Anomaly Detection based on Information-Theoretic Measures and Particle Filtering Algorithms

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ABSTRACT

This paper presents an anomaly detection module that uses information-theoretic measures to generate a fault indicator from a particle-filtering-based estimate of the posterior state pdf of a dynamic system. The selected measure allows isolating events where the particle filtering algorithm is unable to track the process measurements using a predetermined state transition model, which translates into either a sudden or a steady increment in the differential entropy of the state pdf estimate (evidence of an anomaly on the system). Anomaly detection is carried out by setting a threshold for the entropy value. Actual data illustrating aging of an energy storage device (specifically battery state-of-health (SOH) measurements [A-h]) are used to test and validate the proposed framework.

1. INTRODUCTION

Anomaly detection modules (Zhang *et al.*, 2011; Orchard *et al.*, 2011) play an important role within Prognostics and Health Management (PHM) systems since they constitute the first step in the implementation of fault diagnosis and failure prognosis schemes (Orchard and Vachtsevanos, 2009). In most real applications, the anomaly detector requires to perform its task simultaneously minimizing both the false alarm rate and detection time (early detection). The latter is of paramount importance since the setup of online prognostic algorithms, and particularly those based on particle filtering algorithms (Orchard *et al*., 2008; Orchard *et al*., 2009), requires a proper characterization of the initial state pdf to provide adequate estimate of the remaining useful life (RUL) of monitored equipment.

Classical anomaly detection methods rely on a model of the system to measure a discrepancy between the actual measurements and a predetermined pattern of operation. A variety of techniques have been proposed to achieve this task, including tools from estimation theory, failure sensitive filters, multiple hypothesis filter detection, generalized likelihood ratio tests, and model-based approaches (Isermann and Balle, 1997; Ayhan *et al.*, 2006; Zhou et al., 2008; Khan and Rahman, 2009). Other methods focused on statistical analysis techniques, reasoning tools, spectral methods and information theory (Tolani *et al.*, 2005; Zhou *et al.*, 2008; Ibrahim *et al.*, 2008).

In the particular case of the battery state-of-health (SOH) monitoring and prognosis (Orchard et al., 2010; Orchard et al., 2011), there are still issues regarding the proper representation of regeneration (self-recharge) phenomena. Self-recharge phenomena are characterized by sudden, momentary, and occasionally considerable regeneration of the battery capacity that tends to fade in time faster than the typical SOH degradation time constant. These changes, related to physicochemical aspects and temperature/load conditions during charge and discharge cycles, are particularly important in the case of Li-Ion batteries because they often alter the trend of the SOH prediction curve, thus affecting the performance of prognostic modules that depend on Bayesian estimation algorithms to compute initial conditions for their associated predictive models.

This paper presents a solution for this problem that is based on a combination of a PF-based state estimators and information-theoretic measures that allows to detect rare events within the evolution of the fault condition under analysis. The paper is structured as follows: Section 2 introduces the basics on particle filtering (PF) anomaly detection modules, as well as information-theoretic measures applied to sequential Monte Carlo algorithms. Section 3 focuses on describing the case study that is used in this research to illustrate and validate the potential of the proposed detection approach, which corresponds to the analysis of capacity regeneration phenomena in a set of data depicting the battery state-of-health (SOH, [A-h]) degradation. Section 4 presents the proposed anomaly detection scheme and the results obtained for the case study of interest. Finally, Section 5 states the main conclusions.

2. PARTICLE FILTERING, ANOMALY DETECTION AND INFORMATION-THEORETIC MEASURES

Nonlinear filtering is defined as the process of using noisy observation data $Y=\{y_t,\ t\in\mathbb{N}\}$ to estimate at least the first two moments of a state vector $X=\{x_t,\ t\in\mathbb{N}\}$ governed by a dynamic nonlinear, non-Gaussian state-space model.

From a Bayesian standpoint, a nonlinear filtering procedure intends to generate an estimate of the posterior probability density function $p(x_t \mid y_{1:t})$ for the state, based on the set of received measurements. Particle Filtering (PF) is an algorithm that intends to solve this estimation problem by efficiently selecting a set of N >>1 particles $\{x_t^{(i)}\}_{i=1\cdots N}$ and weights $\{w_t^{(i)}\}_{i=1\cdots N}$, such that the state pdf may be approximated (Doucet, 1998; Doucet *et al.*, 2001; Andrieu *et al*, 2001; Arulampalam *et al.*, 2002) by the empirical distribution:

$$\tilde{\pi}_t^N(x_t)=\sum_{i=1}^{N} w_t^{(i)}\delta(x_t-x_t^{(i)}), \tag{1}$$

and the values of the particles weights $w_t^{(i)} \propto w(x_t^{(i)})$ can be computed by:

$$\begin{gathered} w(x_t)=\frac{\pi_t(x_t)}{q_t(x_t)} \propto \frac{p(y_t \mid x_t)\,p(x_t \mid x_{t-1})}{q_t(x_t \mid x_{t-1})} \\ w(x_t^{(i)})=w_{t-1}^{(i)}\cdot\frac{p(y_t \mid x_t^{(i)})\,p(x_t^{(i)} \mid x_{t-1}^{(i)})}{q_t(x_t^{(i)} \mid x_{t-1}^{(i)})}\quad \forall i\in\{1,\cdots,N\} \end{gathered}, \tag{2}$$

where $q_t(x_t)$ denotes the importance sampling density function (Arulampalam *et al.*, 2002; Doucet *et al.*, 2001). The choice of this importance density function $q_t(x_t)$ is critical for the performance of the particle filter scheme. In the particular case of nonlinear state estimation, the value of the particle weights $w_t^{(i)}$ is computed by setting the importance density function equal to the *a priori* pdf for the state, i.e., $q_t(x_t \mid x_{t-1})=p(x_t \mid x_{t-1})$ (Arulampalam *et al.*, 2002). Although this choice of importance density is appropriate for estimating the most likely probability distribution according to a particular set of measurement data, it does not offer a good estimate of the probability of events associated to high-risk conditions with low likelihood. In this sense, this paper explores the possibility of using information-theoretic measures to analyze PF-based estimates of the state pdf in a dynamic system, with the purpose of detecting this type of events in a timely manner.

2.1 Particle Filtering for Anomaly Detection

PF-based anomaly detection modules (Kadirkamanathan *et al.*, 2002; Verma *et al.*, 2004; Orchard and Vachtsevanos, 2009; Zhang *et al*, 2011; Orchard *et al.*, 2011) have been used in the past to identify abnormal conditions in nonlinear, non-Gaussian dynamic systems. The objective in this type of implementations is to fuse the information that is available at a feature vector (measurements) to generate estimates of the *a priori* state pdf that could be helpful when determining the operating condition (mode) of a system and deviations from desired behavioral patterns. This compromise between model-based and data-driven techniques is accomplished by the use of a PF-based module built upon the nonlinear dynamic state model (3):

$$\begin{cases} x_d(t+1)=f_b\left(x_d(t)+n(t)\right) \\ x_c(t+1)=f_t(x_d(t),x_c(t),\omega(t)) \\ \text{Features}(t)=h_t(x_d(t),x_c(t),v(t)) \end{cases}, \tag{3}$$

where f_b, f_t and h_t are non-linear mappings, $x_d(t)$ is a collection of Boolean states associated with the presence of a particular operating condition in the system (normal operation, fault type #1, #2), $x_c(t)$ is a set of continuous-valued states that describe the evolution of the system given those operating conditions, $\omega(t)$ and $v(t)$ are non-Gaussian random variables that characterize the process and feature noise signals, respectively. Since the noise signal $n(t)$ is a measure of uncertainty associated with Boolean states, it is recommendable to define its probability density through a random variable with bounded domain. For simplicity, $n(t)$ may be assumed to be zero-mean i.i.d. uniform white noise.

PF-based detection modules provide a framework where customer specifications (such as false alarm rate and desired probability of detection) can be easily managed and incorporated within the algorithm design parameters. However, the analysis of the relationship that exists between the number of particles and the detection time still depends on general guidelines inspired in empirical experience (for example, “the more particles are used, the longer is the detection time”).

The problem of early detection using PF-based approaches has also been discussed in (Orchard *et al*, 2008), where a Risk-Sensitive PF (RSPF) framework complements the benefits of the classic approach by representing the probability of rare and costly events within the formulation of importance density function to generate more particles in high-risk regions of the state–space. Mathematically, the importance distribution is set as:

$$q\left(d_t,x_t \mid d_{t-1}^{(i)},x_{t-1}^{(i)},y_{1:t}\right)=\gamma_t\cdot r(d_t)\cdot p\left(d_t,x_t \mid y_t\right), \tag{4}$$

where d_t is a set of discrete-valued states representing fault modes, x_t is a set of continuous-valued states that describe the evolution of the system given those operating conditions, $r(d_t)$ is a positive risk function that is dependent on the fault mode, and γ_t is a normalizing constant.

Although the approach presented offered better performance in terms of the detection time, it still required the definition of a risk importance sampling distribution.

In this sense, the use of information-theoretic measures offers an interesting alternative that complements the paradigm of PF-based anomaly detection modules, under the assumption that an anomaly should affect the qualitative behavior of the state pdf estimate. The following section focuses on the most important concepts that need to be taken into account when implementing these measures to analyze and characterize sampled versions of the *posterior* distribution.

2.2 Information-Theoretic Measures Applied to Particle-filtering Algorithms

Several examples that incorporate information-theoretic measures to analyze the outputs of particle filtering algorithms can be found in literature (Ajgl and Šimandl, 2011; Lanz, 2007; Boers *et al.*, 2010; Skoglar *et al.*, 2009). Most of those are related to uncertainty characterization, optimality testing, and evaluation of control strategies. In particular, this research focuses on the widely known differential entropy measure (Cover and Thomas, 1991).

Entropy is a measure of uncertainty that is associated to a probability measure. In particular, the differential entropy H of a probability density function $p(x)$ is given by:

$$H(p) \triangleq -\int p(x)\log\left(p(x)\right)dx \tag{5}$$

Entropy-related applications for particle filtering algorithms generally aim at evaluating how many i.i.d. samples does the filtering algorithm require to represent regions of the state space that accumulate the majority of the probability mass, for a given state pdf estimate $p(x)$. For example, in (Liverani *et al.*, 2006) the authors propose the use of entropy to evaluate the pertinence of resampling procedures in a particle filtering algorithm aimed at estimating the states of a partially observed Markov chain. Instead, it is sought to generate an average weight for sampled particles, which depends on the distance that exists between the estimated and the actual value of the states.

In other applications, such as in (Ryan, 2008), the authors formulate a control strategy for a mobile sensor that intends to track an object, where the merit function depends on particle-filtering-based estimates and information-theoretic measures. Basically this approach uses entropy to characterize the uncertainty of the estimated pdf, and proposes a resampling method that intends to minimize the conditional entropy between the state of the tracked object and observed data, for a given control strategy.

Although the definition of differential entropy introduced in (5) allows straightforward computation in most cases, few considerations are required when trying to compute it in the case of particle-filtering-based estimates of the conditional state pdf's. Indeed, using (5), the differential entropy of the conditional state pdf estimate, given a set of measurements $y_1,\ldots, y_t$, is defined as:

$$H\left(p(x_t \mid y_t)\right) = -\int p(x_t \mid y_t)\log\left(p(x_t \mid y_t)\right)dx_t\,, \tag{6}$$

where the *a posteriori* state pdf estimate can be inferred from the likelihood of measurement y_t, the a priori state estimate $p(x_t|y_{t-1})$, and the probability of acquiring the current measurement using Bayes Theorem:

$$p(x_t \mid y_t) = \frac{p(y_t \mid x_t)}{p(y_t \mid y_{t-1})}\,p(x_t \mid y_{t-1})\cdot \tag{7}$$

Thus, replacing (7) in (6) and applying properties of the logarithm, it is possible to write:

$$\begin{aligned} H\left(p(x_t \mid y_t)\right) &= \log\left(p(y_t \mid y_{t-1})\right) + \cdots \\ &-\int p(x_t \mid y_t)\left[\log\left(p(y_t \mid x_t)\right) + \log\left(p(x_t \mid y_{t-1})\right)\right]dx_t \end{aligned} \tag{8}$$

In addition, given that in this specific case all distributions correspond to particle-filtering estimates, both the *a priori* state estimate and the probability of measured data can be approximated by their corresponding sampled versions, as in (9)-(10):

$$p(x_t \mid y_{t-1}) \simeq \sum_{i=1}^{N} w_{t-1/t-1}^{(i)}\,p\left(x_t^{(j)} \mid x_{t-1}^{(i)}\right), \tag{9}$$

$$p(x_t \mid y_{1:t}) \approx \sum_{i=1}^{N} w_{t/t}^{(i)}\,\delta(x_t - x_t^{(i)}), \tag{10}$$

where $w_{t-1/t-1}^{(i)}$ and $w_{t/t}^{(i)}$ are the *a priori* and posterior weight of the particle (i), respectively. After using (9)-(10) in (8):

$$\begin{aligned} H\left(p(x_t \mid y_t)\right) &= \log\left(p(y_t \mid y_{t-1})\right) + \sum_{j=1}^{N} w_{t/t}^{(j)}\Big[\log(p(y_t \mid x_t^{(j)})) + \cdots \\ &+\log\left(\sum_{i=1}^{N} w_{t-1/t-1}^{(i)}\,p(x_t \mid x_{t-1}^{(i)})\right)\Big] \end{aligned} \tag{11}$$

The term $p\,(y_t \mid y_{1:t-1})$ in (11) can be computed through its sampled version:

$$p(y_t \mid y_{t-1}) \simeq \sum_{i=1}^{N} w_{t/t-1}^{(i)}\,p\left(y_t \mid x_t^{(i)}\right), \tag{12}$$

where $w_{t/t-1}^{(i)}$ are the particle weights. As a final result, the differential entropy of the particle-filtering estimate of the posterior state pdf can be computed as in (13) (Orguner, 2009):

$$\begin{aligned} H\left(p(x_t \mid y_t)\right) &= \log\left(\sum_{i=1}^{N} w_{t/t-1}^{(i)}\,p\left(y_t \mid x_t^{(i)}\right)\right) + \\ &-\sum_{j=1}^{N} w_{t/t}^{(j)}\left[\log\left(p(y_t \mid x_t^{(j)})\right) + \log\left(\sum_{i=1}^{N} w_{t-1/t-1}^{(i)}\,p\left(x_t^{(j)} \mid x_{t-1}^{(i)}\right)\right)\right] \end{aligned} \tag{13}$$

The latter expression will be of use when evaluating the uncertainty associated to online estimates in dynamic processes.

3. CASE STUDY: PF-BASED SELF-RECHARGE DETECTION IN LITHIUM-ION BATTERIES

An appropriate case study has been selected to demonstrate the efficacy of an anomaly detection module based on a PF state estimator and information-theoretic measures. Consider the case of energy storage devices, particularly of Li-Ion batteries, where continuous switching between charge and discharge cycles may cause momentary increments in the battery SOH (capacity regeneration). These sudden increments directly affect RUL estimates in classic prognostic schemes since the state pdf estimate has to be adjusted according to new measurements (thus modifying long-term predictions), while the observed phenomenon typically disappears after a few cycles of operation. Particularly in the case of Li-Ion batteries, the regeneration phenomena can produce an unexpected short-term increment of the battery SOH of about 10% of the nominal capacity.

The analysis of the aforementioned phenomena will be done using data registering two different operational profiles (charge and discharge) at room temperature. On the one hand, charging is carried out in a constant current (CC) mode at 1.5[A] until the battery voltage reached 4.2[V] and then continued in a constant voltage mode until the charge current dropped to 20[mA]. On the other hand, discharge is carried out at a constant current (CC) level of 2[A] until the battery voltage fell to 2.5[V]. Impedance measurements provide insight into the internal battery parameters that change as aging progresses. Repeated charge and discharge cycles result in aging of the batteries. Impedance measurements were done through an electrochemical impedance spectroscopy (EIS) frequency sweep from 0.1[Hz] to 5[kHz]. The experiments were stopped when the batteries reached end-of-life (EOL) criteria, which was a 40% fade in rated capacity (from 2[A-h] to 1.2[A-h]).

Two main operating conditions are thus distinguished: the *normal* condition reflects the fact that the battery SOH is slowly diminishing as a function of the number of charge/discharge cycles; while the *anomalous* condition indicates an abrupt increment in the battery SOH (regeneration phenomena). These phenomena, which are characterized by sudden, momentary, and occasionally considerable regeneration of the battery capacity, are related to physicochemical aspects and temperature/load conditions during charge and discharge cycles. In the case of Li-Ion batteries, the detection of such events is extremely important for a proper implementation of prognostic schemes since they often alter the trend of the SOH prediction curve, thus affecting the performance of prognostic modules based on Bayesian algorithms to estimate the initial conditions of their predictive models.

The study of battery SOH involves the analysis of many different factors, but this research is focused on one of the most critical features associated to it: the life cycle. Life cycle models usually consider a specific term that aims to incorporate part of the phenomenology that is present in the battery degradation process. This term is the Coulomb efficiency, η_c, which is a measure for the amount of usable energy that is expected for the discharge cycle in progress, compared to the capacity exhibited by the battery during the previous discharge cycle (Orchard *et al.*, 2010). Equations (14)-(15) show how this term can be included in a nonlinear dynamic model that can be used for SOH estimation purposes:

$$\begin{cases} x_1(k+1) = \eta_c x_1(k) + x_2(k) x_1(k-1) + \omega_1(k) \\ x_2(k+1) = x_2(k) + \omega_2(k) \end{cases} \tag{14}$$

$$y(k) = x_1(k) + v(k)\ , \tag{15}$$

where k is the cycle index; x_1 is a state representing the battery SOH; x_2 is a state associated with an unknown model parameter that is required to explain minor differences with respect to the expected behavior (which are specific to the monitored battery); $y(k)$ is the measured SOH; ω_1, ω_2 and v are non-Gaussian noises.

Although model (14)-(15) enables the implementation of Bayesian filtering techniques to monitor degradation processes in Li-Ion batteries, it results inadequate when trying to detect and isolate the short and long-term effect of regeneration (self-recharge) phenomena. This fact motivates the development of anomaly detection modules, either based on PF-algorithms as in (Orchard *et al.*, 2011), or information-theoretic measures as the present research proposes.

4. ANOMALY DETECTION MODULE BASED ON INFORMATION-THEORETIC MEASURES AND PARTICLE FILTERING ALGORITHMS

The primary concept behind the proposed anomaly detection scheme is that any sudden abnormal behavior in the system should affect the distribution of the PF-based posterior state estimate. This is caused by the fact that, under abnormal operating conditions, the system model no longer represents the best choice for the importance sampling distribution. As a consequence, the weights associated to particles with low-likelihood undergo strong corrections, increasing the differential entropy of the aforementioned conditional state pdf.

In this sense, the proposed detection module considers a particle filtering algorithm based on model (14)-(15), as state estimator module, and a stage where expression (13) is used to compute the differential entropy of the posterior pdf estimate. The resulting entropy (which is computed at each cycle of operation) corresponds to the output of the detection module. Anomaly detection is carried out by setting a threshold for the entropy estimate. It is of special interest to isolate events where the entropy increases in a sudden manner, or where steadily increases since in both cases it evidences that the particle filtering algorithm is unable to track the process measurements using the predetermined state transition model.

Validation of the proposed scheme is performed on SOH degradation data from an accelerated test at Prognostic Center of Excellence at NASA Ames, where it is of particular interest to detect the moments when battery SOH measurements evidence the existence of capacity regeneration (also known as "self-recharging") phenomena (Orchard *et al.*, 2010). Furthermore, as an additional contribution of the analysis, we will assess what is the actual impact (in terms of early detection) that is associated to an increment in the number of particles in the PF state estimator; taking into account the performance of the proposed entropy-based indicator as the filter uses more particles.

Figure 1 shows the actual SOH degradation data and the results obtained by the proposed detection scheme when 30 particles are used in the implementation of the PF algorithm. In particular, Figure 1 a) illustrates on the difficulty the PF estimator undergoes when the *a priori* transition model (14)-(15) is used to track the degradation of battery capacity in the presence of self-recharge phenomena (for example at the 19^{th}, 30^{th}, and 47^{th} cycles of operation). As it has been mentioned before, the concept behind the entropy-based detection module is to recognize these issues, providing in those cases an indicator that may be used as alarm signal.

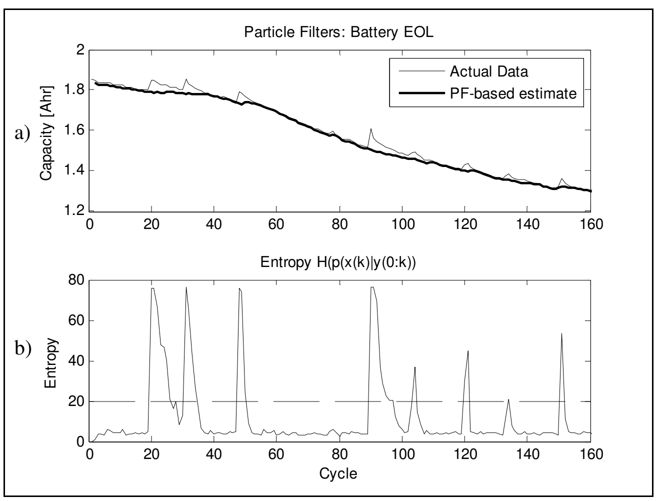

Figure 1. Evolution of the entropy of the posteriori state pdf, using 30 particles within the implementation of the particle filtering algorithm

Figure 1b) depicts the evolution in time of the entropy of the posterior PF-based estimate, for the case of battery SOH degradation. On the one hand, it is important to note that the entropy of the *posterior* state pdf, in absence of self-recharge phenomena, tends to stabilize until it almost behaves like a constant function of time. This stabilization value directly depends on the variance of process and observation noise kernels in equations (14) and (15), which are the actual sources of uncertainty within the implementation of the particle-filtering-based estimator. On the other hand, Figure 1b) also shows that the entropy-based indicator experiences strong modifications on its value in the event of a self-recharge phenomenon (more than eight times in some cases, as in the 19^{th}, 30^{th} and 47^{th} cycle of operation). This fact validates the use of the proposed approach for anomaly detection purposes, triggering the alarm whenever the differential entropy of the posterior state pdf is bigger than a given threshold (e.g., twice the stabilization value for the entropy of the estimate in the absence of capacity regeneration phenomena). However, it is still not clear if an increment on the number of particles would allow computing a lower threshold for the detection module, while simultaneously avoiding the generation of false alarms.

Figure 2 and Figure 3 provide critical information to answer the latter inquire. On the one hand, Figure 2 depicts the obtained results when using N=100 particles in the PF-based estimator, which implies that the computational complexity of the algorithm increases more than three times. On the other hand, Figure 3 shows the case when 500 particles are used.

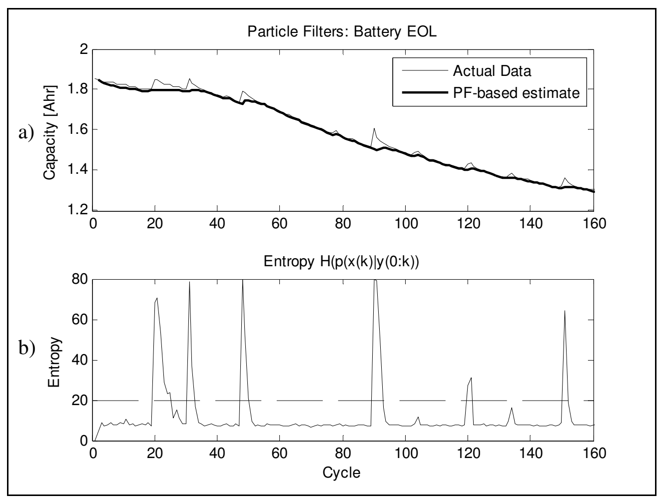

Figure 2. Evolution of the entropy of the posteriori state pdf, using 100 particles within the implementation of the particle filtering algorithm

Although an increment in the number of particles N reduces the amount of time that is required to reach a stabilization value for the entropy of the *posterior* pdf, it does not necessarily increase the capability of the filter to track the evolution of the system in the event of capacity regeneration. As a consequence, the proposed anomaly indicator improves its detection capability (and reduces the probability of false alarms) as the number of particles increases. Moreover, the resulting fault feature (either in the case of N=30 or N=100 particles) allows to easily implement an anomaly detection module based a PF-based detection module (Orchard *et al.*, 2011), which uses the entropy indicator to perform the hypothesis testing and declare the anomaly, for a given false alarm rate.

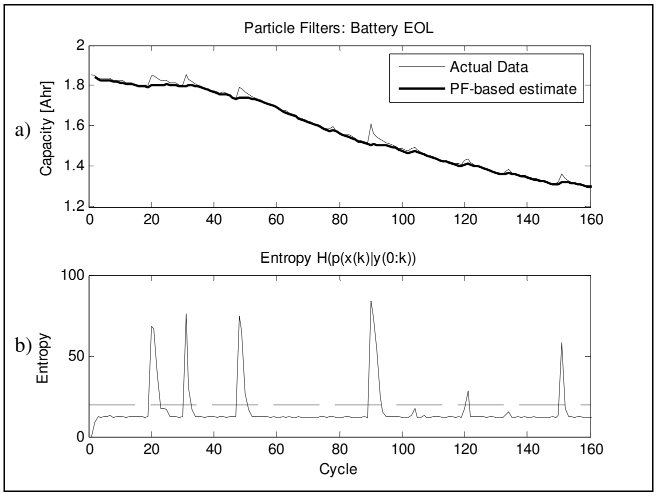

Figure 3. Evolution of the entropy of the posteriori state pdf, using 500 particles within the implementation of the particle filtering algorithm

Finally, it is important to note that a drastic increment in the number of particles (as shown in Figure 3) does not necessarily imply equivalent improvements in the capability of the anomaly detector. Furthermore, this research shows that using less than 100 particles is enough to achieve adequate performance both in terms of detection capabilities and computational effort for the estimation algorithm.

5. CONCLUSION

This paper presents an anomaly detection module that is based on a PF state estimator and information-theoretic measures, which aims at isolating self-recharge phenomena within the SOH degradation process of an energy storage device (Li-Ion battery). From obtained results, we surmise that the proposed anomaly detection approach, which computes a fault indicator from the entropy of the PF-based *posterior* state pdf estimate, is capable of isolating rare and sudden events –such as self-recharge phenomena in the degradation curve– in a simple and efficient manner. Empirical analysis on actual data from acceleration test shows that although an increment in the number of particles within the proposed scheme does improve the detection capability of the proposed approach (also reducing the probability of false alarms), although it does not necessarily compensate the raise on the computational cost of the estimation algorithm. As a result of the aforesaid analysis, an appropriate range for *N* (number of particles) is defined for the case study hereby described.

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REFERENCES

Ajgl J., and M. Šimandl, (2011). “Particle Based Probability Density Fusion with Differential Shannon Entropy Criterion” Proceeding of the 14th International Conference on Information Fusion ISIF, pp. 803-810, Chicago, Illinois, USA.

Andrieu, C., A. Doucet, E. Punskaya, (2001). “Sequential Monte Carlo Methods for Optimal Filtering,” in *Sequential Monte Carlo Methods in Practice*, A. Doucet, N. de Freitas, and N. Gordon, Eds. NY: Springer-Verlag.

Arulampalam, M.S., S. Maskell, N. Gordon, T. Clapp, (2002). “A Tutorial on Particle Filters for Online Nonlinear/Non-Gaussian Bayesian Tracking,” *IEEE Transactions on Signal Processing*, vol. 50, no. 2, pp. 174 – 188.

Ayhan, B., M.Y. Chow, M.H. Song, (2006). “Multiple discriminant analysis and neural network based monolith and partition fault detection schemes for broken rotor bar in induction motors,” *IEEE T. Industrial Electronics*, Vol. 53, No. 4, pp 1298-1308.

Boers, Y., J.N. Driessen, A. Bagchi, and P.K. Mandal, (2010). “*Particle Filter Based Entropy*,” Proceedings of FUSION 2010, Edinburgh, UK.

Cover, T., and J. Thomas, (1991). “Element of Information Theory,” Wiley Interscience, New York, 1991.

Doucet, A., (1998). “On sequential Monte Carlo methods for Bayesian Filtering,” Technical Report, Engineering Department, Univ. Cambridge, UK.

Doucet, A., N. de Freitas, N. Gordon, (2001). “An introduction to Sequential Monte Carlo methods,” in *Sequential Monte Carlo Methods in Practice*, A. Doucet, N. de Freitas, and N. Gordon, Eds. NY: Springer-Verlag.

Ibrahim, A., M. El Badaoui, F. Guillet, F. Bonnardot, (2008). “A new bearing fault detection method in induction machine based on instantaneous power factor,” *IEEE T. Industrial Electronics*, Vol. 55, No. 12, pp 4252-4259.

Isermann, R., and P. Balle, (1997). “Trends in the Application of Model-Based Fault Detection and Diagnosis of Technical Processes,” *Control Eng. Practice*, vol. 5, no. 5, pp. 709-719.

Kadirkamanathan, V., P. Li, M.H. Jaward, and S.G. Fabri, (2002). “Particle filtering-based fault detection in nonlinear stochastic systems,” *International Journal of Systems Science*, 33(4), pp. 259-265.

Khan, M. and M. Rahman, (2009). “Development and implementation of a novel fault diagnostic and protection technique for IPM motor drivers,” *IEEE T. Industrial Electronics*, Vol. 56, No. 1, pp. 85-92.

Lanz, O., (2007). “An information theoretic rule for sample size adaptation in particle filtering,” Proceedings of the 14th International Conference on Image Analysis and Processing (ICIAP), pp. 317–322.

Liverani, S., and A. Papavasiliou, (2006). “*Entropy Based Adaptive Particle Filter*,” 2006 IEEE Nonlinear Statistical Signal Processing Workshop.

Orchard, M., G. Kacprzynski, K. Goebel, B. Saha, G. Vachtsevanos, (2008). “Advances in Uncertainty Representation and Management for Particle Filtering Applied to Prognostics,” 2008 *International Conference on Prognostics and Health Management* PHM 2008, Denver, CO, USA.

Orchard, M., (2009). *On-line Fault Diagnosis and Failure Prognosis Using Particle Filters. Theoretical Framework and Case Studies*, Publisher: VDM Verlag Dr. Müller Aktiengesellschaft & Co. KG, Saarbrücken, Germany, 108 pages. Atlanta: The Georgia Institute of Technology, Diss., 2007.

Orchard, M. and G. Vachtsevanos, (2009). “A Particle Filtering Approach for On-Line Fault Diagnosis and Failure Prognosis,” *Transactions of the Institute of Measurement and Control*, vol. 31, no. 3-4, pp. 221-246.

Orchard, M., L. Tang, K. Goebel, G. Vachtsevanos, (2009). “A Novel RSPF Approach to Prediction of High-Risk, Low-Probability Failure Events,” First Annual Conference of the Prognostics and Health Management Society, San Diego, CA, USA.

Orchard, M., Tang, L., Saha, B., Goebel, K., and Vachtsevanos, G., (2010). “Risk-Sensitive Particle-Filtering-based Prognosis Framework for Estimation of Remaining Useful Life in Energy Storage Devices,” *Studies in Informatics and Control*, vol. 19, Issue 3, pp. 209-218.

Orchard, M., L. Tang, and G. Vachtsevanos, (2011). “A Combined Anomaly Detection and Failure Prognosis Approach for Estimation of Remaining Useful Life in Energy Storage Devices,” *Annual Conference of the Prognostics and Health Management Society 2011*, Montreal, QB, Canada.

Orguner, U., (2009). “Entropy Calculation in Particle Filters,” Signal Processing and Communications Applications Conference.

Ryan, A., (2008). “Information-theoretic tracking control based on particle filter estimate,” AIAA Guidance Navigation and Control Conference

Skoglar, P., U. Orguner, and F. Gustafsson, (2009). “On information measures based on particle mixture for optimal bearings-only tracking,” Proceedings of IEEE Aerospace Conference 2009, Big Sky, Montana, USA

Tolani, D., M. Yasar, S. Chin, A. Ray, (2005). “Anomaly Detection for Health Management of Aircraft Gas Turbine Engines,” 2005 American Control Conference, pp. 459-464, Portland, OR, USA.

Verma, V., G. Gordon, R. Simmons, S. Thrun, (2004). “Particle Filters for Rover Fault Diagnosis,” *IEEE Robotics & Automation Magazine*, pp. 56 – 64.

Zhang, B., T. Khawaja, R. Patrick, M. Orchard, A. Saxena, G. Vachtsevanos, (2009). “A Novel Blind Deconvolution De-Noising Scheme in Failure Prognosis,” *IEEE Transactions on Instrumentation and Measurement*, vol. 58, no. 2, pp. 303-310.

Zhang, B., C. Sconyers, C. Byington, R. Patrick, M. Orchard, and G. Vachtsevanos, (2011). “A Probabilistic Fault Detection Approach: Application to Bearing Fault Detection,” *IEEE Transactions on Industrial Electronics*, vol. 58, no. 5, pp. 2011-2018.

Zhou, W., T. Habetler, R. Harley, (2008). “Bearing fault detection via stator current noise cancellation and statistical control,” *IEEE T. Industrial Electronics*, Vol. 55, No. 12, pp 4260-4269, Dec. 2008.