A state-space model for multi-scale fatigue damage prognosis

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ABSTRACT

[*]Fatigue damage prognosis and diagnosis is of critical importance for the structural health management and is still a challenging problem despite extensive progresses during the last few decades. Traditional fatigue prognosis methods are cycle-based and have some inherent difficulties in the fatigue damage analysis. For example, cycle-based approach requires that the realistic load history needs to be transformed to the cycle history, which makes it impossible to perform the concurrent fatigue damage prognosis at the material and structure level. A novel methodology for concurrent multi-scale fatigue damage prognosis is proposed in this paper. The proposed methodology is based on an incremental time-based fatigue formulation, which is not cycle-based. By coupling the time-based fatigue growth mode with system dynamics, a set of first order differential equations can be setup using state-space concept to solve the structural dynamic response and fatigue crack growth concurrently. Hence, the fatigue damage prognosis can be performed both within an individual loading cycle scale and a loading history scale. Numerical examples for a single degree-of-freedom (DOF) system and a multiple DOF system are demonstrated using the proposed methodology. Coupon-level experimental data under variable amplitude loadings are used to validate and investigate the prognosis performance of the proposed concurrent state-space model.

1 INTRODUCTION

Structural health management is of critical importance to provide the safety, reliability and affordability for structural systems. Fatigue damage is one of the most common failure modes of engineering materials and structures and greatly impacts the long term durability of structures (Nagy *et al.,* 1998). Extensive research on fatigue damage diagnosis and prognosis was made. For example, several diagnostic techniques have been developed to detect the fatigue damage such as: ultrasonic, electromagnetic, and thermographs (Link *et al.,* 2009). Unlike the diagnosis, fatigue prognosis attempts to forecast system performance by assessing the current damage state of the system (Farrar *et al.,* 2006), which usually involves the extraction of the damage-sensitive features from the dynamic response measurements from an array of sensors (Inman *et al.,* 2005) and the estimation of the remaining useful life of a system under the influence of the crack propagation. Two important components need to be carefully addressed to achieve this goal. First, operational loading data and structural response need to be collected from the usage monitoring system. Following this, prediction of the damage based on the information from the first step need to be performed. Most classical fatigue analysis methods are based on the cycle-based damage propagation approach. Several models have been proposed based on this concept, such as: stress-life (S-N) models and crack growth rate approaches (da/dN~ΔK). Earlier study of cycle-based fatigue analysis can be traced back to Wholer (1870) and Basquin (1910) (Schutz, 1996). Fatigue damage is correlated with the range of mechanical driving forces, such as the stress or stress intensity factor range in those cycle-based approaches. However, cycle-based approaches introduce some complexities in fatigue damage prognosis applications. For example, stress ratio effects have to be considered using cycle-based approaches as both applied stress range and mean stress affect the fatigue behavior of materials (Mercer, 1997). Also, cycle-based approaches require cycle-counting techniques to transform the direct stress history to a cycle history before the fatigue damage prognosis can be performed (Suresh, 2006). This transformation adds additional uncertainties to the fatigue damage prognosis and makes it difficult to perform the concurrent fatigue damage prognosis at the material level and structural level. To explore the concurrent analysis of fatigue damage propagation, a new methodology is proposed in this study, which is fundamentally different from cycle-based approaches. An important advantage of the proposed methodology is that it can be coupled with the structural-level dynamic analysis seamlessly, which greatly facilitates the multi-scale fatigue damage prognosis of materials and structures.

This paper is organized as followings. In the first section an introduction of the small time scale crack growth model is given. Next, the proposed small time scale fatigue model is integrated with structural dynamics using the state-space concept to formulate a concurrent state-space prognosis model. Then, two toy problems are given to exemplify the proposed state-space prognosis model. Following this, experimental data are employed to validate the prognosis of the proposed methodology and several conclusions based on the results are drawn.

2 SMALL TIME SCALE CRACK GROWTH MODEL

Traditional fatigue crack growth analysis methods are based on cycle-based crack growth rate curves (e.g., the well-known Paris law (Paris and Erdogan, 1963)), which define a relationship between the crack growth rate and the range of the applied stress intensity factor. In this study, a so called “small scale model” based on the incremental crack growth at any arbitrary time instant during a loading cycle is employed. The key concept of this model is to define the fatigue crack kinetics at any arbitrary time instant. As shown in Figure 1, the crack will extend a distance *da* during a small time scale *dt*. The geometric relationship between the Crack Tip Opening Displacement (CTOD) and the instantaneous crack growth kinetics is shown in Figure 1.

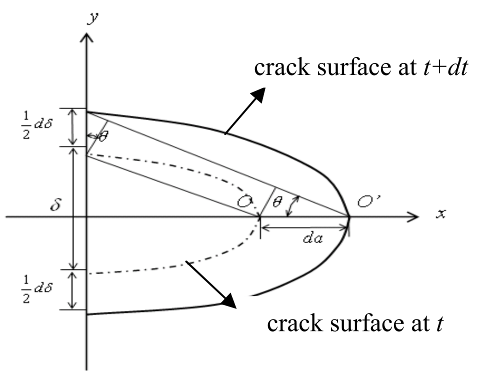


Figure 1: Crack tip geometry

Considering the geometry of crack tips at two time instants (*t* and *t+dt*), the crack growth rate *da/dt* is coded in Eq. (1), where θ is the crack tip opening angle (CTOA).

$$da = ctg\theta \times d\delta / 2 \tag{1}$$

Based on the study in (Janssen, 2004), the CTOD can be approximately expressed as Eq. (2) using the plastic zone model proposed by Irwin,

$$\delta = \left(4K^2\right)/\left(\pi E \sigma_y\right) = \lambda \sigma^2 a \tag{2}$$

$$\lambda = 4/\left(E\sigma_y\right) \tag{3}$$

where E is the Young’s modulus and σ_y is the yield strength. Following the derivation of the small scale model in (Lu and Liu, 2009), the instantaneous crack growth rate in the small scale model at an arbitrary time can be expressed as Eq. (4).

$$\dot{a} = H(\dot{\sigma}) \cdot H\left(\sigma - \sigma_{ref}\right) \cdot \frac{2C\lambda}{1 - C\lambda\sigma^2} \cdot \dot{\sigma} \cdot \sigma \cdot a \tag{4}$$

where H is the Heaviside step function and σ_{ref} is the reference stress level where the crack begin to grow. The crack length at any arbitrary time can be calculated by the direct time integration of Eq. (4) and no cycle-counting is required. Detailed discussion and model validation comparing to traditional methodology can be found in (Lu and Liu, 2009).

One advantage of the proposed small scale model is that it can be used for fatigue analysis at variable time and length scales. The fatigue crack growth analysis under random variable amplitude loading can be performed without cycle-counting. This main advantage makes it possible to couple the fatigue crack model (material level) with system dynamics (structural level) for concurrent analysis.

3 COUPLED STRUCTUAL DYNAMICS AND FATIGUE CRACK GROWTH

The proposed small scale fatigue model is expressed as the first-order time derivative function of the applied stress and its derivative (see Eq. (4)). Mathematically, it can be written as a state-space model coupled with structural dynamics to formulate an integrated state-space prognosis model. To exemplify this prognosis model, two toy problems of single and multiple DOF problems are presented below.

3.1 A single DOF dynamic system

In order to demonstrate the concept of coupling the small scale model with a dynamic system, a single DOF dynamic system is used here for illustration purpose. A schematic presentation of the model is shown in Figure 2.

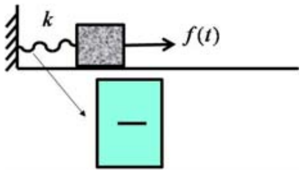


Figure 2: A single DOF dynamic system

The spring is assumed to be a plate specimen with a center through crack. An external force *f(t)* is applied to the system at the end. The governing equation of the dynamic system (Sandor and Richter, 1987) can be express as

$$m\ddot{x} + n\dot{x} + kx = f(t) \tag{5}$$

where m is mass, n is the damping coefficient, k is the stiffness, and x is the displacement of the mass.

A state-space model (Patankar *et al.,* 1998) can then be setup to describe the fatigue crack growth and the structural dynamics. Combining Eq. (4) with Eq. (5), a coupled state-space prognosis model is formulated as Eq. (6),

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = (-k/m)x_1 + (-n/m)x_2 + f(t)/m \\ \dot{x}_3 = H(g(x_1))H(g(x_1) - \sigma_{ref}) \cdot \dfrac{2C\lambda}{1 - C\lambda g(x_1)^2} g(x_2) g(x_1) a \end{cases} \tag{6}$$

where x_1 is the displacement of the mass, x_2 is the velocity of the mass, x_3 is the crack length and E is the Young's modulus. In Eq. (6), the first two state-space equations (structural level) are directly coupled with the third equation (fatigue model at the material-level).The crack length and system dynamic behaviors can be obtained by solving Eq. (6) numerically for any interested time t. For illustration purpose, an example of variable block loading *f(t)* is shown in Figure 3. Figure 3(a) shows the external force on the dynamic system and Figure 3(b) shows the corresponding stress of the specimen. By solving Eq. (6), the displacement of the mass is shown in Figure 3(c) and the crack length variation in the specimen is shown in Figure 3(d). It is clear that the dynamic responses of the system (the displacement and stress) and the crack growth can be obtained concurrently thanks to the coupled state-space prognosis model. The proposed model also shows the potential for real-time structural fatigue damage prognosis since no loading cycle-counting and transformation is needed.

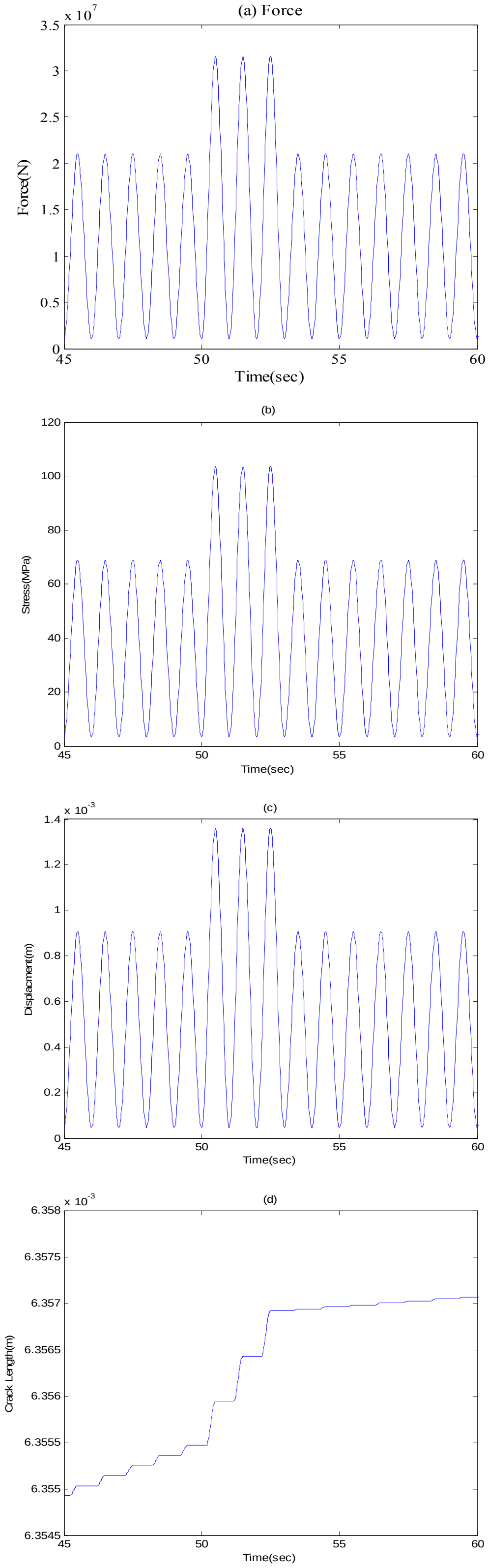


Figure 3: System responses and crack growth

3.2 A multiple degree–of-freedom dynamic system

The single DOF dynamic system in previous section shows the basic concept of coupling the small scale model with the state-space model. However, in practical situations, a structural system is usually a multiple DOF system with multiple initial cracks. In this section, the concept has been extended to a multiple DOF uniform beam system. As shown in Figure 4, the beam has been divided to three parts to solve the dynamic response using finite element method (Riera *et al.,* 2004). At the fixed end of the beam, a through edge crack is assumed. Only open mode under bending is considered in this demonstration example.

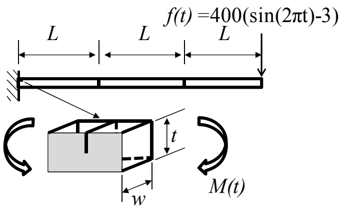


Figure 4: A multiple DOF dynamic system

The structural governing equation under a loading vector *f(t)* can be expressed in the matrix format as Eq. (7),

$$[M]_{n\times n}\{\ddot{x}\}+[N]_{n\times n}\{\dot{x}\}+[k]_{n\times n}\{x\}=\{f(t)\} \quad (7)$$

where M, N, and k are the mass, damping and stiffness matrices, respectively. x is the displacement vector of corresponding mass.

Following the same procedure of the previous single DOF toy problem, the crack growth rate is considered as an additional state variable in the system. The coupled state-space prognosis model can be constructed as Eqs. (8-9),

$$\begin{bmatrix} \dot{x}_1 \\ \vdots \\ \dot{x}_{12} \\ \cdots \\ \dot{v}_1 \\ \vdots \\ \dot{v}_{12} \\ \cdots \\ \dot{a} \end{bmatrix} = \begin{bmatrix} 0 & \cdots & 0 & \vdots & 1 & \cdots & 0 & \vdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & 0 \\ 0 & \cdots & 0 & \vdots & 0 & \cdots & 1 & \vdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{-k_{11}}{m_{11}} & & \frac{-k_{112}}{m_{112}} & \vdots & 0 & \cdots & 0 & \vdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & 0 \\ \frac{-k_{121}}{m_{121}} & & \frac{-k_{1212}}{m_{1212}} & \vdots & 0 & \cdots & 0 & \vdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \vdots & 0 & 0 & 0 & \vdots & f(\vec{x},\vec{v}) \end{bmatrix} \times \begin{bmatrix} x_1 \\ \vdots \\ x_{12} \\ \cdots \\ v_1 \\ \vdots \\ v_{12} \\ \cdots \\ a \end{bmatrix} + f(t) \begin{bmatrix} 0 & \cdots & 0 \\ & & \\ 0 & \cdots & 0 \\ \cdots & \cdots & \cdots \\ m_{11} & \cdots & m_{112} \\ \vdots & \ddots & \vdots \\ m_{121} & \cdots & m_{1212} \\ \cdots & \cdots & \cdots \\ 0 & 0 & 0 \end{bmatrix} \quad (8)$$

$$\begin{bmatrix} y_1 \\ \vdots \\ y_{12} \\ y_{13} \end{bmatrix} = \begin{bmatrix} 1 & \cdots & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_{12} \\ a \end{bmatrix} \quad (9)$$

The structural dynamic analysis and fatigue crack growth analysis can be performed simultaneously using this model. To exemplify the fatigue crack growth prognosis for this multiple DOF system, an artificial loading history in Figure 5(a) is assumed at the fixed end and Figure 5(b) shows the crack growth curve computed by solving Eq. (8). The initial crack size is 6.35*mm*. The crack growth curve clearly shows the crack propagation behavior within an individual loading cycle. Namely, the crack begins to grow when loading amplitude is larger than the reference stress σ_{ref}.

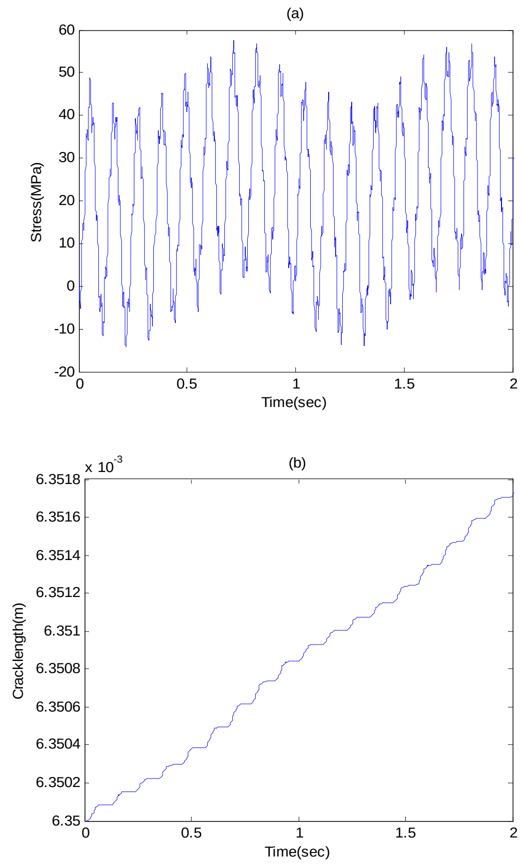


Figure 5: A multiple DOF system responses and crack growth curve prognosis

4 MODEL VALIDATION AND COMPARISON WITH EXPERIMENTAL DATA

Once the coupled state-space prognosis model has been constructed based on system dynamics and external loading, the fatigue damage propagation behaviors can be predicted together with the system dynamic responses as illustrated in previous sections. In order to apply this methodology for actual engineering problems, the proposed model is validated using experimental data on center through crack aluminum alloy 7075-T6 specimens collected by (Porter, 1972). The initial crack size (*2a*) of the specimens is 12.7*mm*. Young’s modulus and yielding strength for this material are 69600MPa and 520MPa, respectively. The applied loading is the blocked variable amplitude loading with different numbers of overload cycles (see Figure 6).

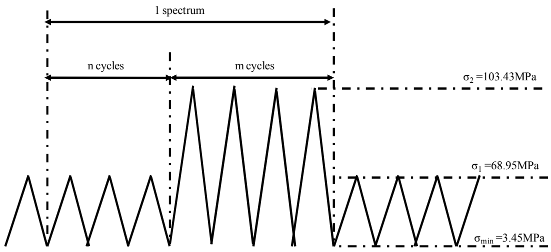
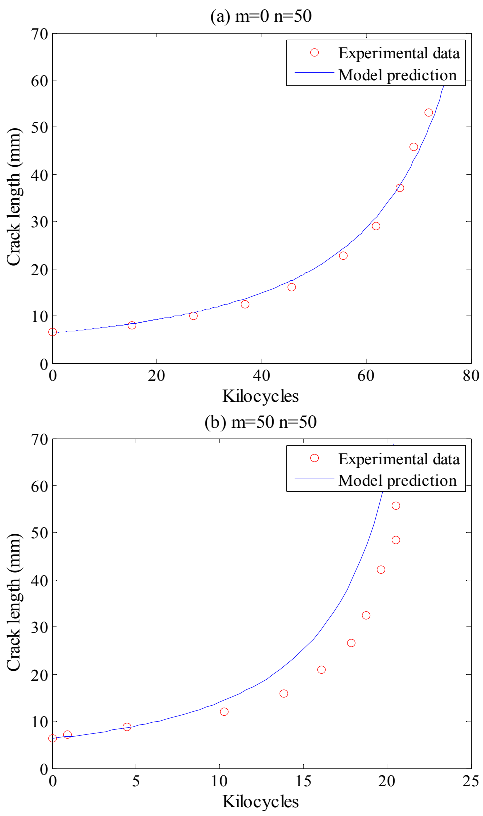


Figure 6: Loading illustration diagram

Several block loadings with different overload cycles (*m*) are considered here to fully validate the model prediction and investigate the prognosis performance. As we can see in Figure 7, model prediction curves are close to the experimental observation in these four cases. An overall satisfactory result is observed.



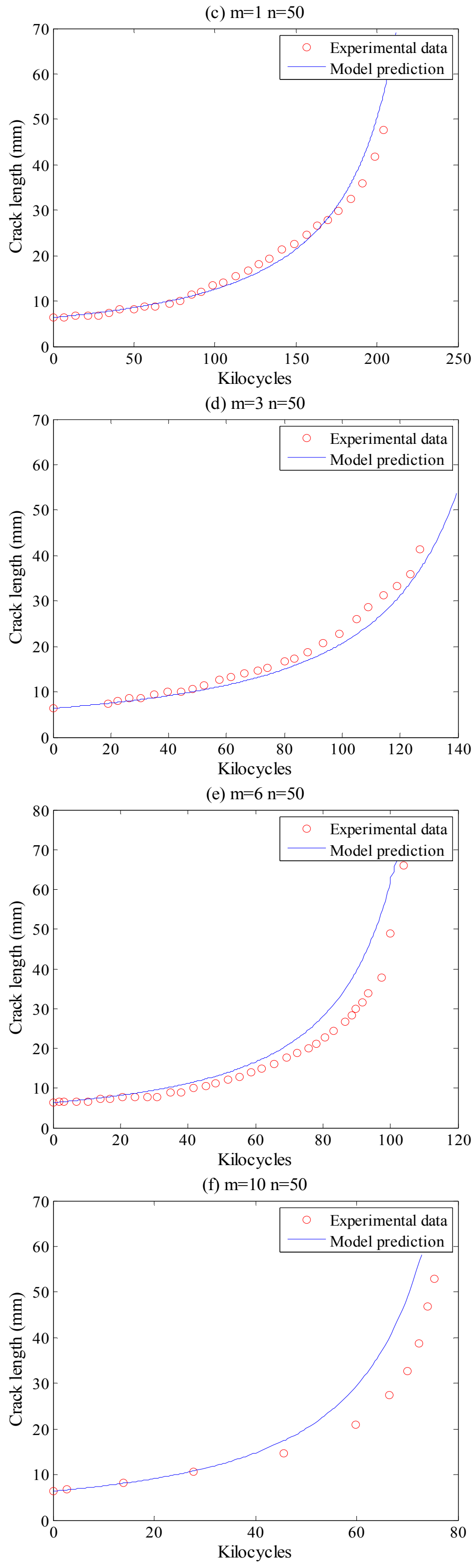


Figure 7: Comparisons of the prediction results with experimental data under block loadings

5 CONCLUSION

In this study, a new methodology for concurrent structural damage prognosis is proposed. A material level incremental fatigue crack model is directly integrated into the structural system dynamics using the concept of state-space model to formulate a coupled state-space prognosis model. Comparing with traditional cycle-based fatigue crack growth models, the proposed model is capable of computing the crack length at any arbitrary time without cycle-counting. Hence it is possible to perform concurrent fatigue analysis for dynamic structural systems. Two toy problems are presented to demonstrate the proposed method. Additionally, the prognosis performance of the proposed state-space prognosis model is validated with aluminum alloy 7075-T6 experimental data under variable amplitude loadings and several conclusions are drawn based on the validation:

1) The small scale model presented in this paper does not suffer from the loading cycle-counting requirements and stress ratio effects comparing to cycle-based crack growth models. Therefore the fatigue propagation behavior can be calculated at any arbitrary time scale, e.g. within a loading cycle.

2) The fatigue prognosis can be directly integrated with health monitoring systems using the proposed couple state-space prognosis formulation. Fatigue propagation prognosis and remaining useful life prediction can be computed and analyzed concurrently with system dynamics thus the proposed method provides a viable approach for real-time fatigue damage prognosis and health management.

3) In this paper, the concept only used to perform the concurrent analysis at the material-level and structure-level. However, the coupled state-space model concept can be extended to concurrent fatigue prognosis at multiple scales.

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NOMENCLATURE

H the Heaviside step function.
σ_{ref} the reference stress level where the crack begin to grow
σ stress
σ_y yield strength
x_1 displacement of the specimen
x_2 velocity of the specimen
x_3 crack length
E Yang's modulus
L length of the specimen
θ crack tip opening angle (CTOA)
M mass matrix
N damping matrix
k stiffness matrix

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