a given path the objective function can be chosen as

$$J = \sum_i \int_{t_0^{(i)}}^{t_f^{(i)}} V(t) \cdot T(t)\, dt. \tag{13}$$

In the present analysis the objective function of the optimization (the performance index in Eq. (6)) was chosen to minimize

$$J = t_f^{(1)} + t_f^{(2)} + t_f^{(3)} + t_f^{(4)} + t_f^{(5)} \tag{14}$$

or maximize

$$J = -(t_f^{(1)} + t_f^{(2)} + t_f^{(3)} + t_f^{(4)} + t_f^{(5)}) \tag{15}$$

transition times between phases.

The dynamical constraints in each phase $t \in [t_0^{(i)}, t_f^{(i)}]$ with $i = 1, ..., 5$ (cf Eq. (7)) were given by

$$\begin{aligned} m\dot{V} &= T\cos\alpha - D - mg\sin\gamma, \\ mV\dot{\gamma} &= (T\sin\alpha + L) - mg\cos\gamma, \\ \dot{x}_e &= V\cos\gamma, \\ \dot{h}_e &= V\sin\gamma, \\ \dot{T} &= (T_c - T)/\tau_T, \end{aligned} \tag{16}$$

Here T_c is the control value of the thrust and the τ_T is the characteristic scale The bounds on the aircraft speed in each phase were given by $V_{max}^{(i)}$ and $V_{min}^{(i)}$ listed above for each flaps configuration.

The results of the minimization of transition times for four phases are shown in Figure 2. It can be seen from the figure that transitions to the configurations gear down, flaps 20 and 25 occur as soon as velocity of the aircraft approaches the corresponding limiting value. The flight path angle γ stays close to the lower bound corresponding to the fast descent. The convergence time was found to be very sensitive to the parameters of the problem and varies between 10 and 50 sec.

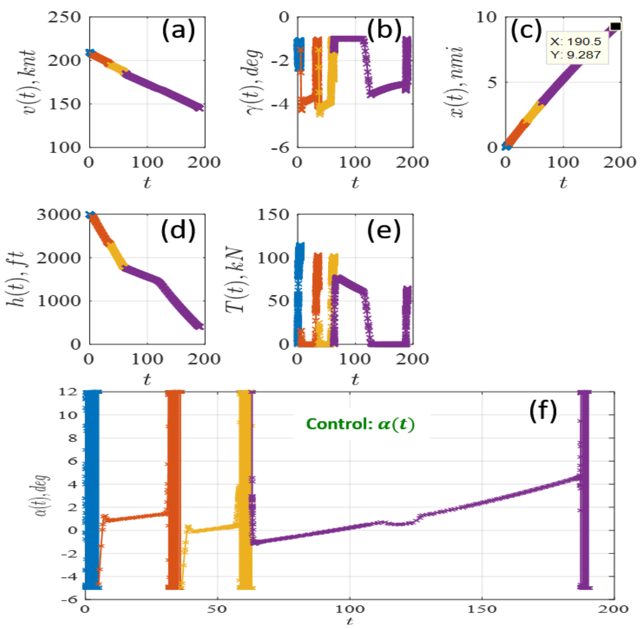

Figure 2. Minimization of transition times for descending trajectory with 4 phases. The figures show the dependence of dynamical variables of the system on time: (a) Velocity; (b) γ; (c) distance; (d) Altitude; (e) Thrust; and (f) Angle of attack. Different colors indicate different phases of the flight and correspond (left to right) to phases (i) to (iv).

The convergence time for maximization problem can vary between 50 and 300 sec and the convergence is not robust. We note that the best reported performance of the optimized implementation of the multiphase pseudospectral algorithm (de Jong, 2014) in C++ using PSOPT package was 30 sec for the whole descent and final approach trajectory.

We therefore conclude that although GPOSP package is potentially very useful for trajectory optimization at present it can only be used for off-line applications. To enable fast online optimization of the multiphase descending trajectory we propose a novel algorithm, which is considered in the next section.

4. FAST MULTIPHASE OPTIMIZATION ALGORITHM

Before we introduce the algorithm, let us provide some estimations for the final ILS approach to RWY 28R at SFO. The starting point of our analysis is the transition from fast to slow deceleration, which may normally happen at the flight level FL100 (10000 ft) and speed $V_{CAS} \leq 250$ kn.

According to the "3:1 rule of descent" this transition point is located $\sim$33 nm from the runway. The final state of the aircraft in our analysis is the state of stabilized approach. At the point of stabilized approach the aircraft speed should be $\sim$140 kn and the altitude is $\sim$500 ft. So during the descent the altitude and speed are reduced by approximately 330 ft and 3.3 kn per each nautical mile.

In addition, the aircraft has to capture the 3° gliding slope from below at the altitude approximately 1800 ft and distance $\sim$5.5 m from the runway, which translates into $\sim$740 ftm vertical speed at the ground speed 140 kn.

The flight plan normally should also accommodate a number of actions including: (i) localizer intercept; (ii) setting a sequence of flaps configurations; (iii) deploying landing gear; (iv) capturing gliding slope; (v) initiating flare; (vi) changing to touchdown phase. All this actions set additional constraints on the flight profile. Note that there can also be multiple air traffic control (ATC) corrections (constraints) to the normal approach procedure that have to be included into the flight profile.

The large number of phases (aircraft configurations), dynamical constraints, and bounds on the control and dynamical variables render trajectory optimization a complex multiphase optimization problem, see Table I with 14 phases defined in (de Jong, 2014; de Jong et al., 2015, 2017).

On the other hand, we see that a large number of phases must be accommodated on relatively short distance along the descending path. This fact can be used to substantially reduce the dimensionality of the problem by approximating segments connecting neighboring transition points with low degree polynomials (in particular, straight lines). This assumption also agrees with the results of the optimization obtained using GPOPS code, see Figure 2 and compare with results of (de Jong, 2014; de Jong et al., 2015, 2017).

4.1. Brief description of the algorithm

Using this approximation we introduce the following algorithm:

1. specify desired phases/configurations of the descent flight, including initial and final stat;
2. determine bounds on the altitude (ALT), distance (DST), and speed (SPD) in each phase;
3. if desired impose non-increasing constraints on speed and altitude;
4. choose arbitrary location of the phase transitions in terms of (ALT, DST, SPD) that satisfy given constraints, i.e. at the centers of the constraint boxes, see Figure 5;
5. build an approximation to the flight path by connecting phase transition points;
6. use obtained flight profile and the corresponding configurations to reconstruct full dynamical trajectory and control variables of the aircraft along the descent path;
7. calculate desired cost function (e.g. arrival time);
8. modify location of transition points according to the search algorithm;
9. stop if converged or go back to step 5

Once the phases of the flight and bounds on the flight profile are defined (see Table 1) the core optimization steps of the algorithm can be described as follows.

4.2. Approximation of the flight path

At the first optimization step an approximation to the vertical flight profile (the altitude and speed as functions of the distance) is constructed by interpolation of the transition points. We note that the distance between these points is only a few miles and the piece-wise linear approximation appears to be quite satisfactory. This approximation is also consistent with the solution obtained by conventional technique, cf. results of (de Jong, 2014; Adler, Bar-Gill, & Shimkin, 2012; de Jong et al., 2015) and discussion in Sec. 3.

An important advantage of this approximation is that the resulting NLP has minimum complexity. Indeed, in our approach the vector of the decision variables includes only speed, amplitude, and distance at the phase transition points. For comparison, in the traditional approach the vector of the decision variables includes both dynamical and control variables at every time step. And each pair of the phase transition points is connected by several time steps. Accordingly, the complexity of the proposed algorithm is at least an order of magnitude lower than in the traditional approach.

The accuracy of the approximation of the vertical flight profile can be further improved by introducing e.g. cubic interpolation of the phase transition points as shown in the Figure 3. It can be seen from the figure that both approximations are very close to each other and the piece-wise linear version was adopted for further analysis.

Only continuous descent and continuous deceleration trajectories are considered in this work. If speed or altitude of the aircraft attends local maxima or minima during the descent the algorithm will have to be modified accordingly.

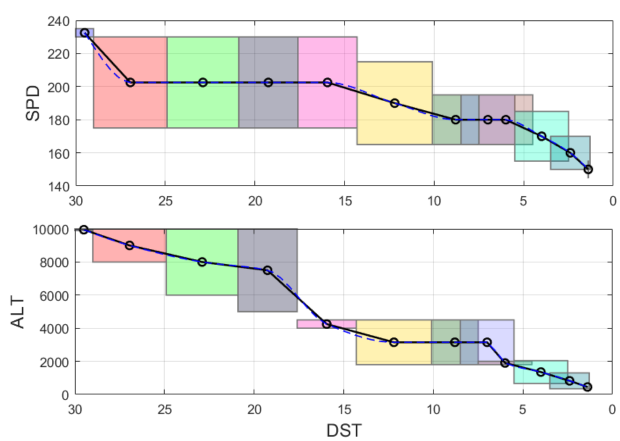

Figure 3. Vertical flight profile obtained by interpolation of the phase transition points: linear interpolation (solid black lines); cubic interpolation (dashed blue lines). Color shaded boxes show bounds on the speed, altitude, and distance introduced in the Table 1.

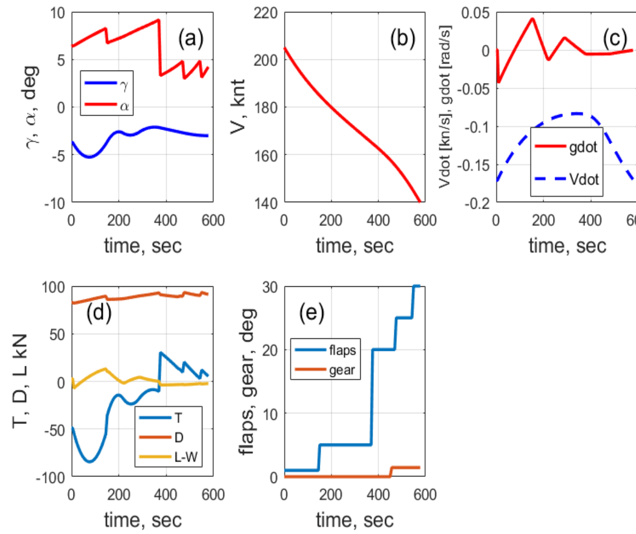

Figure 4. Aircraft dynamics reconstructed from the flight profile defined as $h(x)$ and $V(x)$: (a) $\alpha(t)$ and $\gamma(t)$; (b) $V(t)$; (c) $\dot{\gamma}(t)$ and $\dot{V}(t)$ (d) $T(t)$ - thrust, $D(t)$ - drag, and $L(t) - W$ - lift minus weight; (e) flaps and landing gear configuration.

Using linear approximation of the flight trajectories connecting phase transition points the dimension of the optimization problem is reduced to

$$(N_{tr} - 1) \times D_{tr}. \tag{17}$$

Here N_{tr} is the number of the phases transition points and D_{tr} is the dimension of each transition point (e.g. distance, altitude, speed). In this formulation the decision variables are the locations of the phase transition points.

Furthermore, using this approximation one can avoid dynamical constraints all together by reconstructing all the dynamical and control variables from the piece-wise linear approximations of the flight profile.

4.3. Inferring descent trajectory from the vertical profile

To infer the model dynamics from the flight profile we notice that eqs. (3) and (4) can be rewritten in the form

$$\begin{aligned} u' &= \frac{1}{m \cos\gamma}\left(T \cos\alpha - \rho S u C_D - D - W \sin\gamma\right) \\ \gamma' &= \frac{1}{2mu \cos\gamma}\left(T \sin\alpha + \rho S u C_L - W \cos\gamma\right) \\ h' &= \tan\gamma\,, \end{aligned} \tag{18}$$

using transformation

$$dx = V \cos(\gamma) dt \tag{19}$$

and introducing distance x as a new independent variable along the flight path (Vinh, 1981). In eqs. (18) $W = mg$,

u is the specific kinetic energy $\frac{V^2}{2}$ and the prime refers to derivative with respect to x, e.g. $h' = dh/dx$.

Since V and h are known along a given flight profile, thrust T, angle of attack α, and flight path angle γ can be found as functions of distance using eqs. (18) and then as functions of time using Eq. (19). The results of calculations are shown in Figure 4. In this example the flight profile corresponds to the spline approximation of the initial guess of the trajectory corresponding to the location of transition points shown by the open gray circles in Figure 5.

It can be seen from the figure that all the dynamical and control variables can be successfully inferred from the given flight profile. The obtained results allow one to avoid collocation methods (Hargraves & Paris, 1987) of dynamical trajectory optimization and obtain online solutions to the problems of primary interest for aircraft descent operations including minimization of additional drag and thrust during descent, and enforcing required time of arrival.

To solve these problems we have to combine the algorithm, outlined above, with the optimization algorithm for the location of the transition points.

4.4. Fast optimization algorithm

Using piece-wise linear approximation to the flight profile discussed above, we reduce the problem of multiphase optimization of the descent trajectory to the following standard NLP:

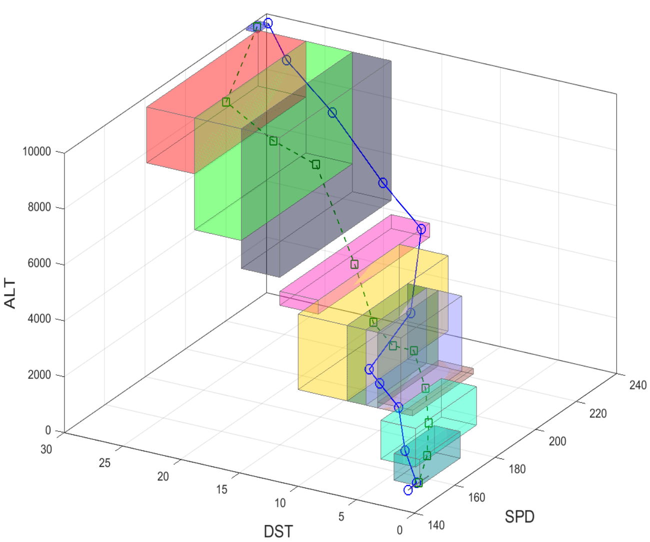

Figure 5. Bounds on the location of the events obtained using Table 1 are shown by colored transparent parallelepipeds in 3D space (V - SPD, x - DST, h - ALT). The initial location of the events is shown by open gray green squares (connected by green dashed line) located at the centers of the parallelepipeds. The optimal solution is shown by the open blue circles connected by the solid blue line.

$$\underset{x}{\text{minimize}} \quad f(x) \tag{20}$$

subject to equality and inequality constraints and bounds on the dynamical variables in the form

$$\begin{aligned} g_i(x) &\leq 0,\ i = 1, \ldots, m, \\ h_j(x) &= 0,\ j = 1, \ldots, n, \\ A \cdot x &\leq b, \\ A_{eq}(x) &\leq b_{eq}, \\ lb \leq x &\leq ub. \end{aligned} \tag{21}$$

Here vector of decision variables x has dimension $N_{tr} \times D_{tr}$ number of phase transition pints times dimension of each point because we allowed for the variation of the initial state.

The performance index (cost function) $f(x)$ can have many different objectives. In the context of the descent trajectory optimization the most common objectives are minimization of the fuel use ($\sim$ minimum thrust) and required time of arrival (RTA) for maximum throughput of a given runway/airport.

In this work we do not consider the change of the phases order. The phases order will be fixed as shown in the Table 1. To enforce the phase order and no-climb, no-acceleration conditions we use inequality constraints in the form

$$A_{eq} \cdot x \leq b_{eq}, \tag{22}$$

where x is the vector of decision variables $\{x_1, \ldots, x_{tr}, h_1, \ldots, h_{tr}, V_1, \ldots, V_{tr}\}$, and the block-bidiagonal matrix A_{eq} is

$$A_{eq} = \begin{pmatrix} -1 & 1 & 0 & \ldots & 0 \\ 0 & -1 & 1 & \ldots & 0 \\ & & \ddots & & \\ 0 & \ldots & 0 & -1 & 1 \end{pmatrix}.$$

By setting vector b_{eq} to zero we ensure that values of the distance x, altitude h, and speed V at a given step are no larger than the corresponding values at the preceding step.

Bounds and the initial guess of the trajectory are shown in Figure 5. It can be seen from the figure that bounds on the speed and altitude of the phase transition points can be both disconnected and overlapping. It can also be inferred from the figure that the initial guess of the descending trajectory was obtained as a set of locations at the centers of the bounding boxes.

In the first test we choose minimization of thrust as the objective of the problem:

$$f(x) = \int_{x_0}^{x_f} T^2(x)\, dx. \tag{23}$$

The solution of the NLP problem in Eqs. (21) - (23) was obtained using a number of optimization solvers including lo-

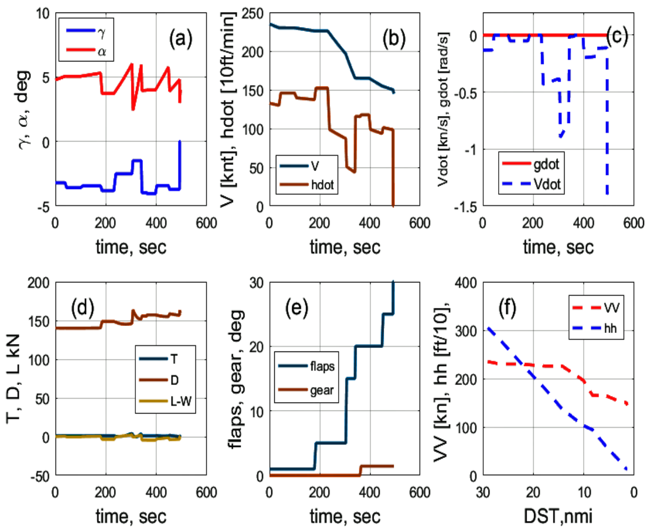

Figure 6. Aircraft dynamics obtained as the result of optimization of the descending trajectory using local IPM solver: (a) $\alpha(t)$ and $\gamma(t)$; (b) $V(t)$ and $\dot{h}(t)$; (c) $\dot{\gamma}(t)$ and $\dot{V}(t)$ (d) $T(t)$ - thrust, $D(t)$ - drag, and $L(t) - W$ - lift minus weight; (e) flaps and landing gear configuration; (f) speed and altitude as functions of the distance.

cal search based on the interior-point method (IPM) (Byrd, Gilbert, & Nocedal, 2000), and global solvers such as genetic algorithm and pattern search (MATLAB Optimization Toolbox, 2016).

The best results were obtained using the local search based on the IPM as shown in Figure 6. It can be seen from the figure that optimized values of the thrust are indeed very small, cf. Figure 4. The corresponding variations of the flight path angle are also small, see Figure 6(a). In addition, it can be noticed from the figure (f) that the optimal vertical descent path is a smooth trajectory respecting the constraints. All these features are expected to be the main features of the descent trajectory with minimum thrust.

We note that the vertical velocity is above 1000 ft/min for most time during descent. This is a quite high value and additional nonlinear constraints

$$| V \cdot \sin(\gamma) | \leq 700\text{ft/min} \tag{24}$$

may have to be imposed to keep vertical speed within predefined limits.

Importantly, the convergence of the local search algorithm was consistently less than 5 sec. This result may suggest that there exist some strong, albeit hidden, convex properties of the objective function.

For comparison, the convergence of the global search algorithms (genetic algorithm and pattern search) was slow. Despite long convergence time the thrust found by the global search algorithms was much larger than the one described above. This fact also indicates that the cost function has strong convex property.

Some insight into the properties of the cost function can be gained by rewriting it in the form (keeping terms $\sim \alpha^2, \gamma^2$)

$$\begin{aligned} \int_{t_0}^{t_f} T^2(t)\, dt &= \sum_{i=1}^{N_{tr}-1} \Delta t_i \left[\left(m\dot{V} + W\gamma \right) \right. \\ & -\rho g S \frac{(V_i+\delta V)^2}{2} + \left(\left(\frac{\tilde{C}_L}{2} - C_{D_\alpha} \right) \alpha \right. \\ & \left. \left. + \left(\frac{C_{L_\alpha}}{2} - C_{D_{\alpha^2}} \right) \alpha^2 \right) - \tilde{C}_D + W\frac{\alpha}{2} \right]^2, \end{aligned} \tag{25}$$

where V_i are the mean value of speed in each phase, δV are the speed variations around the mean value, $\tilde{C}_L = C_{D_0} + C_{D_{fl}}\delta_{fl} + C_{D_{lg}}\delta_{lg}$, and $\tilde{C}_D = C_{L_0} + C_{L_{fl}}\delta_{fl} + C_{L_{lg}}\delta_{lg}$.

From the Eq. (25) one can conjecture that the cost function is convex with respect to "hidden" optimization variables α and δV during the time interval Δt_i corresponding to each phase. In addition, it is expected that there is a smooth continuous dependence of the thrust on the location of the boundaries of each phase. However, the vector of decision variables has 36 components and the full analysis of the convex properties of the cost function is beyond the scope of this work and will be considered elsewhere.

In this work we were primarily concerned with estimation of the earliest and latest transition times for each phase of the flight and accordingly the earliest and latest arrival times. To find these estimations the cost function was chosen as

$$Cost = \sum_{i=1}^{N_{tr}-1} \Delta t_i$$

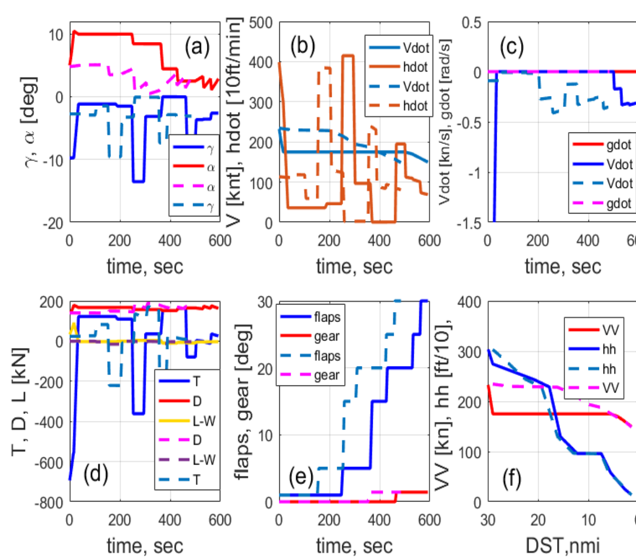

Figure 7. The results of the minimization (dashed lines) and maximization (solid lines) of the required time of arrival. The notations are the some as in Figure 6.

to minimize arrival time and in the form

$$Cost = -\sum_{i=1}^{N_{tr}-1} \Delta t_i$$

to maximize it.

The solutions of the corresponding optimization problems is shown in Figure 7. The obtained arrival times satisfy all the constraints and are in the interval $484 \div 600$ sec. The time windows for the pilot actions during the descent are most clearly seen in the Figure 7 (e) as the time intervals between the solid and dashed lines indicating transitions to the new flaps and landing gear configurations. The corresponding changes in the flight profile can be observed in the Figure 7 (f). It can be seen from the figure that the shortest time corresponds to a more steep descent and a larger speed along the whole profile and quantify the intuitive idea that the aircraft speed during the descent is the key scaling factor for the time windows of allowed pilot actions.

We note that these results were obtained without constraints on the vertical speed of the type (24). It is expected that constraints will reduce the allowed time windows for pilot actions and for the arrival time.

5. CONCLUSIONS

We analyzed the problem of scheduling of the descent and approach of commercial aircraft. The problem was formulated as a general multiphase optimal control problem.

To evaluate the standard approach to the solution of this problem we used Matlab package GPOPS based on the pseudospectral optimization method. We considered simplified scheduling problem with 4 phases due to different flaps and landing gear configurations. It was shown that the package can solve complex multiphase problems in general form. However, its online applications are at present limited by slow convergence time.

To solve the full optimization problem online we proposed novel fast algorithm of scheduling and optimization of the descent trajectory that reduces the original multiphase optimal control problem to the standard NLP problem of low dimension. It was shown that the dimension of the optimization problem can be reduced by

- using algorithm of reconstruction of the full set of dynamical and control variables along the descent path from a low-dimensional vertical and speed profiles;
- avoiding collocation methods of dynamical trajectory optimization;
- choosing the location of the phase transition points as the decision variables;
- approximating flight paths connecting transition points by low-dimensional polynomials.

We note that similar algorithm can be applied to the optimization of the lateral aircraft trajectory.

An additional advantage of the proposed algorithm is the ability to include pilot actions during descent (such as capturing LOC and glide slop, changing flaps and gear configuration, establishing stabilized approach configuration, initializing FLARE and touchdown etc.) directly into the flight plan.

Preliminary testing of the algorithm shows promising results for the future online applications. In particular, the proposed approach paves the way to a development of a general optimization algorithm that combines optimization of arrival times of individual aircrafts with the optimization of runway throughput under e.g. convective weather conditions.

Possible limitations of the proposed scheme are related to its reliance on the convex property of the cost function. The latter property may break down when non-continuous descent operations are considered, i.e. when the aircraft is expected to accelerate or climb during the final approach. A more detailed analysis of such cases is deferred to the future work.

6. NOMENCLATURE

ALT - altitude; ATC - air traffic control; CAS - calibrated airspeed; DST - distance; FL - flight level; FMS - flight management system; GPOPS - general pseudospectral optimization software; GS - gliding slope; LOC -localizer; NOMAD - Nonlinear Optimization by Mesh Adaptive Direct Search (software); OPTI - OPTimization Interface Toolbox; PSOPT - pseudospectral optimal control software package; RTA - required time of arrival; RWY - runway; SFO - San Francisco Airport; SPD - speed; TBO - trajectory based optimization.

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