Point processes for bearing fault detection under non-stationary operating conditions

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ABSTRACT

Bearing faults represent the most frequent mechanical faults in rotational machines. They are characterized by repetitive impacts between the rolling elements and the damaged surface. The time intervals between two impacts are directly related with the type and location of the surface fault. These time intervals can be elegantly analyzed within the framework of renewal point processes. With such an approach the fault detection and identification can be performed irrespective of the variability of rotational speed. Furthermore, we show that by analyzing the entropy of the underlying counting process by means of wavelet transform, one can perform fault detection and identification without any information about the operating conditions. The effectiveness of the approach is shown on a data-set acquired from a two–stage gearbox with various bearing faults operating under different rotational speeds and loads.

1. INTRODUCTION

According to several surveys (MRWG, 1985a, 1985b, 1985c; Albrecht, Appiarius, & Shrama, 1986) one of the most common mechanical failure are bearing faults. Consequently, a variety of techniques for detection of bearing faults have been developed in the past decades. They rely mainly on analysis of vibrational signals acquired from machines operating under constant and known operating conditions. However, such conditions are rarely met in practice. Therefore, in this paper we address the issue of bearing fault detection under variable and presumably unknown operating conditions within the framework of renewal point processes.

In the currently available approaches, fault detection under variable speed is resolved by acquiring precise information about the current speed and load. Most common approach in such a case is time–synchronous averaging (TSA), a method which compensates for the speed fluctuations (Zhan, Makis, & Jardine, 2006; Stander & Heyns, 2005). In the same manner Parker et al. (2000) applied higher order spectra analysis for the detection of various bearing faults under different load conditions. Bartelmus and Zimroz (2009) successfully performed fault detection in multi–stage gearboxes by taking into account the information about both variations in speed and load. Although the proposed approaches give satisfactory results they heavily depend on accurate measurements of the current speed and load of the monitored gearbox.

Can bearing faults be reliably detected in spite of unknown variable load and speed conditions? Poulimenos and Fassois (2006) provided a thorough analysis on modeling and analysis of nonstationary vibration signals in time domain. Padovese (2004) gave a hybrid time–frequency approach for analyzing transient signals. Baydar and Ball (2000) performed detection of gear deterioration under different loads using instantaneous power spectrum by employing Wigner–Ville distribution (WVD). They have successfully realized fault detection of gear faults irrespective of the operating conditions.

Another way of overcoming the difficulties induced by variable operating conditions is to analyze the statistical characteristics of the produced vibrational signals. In case of bearing faults, the most informative source can be found in the distribution of the time intervals between two adjacent impacts occurring between the rolling elements and the damaged bearing surface. By doing so we can employ the framework of point processes in modeling the distribution of these times. The framework of point processes was successfully applied in the areas like modeling the neural spikes, earthquake prediction, describing environmental processes etc. However in the field of fault detection, to the best of the authors knowledge, Antoni and Randall (2003) are the only authors that tried to analyze the distribution of these interevent times by treating them as an ordinary renewal process. However, their analysis was focused only on cases when bearings are operating under presumably constant and known operating conditions.

In our approach we go one step further by removing the limitation of constant and known operating conditions. Furthermore, we will show that the produced bearing vibrations may be modeled as renewal process with Inverse Gaussian interevent distribution. We will show that with such an approach one can construct an unified model for bearing fault vibrations, capable of modeling both single and multiple bearing faults. The statistical properties of the model additionally allow proper modeling under both constant and variable operating conditions. Finally, we will propose one way of performing fault detection based on the statistical characteristics of the renewal process analyzed through wavelet transform.

2. Basics of point processes

The point processes represent a segment of the theory of random processes that are most commonly used for characterizing random collections of point occurrences (Cox & Isham, 1980). In the simplest form, these points usually represent the time moments of their occurrences. This class of point processes is also known as temporal point processes.

Generally it is considered that the observed random points occur at time moments $\cdots, t_1, t_2, t_3, \cdots$. A point process is *simple* if all the observed points are distinct i.e. $t_i \neq t_j$ for $i \neq j$. Additionally the point process is called *orderly* if the number of points N at any moment t and interval length Δt is:

$$\lim_{\Delta t \to 0} \Pr\{N[t, t+\Delta t] > 1\} = 0. \tag{1}$$

Besides the occurrence times t and the number of points N another way of defining a point process is by the interevent times, i.e. the time between two adjacent points. Thus, the nth interevent time is defined as $T_n = t_n - t_{n-1}$.

One general goal is to derive the statistical properties of the mechanism that generates the observed random occurrences. The properties of a point process may be specified in several equivalent ways. The most common approach is to specify the non-negative number $N \in \mathbb{Z}^+$ that specifies the number of observed occurrences between time 0 and time T. Another way to specify the statistical characteristics is through the distribution of the interevent times $\{T_1, \cdots, T_n\}$ where $T_i = t_i - t_{i-1}$. Finally, the approach for describing the statistical characteristics that will be used throughout this paper is based on the frequency with which the events occur around the time moment t with respect to the history of the process up to that particular moment $\mathcal{H}_t$. This statistical property is usually called conditional intensity function $\lambda(t, \mathcal{H}_t)$. Each of these specifications is equivalent and the most appropriate one may be used (Daley & Vere-Jones, 2003a).

For the corresponding conditional density function $f(t|\mathcal{H}_t)$ one can also define its corresponding cumulative function $F(t|\mathcal{H}_t)$. Consequently the conditional intensity function can be defined as:

$$\lambda^*(t) = \frac{f(t|\mathcal{H}_t)}{1 - F(t|\mathcal{H}_t)}. \tag{2}$$

The denominator of (2) is also known as survivor function $s(t)$ (Vreeswijk, 2010):

$$s(t) = \Pr\{\text{event not before } t|\mathcal{H}_t\}. \tag{3}$$

The form of the conditional intensity function completely describes the underlying point process. In general, as shown in Eq. (2), this function depends on both the current time t as well as the complete point process history up to that moment $\mathcal{H}_t$. However, by allowing specific limitations one can define several specific types of point processes. If we let $\lambda^*(t)$ to become independent of $\mathcal{H}_t$, it will define a non-stationary Poisson process. A stationary version is defined by fixing the value of $\lambda^*(t) = const.$ to a specific constant that defines the rate of the underlying Poisson process. With such limitations one can readily show that the interevent times of the Poisson process are independent and distributed with exponential distribution.

A further generalization of this concept is the class of renewal point processes (Lowen & Teich, 2005). Similarly like in the Poisson process, the interevent times of such processes are independent and identically distributed (i.i.d.) but with arbitrary distribution $f(t)$ supported on semi-infinite interval $[0, +\infty)$, i.e. $f(t) = 0$ for $t < 0$. Consequently, the occurrence of a new event becomes dependent only on the time since the previous one.

One can proceed even further by removing the condition of independence of the interevent intervals. If the interevent intervals $\{X_n\}$ form a Markov chain where the length of the X_{n+1} depends only on the length of the previous interval X_n one obtains a so-called Wold process (Daley & Vere-Jones, 2003a). By modeling different transition kernels of the Markov chains one can model various types of point processes (Daley & Vere-Jones, 2003b). The form of the transition directly determines the form of the conditional intensity function (Asmussen, 2003). Therefore, one can define the most suitable transition form of the governing Markov chain that will fit the observed random process. At the same time there is an equivalent opportunity of fitting a specific form of governing chain with respect to an observed history of an arbitrary point process. Such an identification procedure can be implemented by employing well established methods from the area of hidden Markov models.

3. Modeling bearing faults using the point process framework

Generally, the vibrations produced by bearings with localized surface faults have been analyzed in cases of constant and known rotational speed. In such a case the generated vibrational patterns $x(t)$ can be modeled as (Antoni & Randall, 2002; Randall & Antoni, 2011):

$$x(t) = \sum_{i=-\infty}^{+\infty} A_i s(t + \Delta T_i) + n(t), \tag{4}$$

where A_i represent the amplitude of the ith impact, $s(t)$ is the excited impulse response, $n(t)$ is additive background zero-mean noise and $\Delta T_i = T_{i+1} - T_i$ represents the time between two consecutive impacts. The time period ΔT_i contains all the needed diagnostic information.

The intervals ΔT_i can be treated as interevent times of a point process. By imposing a specific distribution of these intervals we can specify a model of the generating point process. Consequently by analyzing the statistical characteristics of such a point process we can infer about the underlying bearing fault.

3.1 Point process model for localized bearing faults

Tandon and Choudhury (1999) specified the characteristic impact frequencies for different bearing surface fault as functions of bearing dimensions and rotational frequency of the rotating ring. Therefore, the interevent times T_i in the model (4) are directly related to the bearing's rotational speed. Thus, in order to model the interevent time distribution we have to specify a suitable condition intensity function. A way to model the rotational speed is by modeling the change in the rotational angle $\theta(t)$ of the rotating ring:

$$\theta(t) = \nu t + \sigma W(t), \tag{5}$$

where $W(t)$ is standard Brownian motion with normally distributed increments with zero mean and some constant variance (Matthews, Ellsworth, & Reasenberg, 2002), ν is directly related to rotational speed and σ accommodate the speed fluctuations. Thus a single evolution occurs when the angle $\theta(t)$ reaches the threshold 2π. A simple realization of such a process is shown in Figure 1. Schrödinger has shown that the distribution of the time needed for a Wiener process (5) to reach a fixed threshold a follows the Inverse Gaussian distribution (Folks & Chhikara, 1978):

$$f(t) = \frac{a}{\sigma\sqrt{2\pi t^3}} \exp\left\{-\frac{(\nu t - a)^2}{2\sigma^2 t}\right\}, \tag{6}$$

usually denoted as $t \sim IG(a/\nu, a^2/\sigma^2)$. Since the parameters ν and σ are constant in time, the resulting point process is stationary with firing rate ν.

3.2 Statistical characteristics of Inverse Gaussian renewal process

Since the Inverse Gaussian renewal process will be the basis of our model we will derive the necessary statistical properties. Besides the conditional intensity function and the interevent times distribution, a point process can be analyzed through its counting process N i.e. the probability distribution $p_N(t)$ of observing N consecutive events within a time interval $[t_0, t)$, where usually $t_0 = 0$. In order to derive the distribution $p_N(t)$ one has to calculate the joint probability distribution $p(t_0, t_1, \cdots, t_N)$.

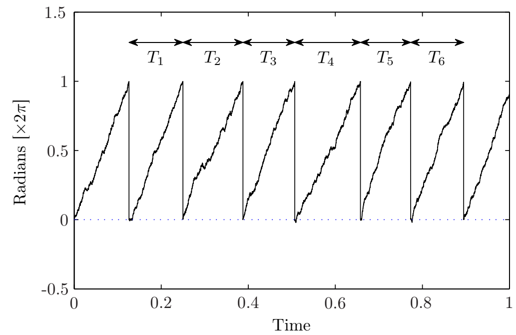

Figure 1. Realization of renewal process with Inverse Gaussian interevent distribution

Firstly, the probability of a single event occurring up to time t_1 is $p_1(t_1) = p(t_1)$, where $p(t)$ is the probability distribution of a single event. The probability of observing N events up to time t_N is:

$$p_N(t_N) = \int_0^{t_N} p_{N-1}(t_{N-1}) p(t_N - t_{N-1}) dt_{N-1}, \tag{7}$$

where $p(t_N - t_{N-1})$ is the interevent probability distribution. The Eq. (7) is a convolution of two p.d.f. defined on the non-negative real line, since both $t_n > 0$ and $t_n > t_{n-1}$, and it can be easily calculated using the Laplace transforms of both $p_{N-1}(t)$ and the distribution of interevent times $f(t)$:

$$p_{L,N}(s) = p_{L,N-1}(s) f_L(s) = f_L^N(s), \tag{8}$$

where $p_{L,N-1}(s) = \mathcal{L}\{p_{N-1}(t)\}$, $f_L(s) = \mathcal{L}\{f(t)\}$ and $\mathcal{L}\{\cdot\}$ stands for the Laplace transform.

In case of Inverse Gaussian interevent times the Laplace transform $f_L(s)$ of (6) is:

$$f_L(s) = \exp\left\{\frac{\nu a}{\sigma^2}\left[1 - \sqrt{1 + 2\frac{\sigma^2}{\nu^2}s}\right]\right\} \tag{9}$$

Calculating then the $\mathcal{L}^{-1}\{f_L^N(s)\}$ we obtain (Tweedie, 1957):

$$f_N(t) = \frac{Na}{\sigma\sqrt{2\pi t^3}} \exp\left\{-\frac{(\nu t - Na)^2}{2\sigma^2 t}\right\}. \tag{10}$$

The obtained result has quite intuitive explanation. Namely, in (6) the threshold for the Wiener process was set at a. Therefore the time t needed to observe N consecutive crossings has the same distribution as if one elevated the threshold up to Na.

4. Bearing fault detection using Inverse Gaussian interevent distribution

Having in hand the statistical properties of the governing renewal process we can now analyze how the model performs under different specific operating conditions. The goal of these analysis is to show that the model is valid for both constant and variable operating speed as well as in cases of single and multiple bearing faults.

4.1 Constant rotating speed

In cases when the rotating speed is strictly constant, the value of σ in (5) and (6) will become zero, hence the distribution becomes Dirac impulse i.e. $f(t;\nu,\sigma=0)=\delta(\nu t-a)$. Consequently, the corresponding point process will be transformed into a truly periodic sequence of impacts.

(Pseudo) Cyclostationarity Small variations in the rotating speed can be accommodated by allowing small values of σ in (6). The autocorrelation function of the stationary renewal process (4) with $\Delta T \sim IG(\nu,\sigma)$ can be derived through its interevent probability distribution. Using (6) as interevent probability distribution it can be readily shown that the autocorrelation function converges to the constant value

$$\lim_{\tau\to\infty} R_{xx}(\tau)=\frac{2\sigma^2}{a\nu}<\infty. \tag{11}$$

As already analyzed by Antoni and Randall (2002), such a process can be treated as pseudo cyclostationary in cases when σ is sufficiently small, i.e. when the speed fluctuations are just a few percent.

4.2 Variable rotating speed

The modeling of completely arbitrary speed variations can be done by allowing both $\nu_{shaft}=\nu(t)$ and $\sigma_{shaft}=\sigma(t)$ in (6) to become time dependent. The resulting process is called doubly stochastic process which in essence is nonstationary process.

Despite the nonstationary characteristics, for cases where $\nu(t)$ varies slowly, one can employ the so–called modified variability measure C_{V2}. This measure is fairly insensitive to variations in the firing rate of the point process and is defined as (Ponce-Alvarez, Kilavik, & Riehle, 2010):

$$C_{V2}=\frac{2|\tau_{i+i}-\tau_i|}{\tau_{i+1}-\tau_i}, \tag{12}$$

where τ_i represents the interevent time between the events $i-1$ and i.

4.3 Single bearing fault

A crucial information when analyzing the bearing faults is the underlying shaft speed. The instantaneous shaft speed can be obtained by differentiation of the random process (5) governing the current angle $\theta(t)$

$$\frac{d\theta(t)}{dt}=\omega_{shaft}=\nu_{shaft}+\sigma_{shaft}\eta(t), \tag{13}$$

where $\eta(t)$ is the governing Gaussian process. The rotational speed of each bearing component is directly related to the speed of the rotating shaft (13) (Tandon & Choudhury, 1999). Consequently, each bearing fault is governed by a random process of form (13) multiplied by a constant C_k. This constant is determined by the geometrical characteristics of the bearing which determine the ratio between the angular speed of the rotating ring and a specific bearing element, i.e. $k\in\{$Inner ring, Outer Ring, Bearing Cage, Ball spin$\}$. Consequently, each bearing fault can be represented by a renewal process governed by Inverse Gaussian distribution with $\nu=C_k\nu_{shaft}$ and $\sigma=C_k\sigma_{shaft}$. Consequently, the distribution of the interevent times for the kth component becomes:

$$t_k \sim IG\left(\frac{a}{C_k\nu_{shaft}},\frac{a^2}{C_k^2\sigma_{shaft}^2}\right) \tag{14}$$

4.4 Multiple faults on different bearing components

As already stated single bearing faults differ in the statistical properties of the governing IG distributions. In cases of multiple bearing faults we can observe the overall produced vibrations as a sum of several random processes each governed by its own IG probability distribution with respect to the underlying fault.

In general case the sum of IG r.v. does not necessarily leads to a result governed by IG distribution. However, the distributions (14) governing the possible bearing faults fulfill the necessary condition that the ratio

$$\frac{Var[t_k]}{E[t_k]}=\frac{a\sigma_{shaft}^2}{C_k\nu_{shaft}^3}\frac{C_k\nu_{shaft}}{a}=\frac{\sigma_{shaft}^2}{\nu_{shaft}^2} \tag{15}$$

remains constant, i.e. independent of C_k. Thus the sum of such renewal processes results into new renewal process with IG interevent distribution:

$$\begin{aligned} S &= \sum_k t_k \\ &\sim IG\left(\frac{a}{\nu_{shaft}}\sum_k C_k,\frac{a^2}{\sigma_{shaft}^2}\left(\sum_k C_k\right)^2\right) \end{aligned} \tag{16}$$

The Eq. (16) comes in hand for the cases of multiple faults. As shown by Eq. (14), distinctive distribution of interevent times governs each type of bearing fault. In such a case the observed vibrations can be regarded as a sum of several repetitive excitations of possibly different impulse responses, unlike the case of single fault as described by (4). Since such a sum fulfills the conditions (15) the resulting point process can be treated in the same manner as the cases with single fault.

5. Detection of impact times using wavelet transform

In order to apply the presented framework for bearing fault detection we should be capable of determining the times ΔT_i from (4) as precise as possible. By analyzing the bearing fault model (4), one can observe that this signal is dominated by sudden excitations of impulse responses positioned at the impact times. The time location of these impacts can be determined sufficiently accurately by analyzing the signal with wavelet transform using a mother wavelet number of vanishing moments v_m higher then the order of the impulse response $s(t)$ in (4) (Unser & Tafti, 2010).

In such a case the selected wavelet will act as a v_mth order differential operator. Consequently, the time moments where the vibration signal $x(t)$ has discontinuities will be marked with wavelet coefficients with higher values. This time moments will coincide with the time moments when the impacts occur.

Therefore by applying Mallat (2008) thresholding process of the calculated wavelet coefficients, one can obtain accurate information about the impact times, i.e. information about the underlying bearing fault. This process is shown in Figure 2.

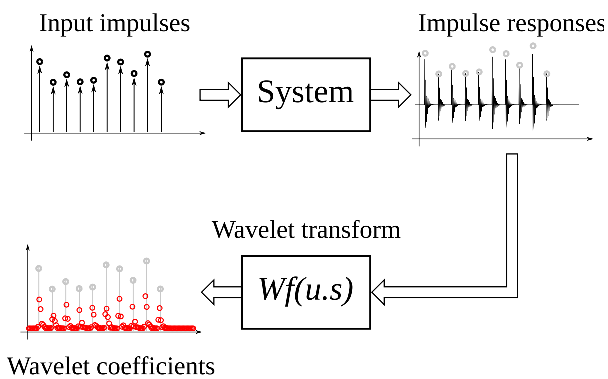

Figure 2. Wavelet as differential operator

Thorough analysis on the influence of the selection of mother wavelet on the accuracy of the decomposition for such signals has been performed by Unser and Tafti (Unser & Tafti, 2010) and Van De Ville, Blu, and Unser (Van De Ville et al., 2005). They have concluded that the crucial parameter is the number of vanishing moments of the mother wavelet rather then the selection of the "optimal" mother wavelet that will closely match the underlying process. By selecting wavelet with sufficiently high number of vanishing moments we can analyze the impulse responses $s(t)$ from (4) regardless of their variable form due to the changes of the transmission path.

5.1 Fault detection procedure

Detecting the impact moments using wavelet transform allows significant simplification in the fault detection process. The calculated wavelet coefficients preserve the statistical characteristics of the probability distribution that is generating the random impulses. Consequently, within a fixed observed window of length T one can use the distribution of the number of impacts N as information that is closely related with the underlying fault.

Due to the orthogonality of the wavelet transform the energy of the observed signal is preserved within the amplitude of the wavelet coefficients. In case when no impacts occur one will observe the wavelet transform just from the noise component $n(t)$ from (4). Therefore, under assumption of Gaussian noise, the energy will be evenly spread throughout the wavelet coefficients. Thus, the entropy of this distribution will be highest. In cases when the impacts are present the bulk of the energy of the signal will be concentrated in a small number of wavelet coefficients coinciding with the impact times, thus the entropy of the wavelet coefficient will decrease. The level of change is directly connected to the number of impulses occurring within the observed time window T. Therefore, by characterizing the distribution $p(N, T)$ of number of impacts N within a time window with length T, one can correlate the changes in the entropy of the wavelet coefficients with a particular bearing fault.

The distribution $p(N, T)$ can be determined by the survivor probability $s_N(t)$ (3). The survivor probability $s_N(t)$ gives a probability of observing the N impact time after a time moment t:

$$s_N(t) = \int_t^{+\infty} f_n(t')dt'. \tag{17}$$

Therefore the probability of observing N impulses within a time window of length T becomes

$$p(N, T) = s_{N+1}(T) - s_N(T). \tag{18}$$

By calculating the Laplace transform of (17) and inserting it in (18) the distribution becomes

$$p(N, s) = \frac{1 - f_L(s)}{s} f_L^n(s), \tag{19}$$

where $f_L(s)$ is the Laplace transform of the IG distribution as defined by Eq. (9). In order to simplify the analysis we will concentrate only on the expected number and the variance of the distribution $p(N, T)$. These values can be approximated by taking into account only a limited number of Taylor expansion terms. Hence for the expected value $E[N, T]$ and the variance $Var[N, T]$ when $f(t) \sim IG(a/\nu, a^2/\sigma^2)$ we obtain

$$\begin{aligned} E[N, T] &= \nu T + \frac{\sigma^2 \nu - 1}{2} \\ Var[N, T] &= \sigma^2 \nu^2 T \end{aligned} \tag{20}$$

As intuitively expected, these two expressions prove that the number of events within a time window depend on the firing rate ν and the variation σ.

However in case of bearing vibrations, as already shown by (16), each bearing fault differ by the factor C_k multiplying the shaft rotational speed and its fluctuation. As a result of this dependence each bearing fault is governed by different interevent distribution $f(t)$, thus the number of expected impulses within a fixed time window of size T will differ among different fault combinations. Consequently, the wavelet energy distribution will be different and the faults will be distinguishable.

Besides the changes caused by different faults, the distribution $p(N,T)$ will change with changes in the rotational speed. As a result of this change the wavelet energy entropy will vary. However, according to (16) the variations in the rotational speed will influence every bearing fault in the same manner, i.e. by adding and additional constant to each coefficient C_k in (16). Consequently, notwithstanding the variations in the speed the entropy values the entropy values for particular bearing fault will be always distinguishable, since the underlying IG distributions will remain different among various bearing faults.

6. Experimental results

The experimental data was acquired on a laboratory two–stage gearbox (PHM, 2009) (cf. Figure 3). The test runs include 7 different fault combinations and one fault–free reference run. From this set we have used the fault runs that contained bearing faults. Each set–up was tested under 5 different rotational speeds of the input shaft: 30, 35, 40, 45 and 50 Hz. Furthermore, two test runs were performed per each combination of different fault and speed.

The detailed list of the introduced faults is listed in Table 1. It should be noted that bearing faults were introduced only on the bearings 1–3, and all the remaining bearings were kept fault–free during the whole experimental runs. Additionally, the shaft imbalance was introduced on the *Input shaft*, whereas the sheared keyway fault was located on the *Output shaft*.

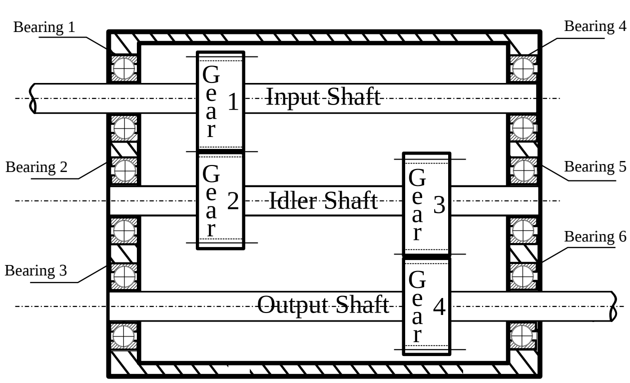

Figure 3. Schematic description of the used two–stage gearbox

6.1 Analysis

Each of the four experimental runs was analyzed using Daubechies8 mother wavelet (Daubechies, 1992). The energy entropies calculated from the corresponding wavelet coefficients are shown in Figure 4. From these results we should note the three key features.

First, the wavelet energy entropy of the fault free run is constant regardless the rotational speed. In absence of fault the observed signal reduces only to background noise $n(t)$ from Eq. (4). Since no information about the machine state is contained in this signal the entropy is constant.

Secondly, the fault 7 shows highest entropy from the other two bearing faults, followed by fault 8 and fault 6 having the lowest entropy. By examining the fault details from Table 1, one can notice that fault 7 contains only a single damaged element, fault 8 two damaged elements and fault 6 with three damaged elements. As already stated in Section 4.4, the occurrence of multiple faults can be treated as sum of several r.v. governed by IG distribution. Thus, according to (16) the resulting random process will have higher firing rate. A higher firing rate in essence contributes to increased number of expected impact occurrences N within a time window T, according to (20). Finally, this effect influences the shape of the wavelet energy distribution in such a manner that the overall entropy decreases.

According to (14), the increase of the rotational speed causes an increase in the firing rate of the IG process, hence decreasing the wavelet energy entropy. This effect has identical influence on all bearing faults. Consequently, as the speed increases the difference among wavelet energy entropies for different bearing faults increases too. Hence, the faults become more distinguishable as the rotational speed increases, as shown in Figure 4.

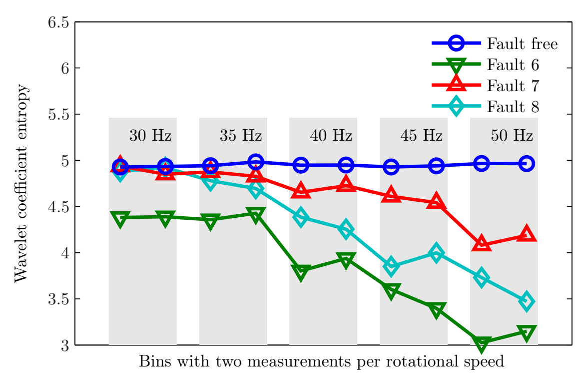

Figure 4. Wavelet coefficients energy entropy for selected bearing faults

6.2 Comments on results and possible improvements

The results support the hypothesis that bearing faults can be detected by employing a statistical model of Inverse Gaussian renewal process and wavelet energy entropy. One of the main assets of the approach is that it is requires no information about the operating conditions. This becomes more evident by comparing the fault detection capabilities of this approach with approaches that incorporate information about the operating conditions. A fine example is the study that we have performed on the same experimental data by applying spec-

Run Number	Gear 1	Gear 2	Gear 3	Gear 4	Bearing[1] 1	Bearing[1] 2	Bearing[1] 3	Shaft fault
#1	Fault Free (FF)							
#6	FF	FF	FF	Broken	Inner	Ball	Outer	Imbalance
#7	FF	FF	FF	FF	Inner	FF	FF	Keyway Sheared
#8	FF	FF	FF	FF	FF	Ball	Outer	Imbalance

[1] Faults were introduced only on Bearings 1–3 (cf. Figure 3). The other three bearings were kept fault–free during all experimental runs.
(Boškoski, Juričić, & Stankovski, 2010)

Table 1. Fault details for each experimental run

tral kurtosis (SK) and envelope analysis (Boškoski & Urevc, 2011). Although the bearing fault isolation capabilities of SK are superior, the fault detection results are comparable, i.e. the set of experimental runs containing bearing faults were accurately detected by both approaches.

Additionally this study provides a possible explanation of the results that we have obtained by the analysis of the same experimental set using a set of entropy functions calculated from the wavelet packet coefficients (Boškoski et al., 2010). Those results showed that based solely on the entropy of wavelet packet coefficients one can perform accurate fault detection of gears and bearings regardless of the operating conditions. The relations (16) and (20) provide an explanation how different bearing faults alter the probability distribution of the wavelet coefficients hence modifying its entropy.

An immediate future improvement to this study would be the application of goodness–of–fit tests that will test the hypothesis that the observed point process is governed by IG distribution. The result of such tests can serve as a starting point for deciding whether bearing faults are causing the changes in the observed probability distribution of wavelet coefficients. Furthermore, with such tests we will be able to quantify the effectiveness of the approach by considering the probability of inaccurate detection.

7. Conclusion

The bearing fault model based on a renewal process governed by Inverse Gaussian (IG) interevent has shown to be capable of modeling the fault vibrational patterns under various operating conditions. This approach provides an unified view on the statistical properties of the produced vibrational signals regardless of the operating conditions. Such a unified concept offers several advantages.

Firstly the rate ν and the variance σ of the IG renewal process contain all the necessary information about the present bearing fault. Furthermore, such an approach allows fairly simple modeling of multiple bearing faults, since the resulting process can be treated as a sum of inverse Gaussian random variables. As bearing faults are related to the shaft rotational speed, the necessary condition is fulfilled so the resulting sum is again governed by Inverse Gaussian distribution.

Secondly, having defined the distribution of the renewal point process we were able to derive the probability of observing N impacts within a time window T. Thus, we have shown that by employing this distribution it is guaranteed that various bearing faults can be distinguished without any knowledge about the geometrical characteristics of the monitored bearings.

Using the distribution of the counting process N we have presented one possible way of using wavelet transform in obtaining an estimate of the number of impacts within a time T by analyzing the wavelet coefficient energy entropy. The results show that various bearing faults can be successfully detected without any knowledge about their geometrical characteristics. Additionally, the behavior of the calculated feature supports the hypothesis that the produced bearing vibrations can be treated as renewal point process with IG interevent distribution.

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