Confidence Assessment in Fatigue Damage Prognosis

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ABSTRACT

This paper presents a methodology for assessing the confidence and the predictive capability of prognosis models, using the application of fatigue crack growth analysis. Structures with complicated geometry and multi-axial variable amplitude loading conditions are considered. Several models – finite element model, crack growth model, retardation model, surrogate model, etc. – are efficiently connected through a Bayes network and the parameters of these models are calibrated after collecting inspection data. The results of the calibration are then used to develop a Bayesian confidence metric to assess the confidence of the models used in fatigue crack growth analysis. Three types of uncertainty are included in analysis: (1) natural variability in loading and material properties; (2) data uncertainty, due to crack detection uncertainty, measurement errors, and sparse data; (3) modeling uncertainty and errors in crack growth analysis, and finite element analysis. The proposed methodology is illustrated using a numerical example of surface cracking in a cylindrical structure.*

1 INTRODUCTION

The scientific community has increasingly resorted to the use of computational models to predict the performance of engineering components and systems so as to facilitate risk assessment and management, inspection and maintenance scheduling, and operational decision-making. Model-based prognosis, i.e. predicting the performance of a system using a physics-based model is promising for health management. However, no model can perfectly represent the system and hence it is necessary to include model form errors and model uncertainty in the prognosis. Secondly, complex engineering systems may have to be modeled using multiple models that interact with one another. In such cases, each model has its own error/uncertainty and the interaction between the errors of multiple models is non-trivial. Some errors are deterministic while some others are stochastic. Systematic methods are needed to quantify the uncertainty and confidence associated with the model prediction. Hence, prognosis methods need to facilitate the integration of multiple models, quantification of different types of uncertainty (physical variability, data uncertainty, and model uncertainty) and assess the confidence associated with the results of prognosis.

This paper uses the fatigue crack growth problem as a foundation to develop this methodology. The objective of this problem is to predict the crack growth in a structural component as a function of number of load cycles. Fracture mechanics-based models such as Paris law (Paris et al., 1961) are calibrated from experimental testing of coupons and used for crack growth analysis. The extrapolation of the results of laboratory testing to practical applications (complicated geometry with multi-axial variable amplitude loading conditions) not only introduces several types of uncertainty but also questions the confidence in the prediction.

There are several issues to be considered while using fracture mechanics-based models for fatigue damage prognosis. The aforementioned fracture mechanics-based models are currently used to predict the growth of the crack only in the long crack region. This paper uses an equivalent initial flaw size (Liu and Mahadevan, 2008) to replace the short crack growth calculations and to make direct use of a long crack growth model. A modified Paris law is used in conjunction with a Wheeler retardation model to account for variable amplitude loading. Three stress intensity factors (mode-I, mode-II, and mode-III) are calculated using finite element analysis and combined to obtain an equivalent mode-I stress intensity factor using a characteristic plane approach. Further, the finite element model is expensive and hence, replaced with an efficient Gaussian process surrogate model. Note that the paper uses modified Paris' law, Wheeler's model, Gaussian process surrogate model, etc. only for the purpose of illustration and any other suitable combination of models may be used.

Fatigue crack growth is a stochastic process and the various sources of uncertainty – natural variability, data uncertainty and model uncertainty – need to be considered in crack growth analysis. Natural variability includes variability in loading, material properties, geometry and boundary conditions. The geometry of the specimen and boundary conditions are considered deterministic in this paper. The uncertainty in experimental data needs to be accounted for. An error term (treated as a random variable) is added to the crack growth law to represent the fitting error since experimental data were used to estimate its coefficients. Further, the model coefficients are also treated as random variables. The discretization error in finite element analysis can be estimated using Richardson extrapolation. This finite element analysis is replaced by a Gaussian process model whose prediction uncertainty can be modeled as a random variable. These different types of uncertainty are summarized in Table. 1.

This paper develops a systematic procedure to (1) connect different models – finite element model, surrogate model, crack growth model, etc. efficiently; (2) quantify the uncertainty in each model; (3) propagate the different sources of uncertainty and calculate the overall uncertainty in the crack growth prediction; and (4) use experimental data to calibrate the model parameters and assess the confidence in the crack growth prediction.

The various models involved in crack growth analysis and the different sources of uncertainty can be connected effectively through a Bayes network, which is useful in two different ways, (1) in an inverse problem (Urbina, 2009), where the crack growth model is calibrated after experimental data (crack size after a number of load cycles) are collected and, (2) in a forward problem (Sankararaman et al., 2009), where the probability distribution of the crack size is calculated as a function of number of loading cycles.

The goal of this paper is to quantify the uncertainty in the crack growth prediction and assess the associated confidence by quantifying "the extent to which available experimental data support the prognosis results." A two-step methodology has been devised for this purpose: (1) Prior probability distributions are chosen for the model parameters and in the presence of experimental evidence (crack size after number of loading cycles), and a Bayesian inference technique is used to update their distributions and calculate posterior probability distributions (2) The prior distributions (before calibration) and the posterior distributions are both used for crack growth prediction (prognosis), thereby resulting in "prior-prognosis" and "posterior-prognosis". These two results are compared and a Bayesian metric is proposed to assess the confidence in the model prediction.

Table 1. Classification of Sources of Uncertainty

SOURCES OF UNCERTAINTY		
Natural Variability	**Data Uncertainty**	**Model Errors and Uncertainty**
1) Loading 2) Material properties 3) Geometry 4) Boundary conditions	1) Sparse data (to construct probability distributions for material properties) 2) Output measurement error 3) Crack detection uncertainty	1) Crack growth law : model form error and parameter uncertainty 2) Finite element discretization error 3) Surrogate model prediction uncertainty

The following section reviews the literature relevant to this topic and motivates the current study. Section 3 discusses the crack growth modeling procedure used in this paper. Section 4 discusses the several sources of uncertainty and proposes methods to handle them. Section 5 outlines the proposed Bayesian inference technique for calibrating the crack growth model parameters. Section 6 develops the Bayesian metric to assess the confidence in model prediction. Section 7 illustrates the proposed methods using an example, surface cracking in a cylindrical crack.

2 PREVIOUS WORK

Numerous studies have addressed probabilistic crack growth and life prediction but focused mainly on natural variability in loading, geometry and material properties. However, the need of probabilistic prediction has been emphasized and the need to assess the confidence in the prediction of models is evident. Several metrics have been proposed to quantify the predictive capability of prognosis models. Saxena et al. (2009) reviewed the literature and discussed the various metrics used for assessing the performance of prognostic algorithms. These metrics are based on the error observed, standard deviation of the quantity observed, sensitivity, reliability and cost-benefit analysis. Also several new metrics such as Prognostics horizons, α-λ performance were proposed. Many of these metrics are calculated on the basis of comparison between the model predictions with the observed set of experiments. However, experimental results may not be available to facilitate this comparison. Sometimes, a current set of experimental results may be available and it may be of interest to assess the prediction of the model in future. There is an evident need to develop a methodology which can use available experimental data to assess the prediction of the model in future.

Rebba and Mahadevan (2006) proposed a Bayesian methodology to extrapolate the confidence in the prediction of the model from one domain to another. Though this was a generic procedure, time-dependent models were not considered and the various sources of uncertainty such as physical variability, data uncertainty and model uncertainty were not clearly delineated. Further, this procedure is not directly applicable if there are multiple interacting models.. In this paper, the approach in (Rebba and Mahadevan, 2006) is extended to overcome these limitations. One important advantage in the proposed methodology is that the various quantities of interest can be connected through a Bayes network, which allows for addition of more models and sources of uncertainty. Further, for use in time-dependent problems, a dynamic Bayesian network can be constructed and the same set of principles developed by Rebba and Mahadevan (2006) can be applied. This approach consists of two steps: (1) model calibration using experimental evidence, and 2) prediction of future behavior and assessment of the prediction.

Model calibration is a well-known statistical problem and several techniques of statistical inference may be employed to solve this problem. As the various quantities of interest in this paper are connected through a Bayesian network, a Bayesian inference-based calibration technique has been employed in this work. The various quantities of interest are assigned prior probability distributions which are then updated to calculate the posterior probability distributions. Such techniques have been employed by various researchers in the past (Urbina, 2009), but they have been used to calibrate either a single parameter (Makeev et al., 2006) or the parameters of a single model (Cross et al., 2007). These studies have considered simple structures under uniaxial constant amplitude loading conditions. In this paper, several different models are combined in crack growth analysis (finite element analysis, surrogate model development, crack growth model) and multiple model parameters are calibrated using the experimental evidence, while accounting for the different sources of uncertainty in a systematic manner.

While fatigue life prediction and probabilistic fracture mechanics are well-known research problems, only a few studies have focused on the various sources of uncertainty involved in crack growth analysis. The "damage prognosis" project at the Los Alamos national laboratory (Doebling and Hemez, 2001; Hemez et al., 2003; Farrar et al., 2004; Farrar and Lieven, 2006) addressed this problem in detail and researchers proposed probabilistic methods as a solution to this problem. Several sources of errors and uncertainty such as discretization error, surrogate model error, crack growth model error, etc. were not included.

Besterfield et al. (1991) combined probabilistic finite element analysis with reliability analysis to predict crack growth in plates. Patrick et al (2007) introduced an online fault diagnosis and failure prognosis methodology applied to a helicopter transmission component. A crack growth model (Paris law) was used for fatigue life prediction. Sources of uncertainty such as data uncertainty and model uncertainty were not considered in these works.

Gupta and Ray (2007) developed algorithms for online fatigue life estimation that relied on time series data analysis. Pierce et al (2008) discussed the application of interval set techniques to the quantification of uncertainty in neural networks. These works only considered the physical variability in input data and other sources of uncertainty were not investigated in detail.

Orchard et al (2008) used the method of particle filters for uncertainty management in fatigue prediction. Papazian et al (2009) developed a structural integrity prognosis system (SIPS) for modeling and simulation. While measurement errors and sensor data were considered in detail, solution errors, variability of model parameters, randomness in loading, etc were not considered.

It is clear from the above survey of existing literature that past studies have mostly considered physical variability in loading and material properties but did not consider other sources of uncertainty such as data and model uncertainty. When approximate computational models are used to predict crack growth in the presence of uncertain data about variable loading and structural properties, there is a clear need to assess the confidence in the prediction of such models before practical application. The existing methods 1) do not account for these sources of uncertainty, and 2) are not suitable when there is no experimental data available for direct comparison. This paper develops systematic techniques for model calibration and fatigue life prediction under various sources of uncertainty – physical variability, data uncertainty, and model uncertainty/errors. A Bayesian metric is proposed to assess the confidence in model prediction. This does not require experimental evidence collected over the entire life of the specimen; data collected at any time instant can be useful in assessing the confidence in the model prediction at a future time. Thereby, this methodology overcomes the limitations of the existing approaches and is suitable for solving practical problems. The various steps in the proposed methodology are explained in the following sections.

3 CRACK GROWTH MODELING

This section summarizes the steps of fatigue damage prognosis for structures with complicated geometry and multi-axial variable amplitude loading conditions. The various models – finite element model, surrogate model, crack growth law, retardation model, characteristic plane model, etc. - involved in this procedure are briefly described and these models are connected systematically to predict the crack growth as a function of number of cycles.

The rigorous approach to fatigue life prediction would be to perform crack growth analysis starting from the actual initial flaw, accounting for voids and non-metallic inclusions. The concept of an equivalent initial flaw size (EIFS) was proposed to bypass small crack growth analysis and make direct use of a long crack growth law for fatigue life prediction. A physics-based formula for the EIFS was proposed by Liu and Mahadevan (2008) in terms of material properties (ΔK_{th}, the threshold stress intensity factor and σ_f, the fatigue limit) and geometric properties (Y).

$$a_0 = (1/\pi)(\Delta K_{th} / Y\sigma_f)^2 \tag{1}$$

The equivalent initial flaw size is used as an initial condition for the long crack growth model. This quantity is a model parameter; Eq. 1 is used to construct the prior distribution of EIFS which is updated after collecting experimental data.

The methods developed in this paper can be applied with any long crack growth low. For the purpose of illustration, this paper uses a modified Paris' law with retardation effects as shown:

$$da/dN = \varphi^r C\,(\Delta K)^n(1- \Delta K_{th}/\Delta K)^m \tag{2}$$

Note that several alternate models (Yuen et al., 2006; Schjive, 1976; Noroozi et al., 2008) have been proposed to tackle variable amplitude loading conditions. This paper uses Wheeler's retardation model (Sheu et al., 1995) only to illustrate the proposed uncertainty quantification methodology, and other appropriate models can also be used instead of the Wheeler model. In Eq. 2, φ^r refers to the retardation parameter (Sheu et al., 1995) , and is equal to unity if $a_i + r_{p,i} > a_{OL} + r_{p,OL}$ where a_{OL} is the crack length at which the overload is applied, a_i is the current crack length, $r_{p,OL}$ is the size of the plastic zone produced by the overload at a_{OL}, and $r_{p,i}$ is the size of the plastic zone produced at the current crack length a_i. Else, φ^r is calculated as:

$$\varphi^r = (r_{p,i} / (a_{OL}+r_{p,OL}-a_i))^{\lambda} \tag{3}$$

In Eq. 3, λ (shaping exponent) is a curve fitting parameter for the original Wheeler model (Yuen et al., 2006). Song. et al. (2001) observed that crack growth retardation actually takes place within an effective plastic zone. Hence the size of the plastic zone can be calculated in terms of the applied stress intensity factor (K) and yield strength (σ) as:

$$r_p = \alpha\,(K/\sigma)^2 \tag{4}$$

In Eq. 4, α is known as the effective plastic zone size constant which is calculated experimentally (Yuen et al., 2006). The retardation model parameters are calibrated for particular experimental conditions, which need to be matched to the problem at hand for proper application (Yuen et al., 2006). The expressions in Eq. 3 and Eq. 4 can be combined with Eq. 2 and used to calculate the crack growth as a function of number of cycles as:

$$\int N = \int dN = \int 1/(\varphi^r \mathrm{C}\,(\Delta \mathrm{K})^{\mathrm{n}}(1-\Delta \mathrm{K}_{\mathrm{th}}/\Delta \mathrm{K})^{\mathrm{m}})da \tag{5}$$

For structures with complicated geometry and loading conditions, the integral in Eq. 5 is to be evaluated cycle by cycle, calculating the stress intensity factor (ΔK) in each cycle of the crack growth analysis. Further, if the loading is multi-axial (for example, simultaneous tension, torsion and bending), then the stress intensity factors corresponding to all three cracking modes need to be taken into account. This can be accomplished using an equivalent stress intensity factor. If K_I, K_{II}, K_{III} represent the mode-I, mode-II and mode-III stress intensity factors respectively, then the equivalent stress intensity factor K_{eqv} can be calculated using a characteristic plane approach proposed by Liu and Mahadevan (2005).

$$K_{eqv} = \frac{1}{B}\sqrt{(k_1)^2 + (\frac{k_2}{s})^2 + (\frac{k_3}{s})^2 + A(\frac{k^{\mathrm{H}}}{s})^2} \tag{6}$$

In Eq. 6, K_1, K_2, K_3 are the parameters associated with mode I, II, and III cracking, respectively. K_H is related to hydrostatic stress. s is the ratio of modes II and I stress intensity factor at a specific crack growth rate (da/dN). A and B are material parameters. The use of the characteristic plane approach for crack growth prediction under multi-axial variable amplitude loading has been validated earlier with several data sets (Liu and Mahadevan, 2005).

In each cycle, the deterministic finite element analysis is substituted with a surrogate model because it is computationally inefficient to use a FEA for cycle-by-cycle integration. The use of a Gaussian process (GP) surrogate model for this purpose has been demonstrated in previous work (Sankararaman et al., 2009; Sankararaman et al., 2010). A few runs of the finite element analysis are used to train this surrogate model and then, this GP model is used to predict the stress intensity factor for other crack sizes and loading cases (for which finite element analysis has not been carried out). The Gaussian process model predicts the mean and the variance of the stress intensity factor in each cycle. Refer to McFarland (McFarland, 2008) for details of this methodology.

The flow of computations through the various models used in this section is shown in Fig. 1.

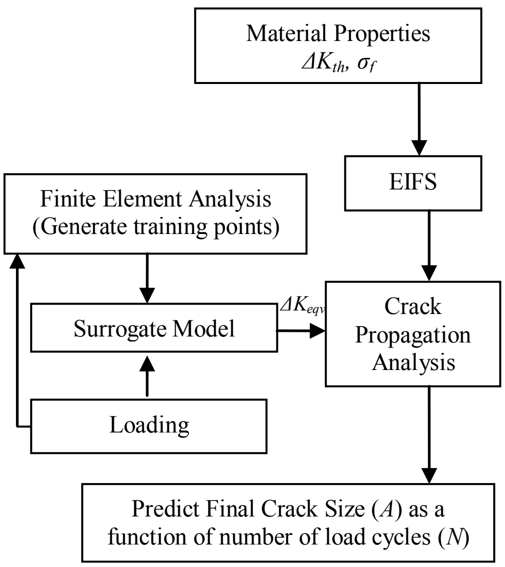

Fig. 1. Crack Growth Analysis

The algorithm in Fig. 1 for crack propagation analysis is deterministic and does not account for errors and sources of uncertainty. The following section quantifies different sources of uncertainty associated with each of the blocks in Fig. 1.

4 SOURCES OF UNCERTAINTY

This section proposes methods to quantify the various sources of uncertainty and errors associated with crack growth prediction. These various sources of uncertainty can be classified as:

I. Physical Variability
- A. Loading
- B. Material properties

II. Data Uncertainty
- A. Material properties
- B. Crack detection uncertainty
- C. Output measurement error

III. Model Uncertainty/Errors
- A. Equivalent initial flaw size
- B. Crack growth law uncertainty (model form and parameters)
- C. Uncertainty in Stress Intensity factor
  - a. Discretization error in FEA
  - b. Uncertainty in surrogate model output

(Note: The material parameters considered in this work are threshold stress intensity factor and fatigue limit. Natural variations in other material properties such as the friction coefficient, Poisson Ratio, Young's modulus, etc. and geometrical properties are assumed to be small and not included in this paper. However, if desired, these can also be included in the proposed framework by constructing different finite element models (for different geometry and boundary conditions) and use these runs to train the Gaussian process surrogate model.)

The following subsections briefly discuss each source of uncertainty and propose methods to quantify them.

4.1 Physical Variability in Loading

The methods developed in this paper are applicable to all generic random load histories and a block loading history is chosen here only for the purpose of illustration. In this paper, the block length is assumed to have a uniform distribution (U(0,500)) and the maximum and minimum amplitudes for each block are assumed to follow normal distributions (N(8,2) and N(24,2) respectively, in KNm) respectively. A sample loading history is shown in Fig. 2.

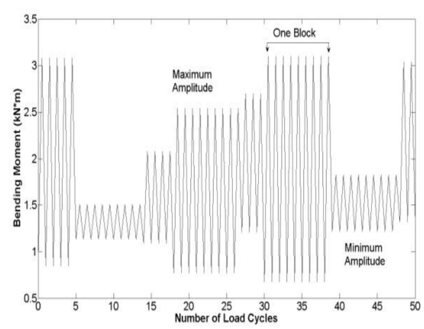

Fig. 2. Sample Loading History

4.2 Material Properties (Natural Variability and Data Uncertainty)

Material properties such as the threshold stress intensity factor (ΔK_{th}) and the fatigue limit ($\Delta\sigma_f$) have natural variability. Experimental data are available in literature to characterize their probability distributions. Sometimes, the available data may not be sufficient to construct a probability distribution (data uncertainty due to sparse data). In order to address this type of data uncertainty, the quantity of interest is resampled and represented using a probability distribution, whose parameters are in turn represented by probability distributions. For e.g., if a quantity X has a distribution $f_X(x|\boldsymbol{P})$, then the distribution of the parameters $\boldsymbol{P}$ can be calculated. This leads to a family of distributions for X which can be integrated using the principles of conditional probability and total probability as:

$$f_X(x) = \int f_{X|P}(x) f_P(P) dP \quad (7)$$

4.3 Crack Detection Uncertainty and Output Measurement Error

Crack growth data measured from laboratory experiments is directly used for the inference of EIFS in the approach followed in this paper. These measurements are affected by several factors, such as environment, equipment, operator skills, etc. and it is necessary to account for these factors include the measurement errors in the inference of EIFS. For an in-service component, non-destructive inspection (NDI) technique is commonly used for damage detection. Several metrics could be used to evaluate the performance of NDI, such as probability of detection (POD), flaw size measurement accuracy, and false call probability (FCP). These criteria are developed from different methods, and they are used to evaluate different aspects of NDI performance. However, Zhang and Mahadevan (2001) showed that these quantities are mathematically related. POD and FCP can be derived from size measurement accuracy, which measures the difference between actual values and observed values of the crack size.

Inspection results may be of two types: (a) crack is detected and measured; (b) crack is not detected. For the case of detecting a crack and also measuring its size, size measurement accuracy could be used to quantify the uncertainty in crack growth inspection data, with the following expression determined by regression analysis (Zhang and Mahadevan, 2001):

$$a_m = \beta_0 + \beta_1 a + \varepsilon_m \quad (8)$$

In Eq. 9, a_m is the measured flaw size; a is the actual flaw size; β_0 and β_1 are the regression coefficients; and ε_m represents the unbiased measurement error, commonly assumed as a normal random variable with zero mean and standard deviation σ_τ. The value of σ_τ is different for each inspection technique used.

4.4 Equivalent Initial Flaw Size

As mentioned earlier, this paper treats the equivalent initial flaw size (initial condition of the differential equation) as a model calibration parameter. However, a prior probability distribution can be estimated using Eq. 2, and can be updated using data. This prior distribution is calculated using the probability distribution of material properties such as the threshold stress intensity factor (ΔK_{th}) and fatigue limit (σ_f).

4.5 Uncertainty in Crack Growth Model

In this paper, a modified Paris law has been used for illustration, however, the methodology can be implemented using any other crack growth model. The uncertainty in crack growth model can be subdivided into two different types: crack growth model form uncertainty, and uncertainty in model parameters. In each cycle, a normally distributed random variable ε_{cg} is added to the crack growth rate equation in Eq. 2 to account for model form uncertainty. Prior probability distributions can be assumed for model coefficients and model form error term ε_{cg}, and updated after collecting evidence.

4.6 Uncertainty in Stress Intensity Factor

The calculation of stress intensity factor ΔK is done in two stages. First, a few finite element analysis runs are required to train the GP model. Second, the GP model is used to predict the stress intensity factor. Finite element solutions have discretization errors which are deterministic and surrogate model prediction uncertainty is stochastic. First, the errors in finite element analysis are calculated and the solutions are corrected before training the Gaussian process surrogate model. The surrogate model prediction is stochastic, and hence the surrogate model uncertainty is addressed through sampling techniques.

The finite element discretization error is calculated based on Richardson extrapolation in this paper. This method is based on convergence analysis and calculates the error based on solutions from three different mesh sizes. For further details of this methodology, refer can be found in Sankararaman et al. (2010).

Once the finite element solutions are corrected for discretization error, they are used to train the surrogate model. The prediction of the Gaussian process model is a normal distribution with mean and variance (McFarland, 2008). Hence, in each cycle, a value of stress intensity factor is sampled from this normal distribution and used in crack growth analysis.

5 INSPECTION DATA AND CALIBRATION

Several prognosis metrics have been proposed based on comparison between available inspection data and model prediction. However, inspection data may be available only at a particular time and it may be necessary to assess the prognosis confidence at a future time instant when there is no data available. In order to accomplish this, a two step methodology is proposed: (1) calibrate the model parameters using available data, and (2) use the calibration results to assess the prognosis at a future time. This section describes the calibration methodology while the proposed confidence metric is discussed in Section 6.

Inspection data can be in three forms: (1) Crack is not detected after N cycles; (2) Crack is detected after N cycles but size not measured; (3) Crack is detected and the size is measured after N cycles. There are several possible quantities that can be calibrated; these include (1) equivalent initial flaw size; (2) parameters of modified Paris's law (C, m); (3) error (ε_{cg}) of the modified Paris law; (4) material properties such as threshold stress intensity factor (ΔK_{th}), and fatigue limit (σ_f); and (5) output (final crack size) measurement error (ε_{exp}). All of these parameters can be connected through a Bayes network as shown in Fig. 3. Note that this is a dynamic Bayes network, which connects the variables in a particular load cycle (i) to the next load cycle ($i+1$).

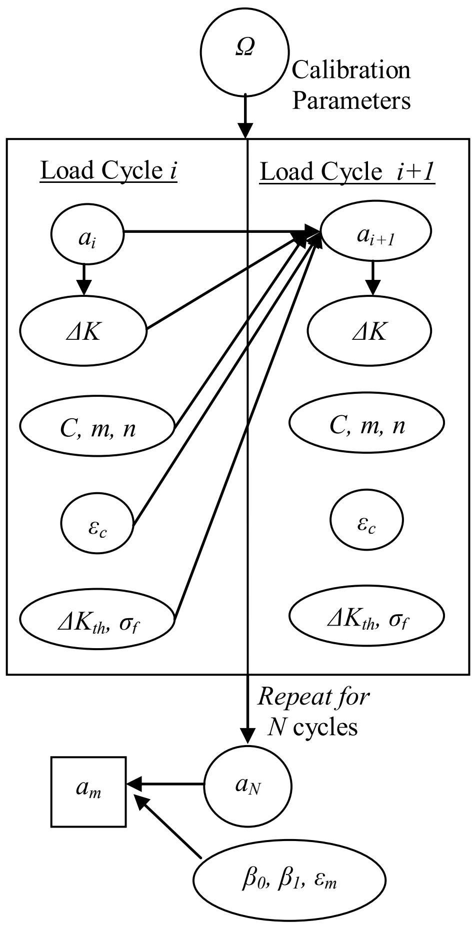

Fig. 3. Dynamic Bayes Network

Let $\boldsymbol{\Omega}$ denote the vector of quantities that are selected for calibration. Assume that there is a set of m experimental data points (A_i, N_i, i = 1 to m), i.e. the measured crack size A after N loading cycles, and n "no crack detections" after A_i (i = 1 to n) number of cycles. Using this information, it is possible to calculate the probability distributions of $\boldsymbol{\Omega}$ using Bayes theorem. In this paper, experimental data have been simulated by assuming true distributions for $\boldsymbol{\Omega}$.

Bayesian updating is a three step procedure: (1) Prior probability distributions are assumed for each of the parameters in $\boldsymbol{\Omega}$. (2) The likelihood of $\boldsymbol{\Omega}$ is calculated as being proportional to the probability of observing the given data conditioned on $\boldsymbol{\Omega}$; and (3) The prior and likelihood of $\boldsymbol{\Omega}$ are multiplied and normalized to calculate the posterior probability distribution. Finally, the joint distribution of $\boldsymbol{\Omega}$ is used to calculate the individual distributions of the parameters, i.e. equivalent initial flaw size (a_0), model parameters (C) and crack growth model error (ε_{cg}).

The likelihood in Step 2 is a function of $\boldsymbol{\Omega}$ and it needs to account for the other sources of uncertainty explicitly. For every realization of $\boldsymbol{\Omega}$ a Monte Carlo analysis is required for the calculation of the likelihood function. The various steps in the construction of the likelihood function are as follows: (1) Design experiments for trainings points and construct the Gaussian process surrogate model after correcting for discretization errors. The likelihood function then needs to be constructed as a function of $\boldsymbol{\Omega}$. Hence, for a given value of $\boldsymbol{\Omega}$, (2) Generate a loading history (N cycles); (3) Use the deterministic crack growth analysis algorithm in Fig. 1 to calculate the final crack size at the end of N_i (for i = 1 to m) cycles; (4) Repeat steps 2 and 3 and calculate the probability distribution of crack size at the end of N_i (for i = 1 to max(m,n)) cycles by sampling the stochastic quantities not contained in $\boldsymbol{\Omega}$. Let this distribution be denoted by $f(a)$. Use Eq. 9 to calculate $f(a_m|a)$. This probability density function can be used to calculate the likelihood of $\boldsymbol{\Omega}$.

If no crack is detected, then the likelihood function can be calculated as being "proportional to the probability of not detecting a crack" based on $f(a_m|a)$.

$$L(\Omega) \propto \int_{a_m<0} f(a_m \mid a) f(a \mid N_i) da \tag{9}$$

If a crack is detected but size not measured, the likelihood function can be calculated as:

$$L(\Omega) \propto \int_{a_m>0} f(a_m \mid a) f(a \mid N_i) da \tag{10}$$

If a crack is detected and the size is measured, then the likelihood function can be calculated as:

$$L(\Omega) \propto f(a_m = A_i \mid N_i) \tag{11}$$

Combining all inspection data, i.e. "no detection", "crack detections without size measurement", and "crack detection with size measurement", the overall likelihood function can be calculated by multiplying Eq. 9, Eq. 10, and Eq. 11 for each inspection.

Finally, the likelihood is multiplied with the prior and normalized to calculate the posterior distribution. This joint posterior distribution can be used to calculate the individual posterior distributions of the parameters. The following section describes how these results can be used to assess the confidence in prognosis.

6 PROGNOSIS CONFIDENCE ASSESSMENT

This section proposes a methodology to assess the confidence in the future prediction of the fatigue crack growth model using current experimental data. Rebba and Mahadevan (2006) developed a method using which the confidence in the model prediction can be extrapolated from experimental conditions to actual usage conditions. The proposed methodology uses this concept and extends their work to time-dependent fatigue crack growth problems, where the confidence in the model prediction is extrapolated from the current instant to the future.

Consider the crack-growth algorithm discussed in Section 3. The probability distribution of the crack size (A) can be calculated as a function of number of load cycles (N) after accounting for the various sources of uncertainty in a systematic manner. For complete details of this algorithm, refer Sankararaman et al. (2009). The overall aim is to assess the confidence in this prediction. Rebba and Mahadevan (2006) used the concept of Bayes factor (B) for confidence assessment. The Bayes factor can be expressed as the ratio of likelihoods of "the model being correct" and "the model being incorrect" (Zhang and Mahadevan, 2003) as:

$$B = \frac{P(Data \mid Model\_is\_correct)}{P(Data \mid Model\_is\_incorrect)} \qquad (12)$$

In the presence of uncertainty, the quantity of interest (i.e. the quantity predicted by the model, for e.g. crack size) can be represented using a probability distribution. When experimental data is available for validation, the distribution of the quantity of interest is updated to obtain a posterior distribution. Rebba and Mahadevan (2006) showed that the Bayes factor can be expressed the ratio between the prior distribution and the posterior distribution of the quantity of interest, and the associated confidence can be calculated as $B/(B+1)$. The similar approach is used here to calculate the confidence in crack growth prediction. Here, the Bayes factor is a metric for validating the crack growth model using experimental data. This is then used to calculate the confidence in the prediction in future.

Jiang and Mahadevan (2006) derived a risk-based measure for the threshold Bayes factor value for model acceptance, based on the costs involved in type-I and type-II errors in hypothesis testing. If the Bayes factor is equal to one, then the associated confidence is equal to 50%, thereby stating that the two hypotheses "the model is correct" and "the model is incorrect" are both equally likely. Note that the term "confidence" used in this paper, refers to the posterior probability of the "model being correct" and is not related to confidence intervals in statistical hypothesis testing..

First, the prior distributions of the quantities $\boldsymbol{\Omega}$ (defined earlier in Section 5) are used to calculate the prior distribution (f') of the crack size (A) as a function of the number of load cycles (N). Then, the posterior distributions of the quantities $\boldsymbol{\Omega}$ are used to calculate the posterior distribution (f'') of the crack size (A) as a function of the number of load cycles (N). Then the Bayes factor (B) is calculated as the ratio between the posterior and the prior distributions. However, in this case, the Bayes factor can be measured at any crack size and hence it is not clear where to calculate the Bayes factor. Two options may be explored: (a) Bayes factor at the mean of the posterior distribution, and (b) Expected value of the Bayes factor. The first metric is a first-order approximation and hence is preliminary, whereas the second metric is more rigorous.

If A_m is the mean of posterior distribution of the crack size (A), then the first metric can be calculated as:

$$B(A_m) = \frac{f''(A = A_m)}{f'(A = A_m)} \qquad (13)$$

The second metric, i.e. the expected value of the Bayes factor can be calculated as:

$$E(B) = \int \frac{f''(A)}{f'(A)} f''(A) dA \qquad (14)$$

The confidence associated with the prediction can be calculated as $B/(B+1)$. Higher the Bayes factor value, higher is the confidence associated with the model prediction. According to Jiang and Mahadevan (2006), a higher confidence implies a lower risk in terms of decision making.

In summary, Sections 4 and 5 proposed a Bayesian methodology for (1) calibration of model parameters using inspection data (crack size measurements after number of cycles, including cases where no cracks were detected), and (2) using the results of calibration to assess the confidence in prognosis. The following section illustrates the proposed methodology using a numerical example.

7 NUMERICAL EXAMPLE

A two radius hollow cylinder with an elliptical crack in fillet radius region is chosen for illustrating the proposed methodology. An initial semi-circular surface crack configuration is assumed and the crack shape develops over time into a semi-elliptical planar, surface crack under biaxial loading.

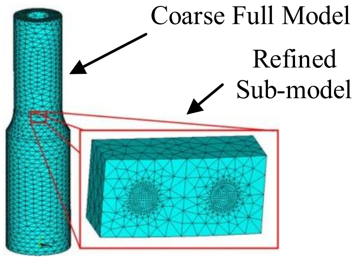

Fig. 4. Surface Crack in a Cylinder

The commercial finite element software ANSYS (ANSYS, 2007) is used to build and analyze the finite element model, shown in Fig. 4. A sub-modeling technique is used near the region of the crack for accurate calculation of stress intensity factors. Sankararaman et al. (2009) describe in detail the modeling of the specimen and also provide the material and geometrical properties.

The geometry of the structure, Young's modulus, Poisson ratio, boundary conditions, friction coefficient between crack faces, etc are treated to be deterministic in this paper. This finite element model is run for 10 different crack sizes and 6 different loading cases and these results are used to train the Gaussian process surrogate model for the calculation of the stress intensity factor.

Experimental evidence of 20 data points is simulated and used for calibration and confidence assessment as explained in Section 4. In this paper, for the purpose of illustration, the equivalent initial flaw size (θ), model parameters (C) and crack growth model error (ε_{cg}) are chosen for calibration. The prior distribution of EIFS is constructed using Eq. 2. A non-informative prior is used for the standard deviation of crack growth model error (ε_{cg}). A zero mean is assumed for this quantity because it represents the fitting error and only the variance is updated. The prior distribution of C is obtained from Liu and Mahadevan (2008).

The results of prognosis using these probability distributions are shown in Fig. 5. The mean of the crack size and the 90% bounds are plotted as a function of number of cycles.

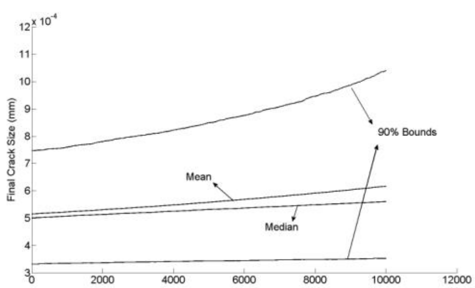

Fig. 5. Mean, Median, and 90% Bounds

The results of model calibration using Bayesian inference are shown in Table 2.

Table 2. Results of Parameter Estimation

Distribution	a_0 (mm)	C x10^{-13} m/cycle	ε_{cg} (mm)
True			
Mean	0.375	6	0
Std. Deviation	0.025	1	0.001
Prior			
Mean	0.5	6.5	0
Std. Deviation	0.1	4	0.01
Posterior			
Mean	0.37	6.1	0
Std. Deviation	0.024	1.3	0.005

From Table 2, it is seen that while the distribution of EIFS is almost the same as the true distribution, the distributions for the other parameters are not identical to the true distributions. During calibration, the equivalent initial flaw size is used only once to start the crack growth propagation procedure, whereas the model parameter C and the error ε_{cg} are used in every loading cycle, thereby adding uncertainty with every additional loading cycle. This uncertainty may be decreased by frequent inspections and collecting more data for calibration.

The ratio of the prior and the posterior distributions of the crack size is used to calculate the Bayes factor, and hence the confidence in prediction. These results are shown in Fig. 6 and Fig. 7 respectively.

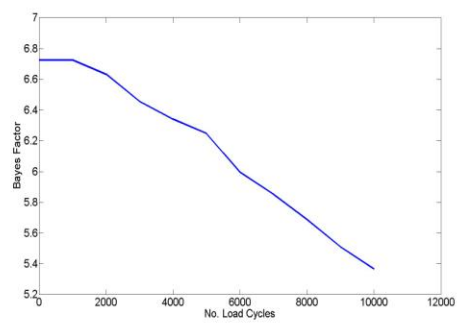

Fig. 6. Expected Value of Bayes Factor

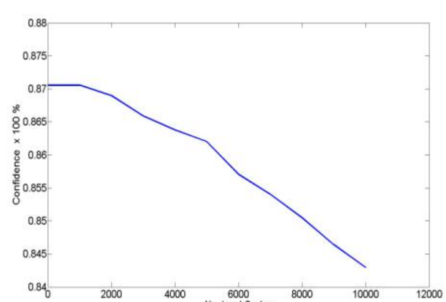

Fig. 7. Confidence in Model Prediction

From Fig. 6, it is observed that the Bayes factor decreases steadily, thereby indicating a decreasing confidence in the prediction made by the model as the number of cycle increases. With each additional loading cycle, there is more uncertainty added to the crack growth analysis. Future loading conditions, crack growth errors, etc. are unknown and hence, the confidence in the model prediction decreases with time.

8 CONCLUSION

This paper presented a methodology for model calibration and confidence assessment in fatigue damage prognosis. Structures with complicated geometry and multi-axial variable amplitude loading conditions were considered. The finite element analysis used for the calculation of stress intensity factor was replaced using a Gaussian process surrogate model, thereby reducing the computational effort. Different sources of uncertainty – physical variability, data uncertainty, and model error/uncertainty – were included in the crack growth analysis. Different types of model errors – discretization errors, crack growth errors, surrogate model uncertainty – were considered explicitly. Deterministic errors were corrected and stochastic errors were addressed using sampling techniques during uncertainty propagation.

A Bayesian inference-based methodology was proposed where the parameters of different models are calibrated using inspection results (crack size after number of cycles, including no crack detected). The results of prognosis using prior distributions, and the results of prognosis using posterior distributions are compared to compute a Bayesian confidence metric to assess the confidence in the model prediction. The prior and posterior distributions of the predicted crack size were used to assess the confidence in model prediction. The general trend observed was that the confidence in prediction decreases steadily over time, as expected.

Note that this approach is applicable to prognosis in several engineering disciplines and the problem of crack growth analysis was used as an illustration to develop the methodology. In general, the proposed methodology provides a fundamental framework in which multiple models can be connected through a Bayes network and the predictive capability of the overall prognosis model can be assessed quantitatively. The use of a Bayes network also facilitates the systematic treatment of various sources of uncertainty for both model calibration and prognosis. The proposed Bayes factor metric provides a measure of confidence in the model prediction. If the confidence in the model prediction reduces below a critical level at a particular time in future, then it would be desirable to inspect the structure at that time instant. Therefore, the proposed method can also guide maintenance planning and inspection scheduling.

ACKNOWLEDGMENT

The research reported in this paper partly supported by funds from the NASA ARMD/AvSP IVHM project under NRA Award NNX09AY54A (Project Monitor: Dr. K. Goebel, NASA AMES Research Center) through subcontract to Clarkson University (No. 375-32531, Principal Investigator: Dr. Y. Liu). The support is gratefully acknowledged.

NOMENCLATURE

A	Crack size
N	Number of loading cycles
a_0	Equivalent initial flaw size
ΔK_{th}	Threshold stress intensity factor
ΔK	Stress intensity factor in each cycle
σ_f	Fatigue limit
Y	Geometry factor
φ^r	Wheeler's retardation coefficient
C, m, n	Parameters of modified Paris' law
B	Bayes Factor

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