Rotor Track and Balance Improvements

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ABSTRACT

Vibration derived from the main rotor dynamics and imbalance causes premature wear to the aircraft components, and can cause pilot fatigue. While improvements have been made in rotor track and balance (RTB) techniques; there is room to enhance the quality of the recommended RTB adjustments.

One aspect that limits the development of RTB algorithms is the difficulty in quantifying the performance of new algorithms. This is because there are limited data sets to work on, and no agreed upon metrics on which to measure RTB performance.

This paper develops a methodology to simulate the vibration due to injecting a fault into the rotor system, and demonstrates metrics to evaluate the performance of a RTB algorithm. A new Bayes RTB method is evaluated against a standard least squares technique. In addition, a technique is presented to automate the selection of active adjustments.

1. INTRODUCTION TO ROTOR TRACK AND BALANCE

Vibrations in helicopters result in:

- Crew fatigue,

- Increased fatigue of mechanical parts,

- Higher probability of avionics malfunctions,

- And potential limits on the operational envelope (Rosen and Ben-Ari, 1997).

Failure rates for components in fixed-wing aircraft are lower than the rates for similar components installed in rotary-wing aircraft. The impact of vibrations on overall aircraft health was demonstrated in a study conducted by Sikorsky

for the U.S. Army Air Mobility Research and Development Laboratory (Veca, 1973). In the study a squadron of H-3 helicopters were configured with rotor-mounted bifilar vibration absorbers and compared to a similar squadron that did not have the device. The two squadrons were similar in size and mission and flew approximately the same number of flight hours over the period of the study. The results were significant: overall helicopter failure rate and corrective maintenance requirements were reduced by 48% and 38.5%, respectively. Additionally, life-cycle cost showed a significant reduction of approximately 10% for the overall aircraft.

Vibration in helicopters is divided three general categories:

- High frequency vibration associated with the engine/gearbox and drivetrain. Typically, the frequencies are between 100 Hz to 10,000 Hz. Improvements to engine/transmission mounts and improved gear designs have greatly reduced this source of vibration.

- Medium frequency vibration, associated with the tail rotor, and to a lesser extent, high harmonics of the main rotor, are the main source of these vibration.

- Low frequency vibration, caused exclusively by the main rotor. This has the most severe effect on flight crew and equipment fatigue.

Main rotor vibration can be characterized as either vibration that is inherent due to the asymmetric nature of rotor dynamics in forward flight (present even with identical blades), and vibration due to the non-uniformity of the blades. The non-uniformity is due to the variation in manufacturing, and uneven wear/fatigue of the blade as a result of usage.

The vibration caused by non-uniformity results in ongoing maintenance and inspection by ground crews, and is the

focus of the development of improved Rotor Track and Balance algorithms.

2. ROTOR PHENOMENOLOGY

A rotor blade rotates with a constant angular rate Ω, with the root of the blade attached to the hub. The blade position for the kth blade is: Ψ_k. The motion of the blade includes a flapping angle β_k, a lead-lag angle ζ_k, and a pitch angle θ_k, where k is the index of the blade. If elastic deformations are small, then β_k practically determines the blade tip path (Figure 1).
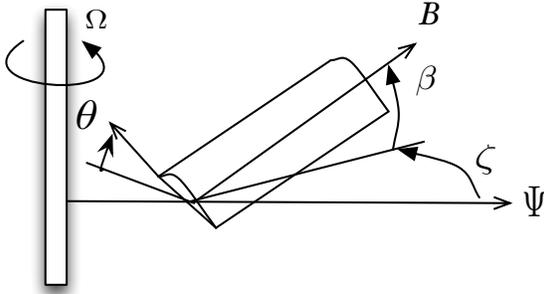

Figure 1 Blade Motion and Coordinates

The loads from the blades are transferred to the hub. If the blades are articulated, then moments acting on the hub are theoretically negligible. The force of the kth blade on the hub is then:

$$F_k^H = X_\alpha sin(\psi_k) + X_\alpha cos(\psi_k) \qquad (1)$$

where X_α are the loads along the aircrafts x,y and z axis and the force due to blade k on the hub is F_k^H. In the case of identical blades, the sum of all forces on the hub would then be:

$$F^H = \sum_{k=0}^{b-1} F_k^H = 0 \qquad (2)$$

where b is the total number of blades in the rotor system. Deviation from a nominal blade will result in a non-zero force, which is measured as accelerations in the helicopter.

The relationship between perturbations between blades and the resulting track deviation and vibration is complex. Consider the case where the mass balance of all blades is identical, but the flapping angle, β_k, is different. By adjusting pitch of the kth blade, an identical track/flapping angle could be reached, for a given helicopter airspeed. However, a change in pitch of that blade would:

- Affect that blade's lift and drag,

- Which would change the blade's lead/lag,

- That would in turn change the mass balance of the hub,

- Resulting in accelerations that increased vibration.

Initially, all efforts to decrease the non-uniformity of the blades started as an effort to reduce track split errors. But as many maintainers know, a flat track does not always result in a smooth helicopter. Since the primary goal of rotor track and balance is to reduce vibration, solving the problem efficiently is an underlying motivation.

2.1. Modeling Helicopter Vibration

The non-uniformity of the blades results in aerodynamic, mass imbalance, and track errors. To correct for these non-uniformities, rotor blades are manufactured with devices to purposely induce non-uniformity to cancel the effect of the naturally occurring blade errors. These devices include:

- Weights (WTS), which are attached at specific locations (hub and rotor tip) to change the blade moment,

- Pitch control rod (PCR) setting, which by changing length of the pitch rod, changes the angle of attack of that blade relative to the other blades, and

- Trailing edge tabs (TAB), which effectively change the blade's camber when bent. This in turn affects the aerodynamic loads/moments on the blade.

The acceleration due to blade induced vibration is measured at specific points in the aircraft, such as the:

- Pilot/Copilot vertical acceleration. These can be combined vectorially to derive cockpit vertical (A+B) or cockpit roll (A-B),

- Cabin Vertical

- Cabin Lateral

or other location where vibration deleteriously effects equipment or passengers. The levels of vibration will also be affected by the regime (airspeed) of the helicopter. For example, there is no flapping motion (β_k) when the helicopter is in ground or hover. Typical regimes for helicopter might be: Ground, Hover, 90 knots, 120 knots and 150 knots. The Fourier coefficients to describe the change in vibration then need to account for: adjustment type (a), sensor location (s), aircraft regime (r) and order (e.g. harmonic order, o).

For b blades, the Fourier representation of an adjustment, A_a, is the multiplication of the time domain representation of the adjustment (e.g. blade k) multiplied by the discrete Fourier transform matrix $\boldsymbol{D}_{k,o}$,

$$\boldsymbol{D}_{k,o} = exp(-i2\pi \times k \times o \ / \ b) \qquad (3)$$

And

$$A_o = \boldsymbol{D}_{k,o} \times \boldsymbol{a} \qquad (3)$$

For example, the Fourier representation of an adjustment of 2 on blade 1, and 3 on blade 2, of a 3 bladed rotor is:

$$\begin{bmatrix} 2 + 1.73i \\ 2 - 1.73i \\ 5 \end{bmatrix} = \begin{bmatrix} -.5 - .86i & -.5 + .86i & 1 \\ -.5 + .86i & -.5 - .86i & 1 \\ 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix} \quad (5)$$

The vector A_o, is indexed by order: the 1st index is first harmonic (e.g. shaft order 1), the 2nd index is the 2nd harmonic, while the 3rd index is DC (static value, which for WTS in the sum of all weights, while for PCR/TAB is the coning angle of the blades). However, Equation (1) is by a given order, for an adjustment type. Measured vibration is for a given order, over sensor and regime. This means that an adjustment vector, over adjustment type, is built by calculating the DFT adjustment for a given adjustment type, then building and adjustment vector for a given order.

Consider a WTS adjustment of [1 1 0], a PCR adjustment of [0 2 3], and a TAB adjustment of [-5 5 0]. Then the DFT adjustment for order 1 is:

$$A_1 = [-1 - 0i \quad 2 + 1.73i \quad 0 + 8.66i] \quad (6)$$

In other words, the first term of each A_o vector calculated by multiplying $D_{k,o}$ with each adjustment type vector, is combined into a new vector A_1.

The acceleration, for a given sensor location and regime for order 1, in the matrix representation of Equation (1), is expressed as:

$$F_1 = X_1 \times A_1 \quad (7)$$

Note that this is a linear model. It is assumed that the perturbation induced by A_o, is small relative to the nominal blade, such that the Taylor series of X_o is dominated by the first derivative (e.g. slope). The concept that adjustment coefficients are linear has, been presented by other researchers (Ferrer, 2001., Dimarco, 1990).

Equation (7) explicitly describes a system of equations that can generate vibration, for a given set of adjustment types, over a given order. This also suggest that:

- Implicitly, this means that there is no control over the bth order vibration (e.g. forth harmonic of a 4-blade rotor cannot be controlled passively).

- That vibration is operated on by order (e.g. one cannot solve a system of equations for order 1 and order 2 simultaneously. Meaning, when implementing a 2 blade solution on a 4 bladed rotor for vibration on the 1st order, if the vibration coefficients are not zeros for the 2nd order, the 2 blade solution affects the 2nd order vibration.

## 2.2. Development of a Vibration Simulation

Simulation provides a power tool to for RTB research. Because of restriction concerning software developed (FAA 2008), it is difficult to develop, test and mature algorithms,

such as RTB, on aircraft. By modeling the vibration associated with the rotor, it is possible to test algorithms without the large expenses associated with on-aircraft development. This allows for quicker deployment of new features, and reduces risk of associated with deviating from an existing practice. Additionally, it allows testing that would be deemed to risky for on aircraft use.

Further, simulation will allow the development of metrics for algorithm performance evaluation. Consider a typical scenario to test a new RTB algorithm. Using Equation (3) and Equation (7), a known adjustment will derive a known vibration F, plus measurement process noise. The test algorithm generates a solution, from which a residual error is derived (e.g. input adjustment – calculated adjustment, or difference in measurements prior to the adjustment and after the adjustment). This experiment can be run in Monte Carlo fashion to derive performance statistics. Hence, one can now develop probabilistic models on how well one RTB algorithms perform against another algorithm.
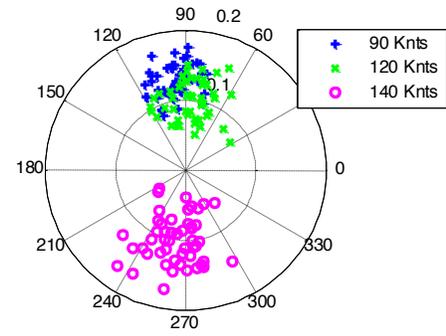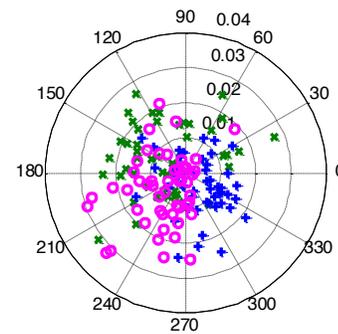
Figure 2. Simulated Pilot Sensor Vibration, 1st and 2nd Orders

Of course, simulation is only as good as the data that drives it. In this study, rotorcraft data from a 4-blade helicopter

was used to model the vibration coefficients, X_1, X_2, with measured process noise. Process noise was modeled as a stochastic process, where a Gaussian random variable, $N(0, \sigma_{s,r,o})$ was added to the real and complex values of the vibration coefficients. These process noises where estimated from flight data. Figure 2 shows the simulated vibration for the pilot sensor at 90 knots, 120 knots and 140 knots, for orders 1, and 2. This is the estimated vibration as a result of injecting the these blade faults on a 4-blade rotor

- WTS: [0 5 10 0]. Because this is Hub WTS, there is no effect on the 2nd order, hence only 2 blades.

- PCR: [5 7 2 0]

- TAB: [5 0 3 -3]

3. ROTOR TRACK AND BALANCE SOLUTION STRATEGIES

RTB solutions present an unusually difficult challenge in solving. While optimization on Equation (7) using least squares or some other methodology, the solution is in the Fourier domain. In the conversion from Fourier to "time domain" or "real solution", the result has multiple equivalent solutions. Consider an order 1 solution for a 4 blade rotor of: -8 -4i. There are four possible real solutions:

- [-8 -4 0 0], [0 -4 8 0], [-8 0 0 4] or [0 0 8 4]

These four solutions are equivalent in the Fourier domain. The best solution would be based externalities, such as: if an adjustment can be pulled off the blade, or an adjustment that minimizes track, or the preference of the maintainer.

3.1. Details on Converting the Adjustment from Fourier to Time Domain

For this discussion, the following convention for blade identification is used for a notional, 4 bladed rotor:

Black $\Leftrightarrow k = 0$, Yellow $\Leftrightarrow k = 1$, Blue $\Leftrightarrow k = 2$, Red $\Leftrightarrow k=3$

Expanding on the prior blade solution example, assume that the order 2 solution was: 2.0. From Equation (3), one should observe that the order 2 solution for a 4-blade solution is always real, and that the resulting time domain solution is: [2 -2 2 -2] . The implemented adjustments are the superposition of the order 1 and order 2 solutions:

Black	Yellow	Blue	Red
-8	-4	0	0
2	-2	2	-2
Adj: 6	**-6**	**2**	**-2**

However, no maintainer would implement this adjustment, as it is equivalent to an [8 -4 4 0] blade adjustment. Why touch 4 blades when 3 blades will do? In effect, the 4 blade solution captures DC in the Fourier domain, but in time, adds nothing to reducing the order 1 and order 2 vibrations, hence it should be removed. The DC component would,

however, affect the helicopter rigging for such things as auto rotation, which is not a desired result of an RTB event.

3.2. A Procedure to Develop Real Blade Adjustments

Multiplying the inverse of Equation (3) to solve for the real blade adjustments results a solution with a DC component. In order to get a solution that would be implemented, an automated procedure must be used in order to provide appropriate adjustment solution. This will depend on the adjustment order and type.

A balance solution (either vibration or track) requires solving for Equation (7) for the number of blades–1 orders (recall that in the DFT, only the number of blades–1 order are available, as k^{th} blade is DC). Additionally, the solutions are conjugate (Ventres, 2000): the order 3 solution is the conjugate to the order 1 solution on a 4-blade rotor. For a 5-blade rotor, order 1 and 4 are conjugate, just as order 2 and 3 are conjugate. Thus, the RTB analysis calls the for the solution of Equation (7) for order 1 and order 2 (assuming a 4 blade rotor) then sets order 3 as the conjugate of the order 1 solution.

For WTS solution, since there is no flapping motion, there is no order 2 solution. The real blade solution is the set of all possible 2-blade solutions. This is found by multiplying the DFT solution of Equation (7) by the partitioned inverse of (3).

Set of possible 2-blade combinations:

- $B_1 = [1\ 2]$, $B_2 = [1\ 4]$, $B_3 = [2\ 3]$, and $B_4 = [3\ 4]$

Note that solutions such as [1 3] or [2 4] do not exist, as this is equivalent to adding weights on opposing blades. Since there is no order 2 solution, for each set (e.g. $i = 1$ through 4), the real blade solution would be:

$$a[B_i]_i = D \begin{bmatrix} 1 \\ 3 & B_i^{\ T} \end{bmatrix}^{-1} A \tag{8}$$

Recall that $A[2] = A[1]^*$ and that a_i is a real valued vector, where the index B_i is the blade adjustment value (say -8 on the black blade, and -4 on the yellow blade, for solution B_1).

For a three-blade solution, which is appropriate for adjustments that are affected by blade flapping in forward flight, the set of all possible solutions is:

- $B_1 = [1\ 2\ 3]$, $B_2 = [1\ 2\ 4]$, $B_3 = [2\ 3\ 4]$, and $B_4 = [1\ 3\ 4]$

The order 2 solution is real: for each set (e.g. $i = 1$ through 4), the real blade solution would be:

$$a[B_i]_i = D \begin{bmatrix} 1 \\ 2 \\ 3 & B_i^{\ T} \end{bmatrix}^{-1} A \tag{9}$$

Here $A[3] = A[1]^*$ and that, again, a_i is a real valued vector. A comment on the values of a in both Equation (8), and

Equation (9) is that generally, adjustments for weights are an integer values. Similarly for PCR (number of "clicks" or "notches"), TABS are in mils of bend against a jib or dial caliper fixture. Thus, it is implied in Equation (8) and Equation (9) that the values are rounded to the nearest integer.

3.3. The Least Square Solution

The least squares solution (LSS) is a relatively simple solution strategy. The optimization object is to minimize the sum of squares residual error. In effect, this is the dual problem to the solution strategy implement by (Bechhoefer, 2011), in which the objective function was to minimize the adjustment size given a constraint on allowable vibration after the adjustment.

The LSS is a naïve implementation, in that is sensitive to outlier data, especially at the "end points". For non-Gaussian residuals (difference between the measured and predicted vibration), this could be problematic. That said, the solution to Equation (7) is simply implemented as:

For Each Order, i:

$$A_i = (X_i^T X_i)^{-1} X_i^T F_i \qquad (10)$$

Then the set of real blade adjustments are calculated as per Section 3.2.

3.4. The Bayes Least Square Solution

One strategy to add robustness to Equation (10) is to weight the coefficients by some appropriate metric. One method would be to weight F_i by the Fisher's information matrix, which is a measure of the information carried in F_i (Fukunaga, 1990). This becomes Bayes least squares solution, where:

For Each Order, i:

$$A_i = (X_i^T \Sigma_i^{-1} X_i)^{-1} \Sigma_i^{-1} X_i^T F_i \qquad (11)$$

And Σ is the measured covariance of F_i.

3.5. Quantifying Solution Strategies

Given these solutions strategies, Equation (10) or Equation (11), one can now determine, stochastically, which algorithm will give the best performance given some objective. For this experiment, the norm residual vibration for order 1 and order 2 will be used. The scenario consists of 10 acquisitions for the 3 sensors, at 90 knots, 120 knots and 140 knots. The experiment will be run for 500 trials, and the PDF of the norm residuals well be evaluated. The norm residual is calculated by estimating the vibration given a proposed adjustment solution. That solution will use integer value adjustments from Equation (8) or Equation (9), where the estimated vibration of the solution is calculated using

Equation (3) and Equation (7). The results are given in Figure 3.

Clearly, the Bayes solution strategy provides a more robust solution, as both the order 1 and order 2 norm residual vibration error is approximately 40% smaller.

3.6. Issues with Track

Typically, the object of RTB is to reduce vibration, and as noted, a flat track does not mean low vibration. However, there are cases where minimizing Track split is a requirement. For example, after a blade change and prior to flight, a flat track maintenance event is performed. This is primarily the result of established procedures but also serves the purpose of providing a better field of view for the pilots.

The solution strategy for track is identical to vibration. This is done by converting the track into its Fourier representation (T) using Equation (3), replacing a in Equation (3) with blade track height, then replacing F in Equations (10) and Equation (11) with T, and solving for the time domain adjustment per Section 3.2. Track is in fact a simpler solution. This is because for track, there is always only one sensor. Care must be taken in that, for one regime (ground), only one adjustment can be solved (typically a PCR adjustment).
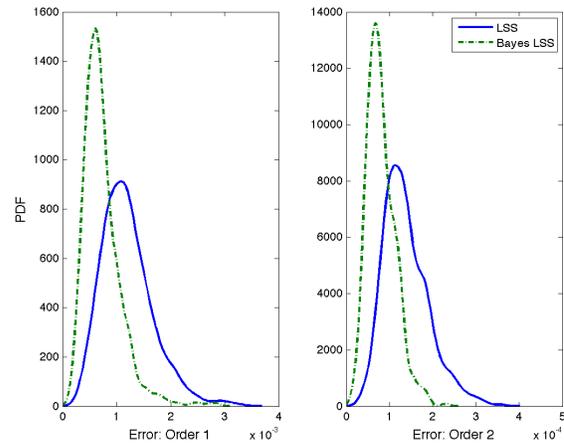

Figure 3. Comparison of LSS to Bayes LSS, for Order 1 and Order 2 Norm Error

4. IMPROVING USER EXPERIENCE

In addition to reducing vibration, the RTB algorithm should present the maintainer with a solution that is easy to implement. Most commercial systems (Renzi, 2004) provide only a 2-blade solution (as they only solve for order 1 vibration). For track, a 2-blade solution can introduce some additional complexity in attaining a flat track in 1 adjustment (Keller, 2007). Additionally, order 2 or higher harmonics do occur and require maintenance adjustments to restore the helicopter into normal operational limits. Ideally,

the RTB algorithm should be able to determine the most appropriate solution based on the measured vibration or track.

In (Bechhoefer, 2011), an expert system was developed in an attempt filter the options used by the RTB algorithm, based on the current set of measurements. The solution was not ideal in that it required an extensive library of *a priori* data. Essentially, configuration was needed to model to decision space, which selected the adjustment type (WTS, PCR, in board TAB, out board TAB), and adjustment order (1, or 1 and 2). The decision space encompassed 27 sets of configuration items.

An alternative method is proposed for the selection of adjustment type and adjustment order based on the estimated outcome of an adjustment. Because one can use Equation (7) to predict the vibration as a result of an adjustment, it is possible to estimate the residual vibration error post adjustment. This allows hypothesis testing for the adjustment/order options.

Consider that a full adjustment (WTS, PCR, TAB) is selected, and vibration order 1 and 2 are solved for (assuming a 4-bladed rotor). The residual error variance is then calculated. Say that an alternative adjustment is selected, in which only an order 1 (2-blade solution) is selected. Then one can test the hypothesis that the error variances are the same. If the test fails to reject the null hypothesis, then the simpler (2-blade solution) is selected over the 3-blade solution. Formally, as per (Wackerly, 1996), the test is derived as:

$$H_0: \sigma_1^2 = \sigma_2^2$$
$$H_a: \sigma_1^2 < \sigma_2^2$$

where the test statistic is:

$$F = \frac{S_2^2}{S_1^2}$$

The rejection region of the test is: $F > F\alpha$, where $F\alpha$ is chosen so that $P(F > F\alpha) = \alpha$ when F have $v_1 = n_1 - 1$ degrees of freedom in the denominator, and $v_2 = n_2 - 1$ degrees of freedom in the numerator. This test is easily performed online, and requires only the selection of the probability of false alarm, α, which was set at 0.05.

Given the simulation capability developed in Section 2.0, and the vibration generated by the adjustments used in Figure 2, the probability distributions were calculated for order 1 norm error, order 2 norm error, and the track split. Multiple hypothesis test were conducted, where the null hypothesis was a full adjustment: [WTS/PCR/TAB], and the alternative hypothesis were reduced adjustment sets: [WTS/PCR], [WTS/TAB], [PCR/TAB] or WTS alone.
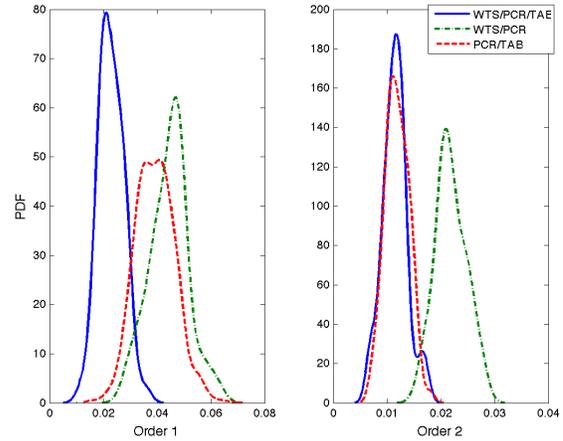

Figure 4. Order 1 and Order 2 Residual Vibration for Different Adjustments

The scenario assumed 10 acquisitions at 90 knots, 120 knots and 140 knots. Figure 4 shows that the algorithm selected between WTS/PCR/TAB (13% of the trials), WTS/PCR (75% of the trials) and PCR/TAB (12% of the trials). Figure 5 shows the difference in Track Split between the different adjustment sets.
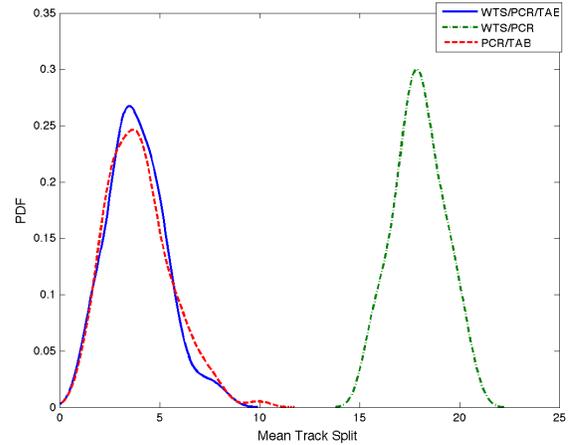

Figure 5. Track Split for Different Adjustment Sets

The algorithm did not select WTS alone, or a WTS/TAB solution, which as a general practice, reflects reality. We can note that the full adjustment results in a lower vibration in both vibration orders. The order 2 results are sensitive to the presence of a WTS solution. This is similar to the track performance issue – seeing as WTS has no effect on track, when the PCR or TAB adjustment is removed, the track split is larger. This is an important observation: improving Order 2 reduced the track split, even though optimization objective was vibration and not track. This suggests that a 2-blade solution (no reduction in Order 2) will always result in larger track split than, as seen in Figure 6.
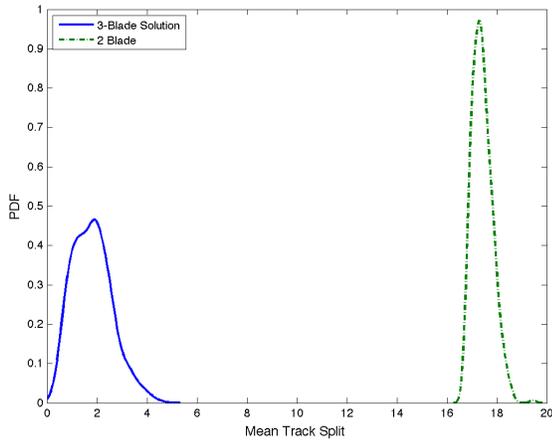
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Figure 6. Effect of 2-Blade vs. 3-Blade Solution on Track Split

Given how the adjustments are selected, when the estimate of vibration is poor (due to stochastic nature of vibration), adding an adjustment, statically, does not improve the results. This hypothesis was tested by increasing the number of acquisitions per trial to 50 (Figure 7), as increasing the sample size improves the estimate by sqrt(n). This suggests that increasing the number of acquisitions in give time period will improve the overall quality of the adjustment and lower overall vibration.
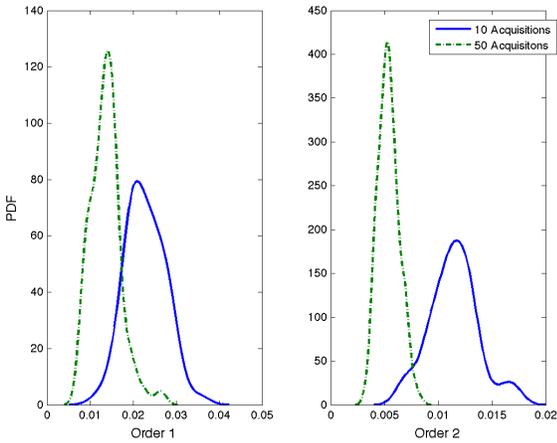

Figure 7. Order 1 and 2 Vibration for 10 vs. 50 Acquisitions per Trial

In the 50 acquisitions per trial case, because the estimate of the vibration was improved, the calculated adjustment results in a lower residual vibration error. Additionally, because the information was better, adding an adjustment improved the solution. This was seen in that the full adjustment set was selected 97% of the time, vs. 13% when only 10 acquisitions were used.

4.1. Methods to Reduce "Selection Fatigue"

Because each adjustment type has a large number of equivalent adjustments (see example in Section 3.0), even a WTS/PCR adjustment presented too many options for most maintainers. In some cases, it caused confusion and "selection fatigue". Additionally, both the helicopter manufacturer and the operator may have preferences as to what is a good adjustment. Subjectively, a good adjustment:

- Touches as few blades as possible

- Tries not to change the rigging of the helicopter

- Does not recommend adjustments which are too small to implement (e.g. minimum TAB is greater than 3 mils)

These preferences need to filter the adjustment such that the initial view to the maintainer is one set of WTS, PCR and TAB, which encompasses the rules or preference of the maintainer. A proposed rule set would be:

- Minimum DC offset on PCR. This ensures that, over time, the changes in PCR does not effect the helicopter rigging, and therefore the main rotor RPM during autorotation.

- Minimum TAB of +/- 3 mils. If mathematical solutions are less than 1.5 mils, zero the adjustment, if greater than +/- 1.5 and less than +/- 3 mils, round to 3 mils (sign appropriate)..

- Only add WTS. For a 4-blade rotor, since adding weights on one blade is the same as removing weights on the opposing blades, it's relatively easy for the maintainer to implement this.

- If there are two equivalent sets for an adjustment type, pick the adjustment set that intersects with a set of another adjustment type. This attempts to minimize the number of on which maintenance is performed.

Example: Generated Adjustments for WTS/PCR/TAB

WTS	Set 1	Set 2	Set 3	Set 4
Black	8	8	0	0
Yellow	-4	0	-4	0
Blue	0	0	-8	-8
Red	0	4	0	4

PCR	Set 1	Set 2	Set 3	Set 4
Black	-6	-2	2	0
Yellow	-8	-4	0	-2

Blue	-4	0	4	2
Red	0	4	8	6

TAB	Set 1	Set 2	Set 3	Set 4
Black	-13	-2	-8	0
Yellow	-5	6	0	8
Blue	-11	0	-6	2
Red	0	11	5	13

For PCR, the DC Offset is the sum of blade adjustments by set:

PCR	Set 1	Set 2	Set 3	Set 4
DC Offset	-18	-2	14	6

Set 2 for PCR affects the rigging the least, and touches the Black, Yellow and Red blades. For WTS, the positive adjustments are on the Black and Red blades. For TAB, corresponding adjustments are Black: -2, Yellow: 6, and Red: 11. Because the Black is -2, and violates the minimum adjustment for TAB rule, it is rounded to -3 with little effect on the vibration. Thus, the "best" adjustment presented to the maintainer is:

Adjustments	Black	Yellow	Blue	Red
WTS	8	0	0	4
PCR	-2	-4	0	4
TAB	-3	6	0	-4

5. CONCLUSION

In the paper, we present a methodology to simulate vibration on a helicopter for the purpose of developing, testing and, ultimately, improving Rotor Track and Balance (RTB) performance. Low frequency (e.g. order 1 and order 2, corresponding to the first and second harmonics of the main rotor) vibration is known increase the rate of component failure and to cause pilot fatigue. RTB maintenance is designed to reduce these vibrations.

Two potential solver strategies were presented, and using simulation procedure that was developed: the Bayes Least Squares solution was found to be superior to the Ordinary Least Squares in reducing vibration. Techniques were presented to automatically select the best adjustments based on the measured vibration. Additionally, the relationship

between 2nd order vibration (e.g. the second harmonic of the main rotor) and blade track split was observed.

Most importantly, it was observed that increasing the number of acquisitions used in an adjustment reduced the post adjustment vibration. This could impact future RTB design requirements. Instead of sampling helicopter vibration once every 6 to 10 minutes (a limit imposed by the processing power of the onboard vibration monitoring system), the key to a rotor tuning may be sampling the helicopter once per minute. Simulation results showed that increasing the number of samples from 10 to 50 acquisitions reduced mean vibration error and track split by 45%.

NOMENCLATURE

- Ω angular rate of the main rotor shaft
- β blade flapping angle
- ζ blade lead-lag angle
- θ blade pitch angle
- B blade tip path
- b number of blades in the rotor system
- F^H force exerted on the rotor hub
- $D_{k,o}$ Fourier transform matrix
- a time domain adjustment
- A Fourier domain adjustment
- X vibration adjustment coefficients
- F measured vibration over regimes and sensors
- DC   static load (sum of weights for WTS adjustment or conning angle for PCR/TAB adjustment).

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Eric Bechhoefer received his B.S. in Biology from the University of Michigan, his M.S. in Operations Research from the Naval Postgraduate School, and a Ph.D. in General Engineering from Kennedy Western University. His is a former Naval Aviator who has worked extensively on condition based maintenance, rotor track and balance, vibration analysis of rotating machinery and fault detection in electronic systems. Dr. Bechhoefer is a board member of the Prognostics Health Management Society, and a member of the IEEE Reliability Society.

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Ephrahim Garcia is a Professor of Mechanical and Aerospace Engineering at Cornell University, College of Engineering. Dr. Garcia's is interested in dynamics and controls, especially sensors and actuators involving smart materials. Current projects include: Modeling and Analyses of Flapping Wings, Design and Control of Nanoscale Smart Material Actuators, Control of Reconfigurable Morphing Aircraft , Energy Harvesting for Biological Systems: Lab-on-a-Bird,Aeroelastic Energy Harvesting Modeling with Applications to Urban Terrain, Artificial Muscles for a Bipedal Walking Robot, and Mesoscale Hydraulics for Bio-inspired Robots