

Pricing full-service maintenance contracts: a data analytics approach

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ABSTRACT

A full-service maintenance contract covers all future costs of both preventive and corrective maintenance over a predetermined time horizon in exchange for a fixed upfront price. Due to the stochastic nature of the maintenance costs the determination of the correct break-even price of such a contract is a key challenge. We set out a data-driven methodology to provide insight in the future maintenance costs within a full-service contract. This methodology involves building predictive models for the frequency of failure and the associated costs taking into account machine and customer characteristics. Not only will our approach lead to a break-even price driven by the analysis of relevant historical data, it also leads to a classification of the customer base. This classification may in turn enable price discrimination of future service contracts.

1. INTRODUCTION

Full-service maintenance contracts are common practice in industry involving the maintenance of capital goods. These contracts cover all future costs of (preventive and corrective) maintenance and potentially also down-time compensation over a predetermined time horizon. In this paper we consider these contracts from the viewpoint of the seller of the contract, being the service provider. The buyer of the contract is the user of the equipment and can be considered the customer of the service provider. The main advantage of a full-service maintenance contract for customers is that they no longer bear the risk of stochastic maintenance cost but only pay a fixed service fee. The risk of facing maintenance, be it preventive or corrective, and its associated costs is transferred to the service provider. Moreover, to protect against the moral hazard of being served later than on-call customers, guaranteed repair

times can be added to the contract and compensation is paid by the service provider in case these are not fulfilled (Huber & Spinler, 2012). If this compensation is not present, the service agent has the opportunity to postpone the maintenance for his full-service clients in favour of the maintenance visits for on-call clients since the latter generates new revenue.

Alternatives for a full-service maintenance contracts are on-call service and performance-based contracts. On-call service provides maintenance on-call, in other words the customer contacts the service provider in case of failure and a corrective maintenance will result. In contrast to the full-service maintenance contract, there is no risk transfer to the service provider and consequently the customer bears all the maintenance risk. On-call service and full-service maintenance contracts focus on the time and materials spent to determine their price and as such can be considered resource-based contracts (RBC). Contrarily, performance-based contracts (PBC) (Kim et al., 2017) do not guarantee parts, labor or other resources, however they insure availability of the equipment. The compensation for these types of contracts is based on performance of the underlying product. The literature comparing resource-based contracts and performance-based contracts is quite vast (Kim et al., 2017; Bakshi et al., 2015; Kim et al., 2010) and relies heavily on game-theory.

Accurate price setting is a major challenge for full-service maintenance contracts. The stochastic nature of the costs covered by the full-service contract requires a more sophisticated methodology than the on-call service contracts, which is simply priced based on materials used and time spent. A simple approach to pricing full-service maintenance contracts could be based on expected total historical costs incurred over the contract period plus a safety margin, e.g. a factor reflecting the risk-averseness of the service provider times the standard deviation of the historical total costs. Such an approach does not depend on careful statistical analysis of data collected on

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historical failure events, their corresponding corrective maintenance costs and the characteristics of the machine and the customer. To describe the total costs incurred during the contract period, often pricing relies on a separate analysis of the building blocks of the total costs, that is the frequency of failures and their impact or severity (Huber & Spinler, 2012; Luo & Wu, 2018b,a). However, these authors do not analyze data on historical failures and associated costs, nor do they take characteristics of the customer or the machine into account. As a consequence, price differentiation is impossible and all customers will pay the same price for a contract with the same conditions. This may lead in turn to a loss of customers, since good customers, who take good care of their equipment, will pay too much. Moreover, because of the complexity of a full-service, a naive pricing strategy based on the overall average cost realized on historical contracts could be detrimental to the profitability. This particularly holds true in an era of big data and data analytics where the collection and statistical analysis of data provides useful insights to many decision support systems, including tariff plans of full service contacts. Inadequate pricing will make it hard to exploit the financial potential of extended service business (Rapaccini, 2015). Market research on service contracts suggests that only 50% make modest benefits and, even worse, 25% lose money (Hancock et al., 2005; Ulaga & Reinartz, 2011).

A good pricing methodology should take into account at least the basic underlying price drivers: frequency or arrival of failure, their recurrence over time, types of failure and associated costs. A more involved approach would allow for the integration of customer information, e.g. industry sector and country of residence, and machine characteristics, e.g. temperature measurements on connected machines. Using such risk factors then allows for machine and customer specific pricing based on a proper risk assessment. A pricing strategy driven by data analytics is inspired by the practice of insurance pricing, where predictive models are extensively used to analyze the number of claims (frequency) and their corresponding impact or cost (severity), in the presence of risk factors (Henckaerts et al., 2018).

Rapaccini (2015) presents a general overview of the literature on pricing service contracts. He distinguishes three approaches to price such a contract: first, the price can be based on the predicted costs under the contract; second, it can be based on the perceived value of the full-service contract; third, it can be benchmarked against the price of similar contracts offered by competitors. We focus on cost-based pricing, where the service agent determines the price based on the expected cost of maintenance and repair plus a desired profit margin. However, the stochastic nature of the frequency of failures and the severity makes this a non-obvious exercise.

With focus on pricing of full-service maintenance contracts, Huber & Spinler (2012) describe a value-based pricing ap-

proach for a full-service repair contract, which doesn't include preventive maintenance. First, they focus on the failures and their associated costs, which are modelled independently with a non-homogeneous Poisson process and a distribution with finite support respectively. The latter is chosen since the repair cost will always be bounded from below by a minimum cost, the basic diagnostic cost, and from above by a maximal cost, the cost of replacement. This leads to cost-based price. Then, they determine the value-based price using a mean-variance utility optimization scheme.

A full-service maintenance contract bears significant resemblance to the non-renewing free replacement warranty (NFRW) policy, where the manufacturer provides repair or replacement of a product at no cost. Luo & Wu (2018b,a) optimize the warranty policy, by determining an optimal warranty price and the optimal length of the warranty period, for different product types collectively. Their approach is inspired by mean-variance portfolio optimization (Markowitz, 1952). They incorporate dependencies between different product types, e.g. different car models by Ford, to deal with shared components, e.g. a particular type of engine, similar design and same production lines using copula theory. Analogous to Huber & Spinler (2012) they consider the incoming claims and their associated costs to be statistically independent.

A full-service maintenance contract can be considered an insurance covering the maintenance, both preventive and corrective, costs during a certain period of time. The actuarial literature therefore provides a rich source of statistical techniques which are inspiring for the case of pricing of full-service maintenance contracts. A first key concept in insurance pricing that is equally valid for full-service maintenance contracts, is the frequency-severity approach of handling claims or incurred costs, see e.g. Henckaerts et al. (2018) and Verbelen et al. (2018). A second element of insurance pricing is the use of risk factors to reflect the heterogeneity of the risks in the portfolio (Henckaerts et al., 2018). To avoid lapses in a competitive market, many rating factors (e.g. age, gender, postal code area) are used to classify risks and differentiate prices of an insurance product. Pricing through risk classification is the mechanism for insurance companies to compete and to reduce the cost of insurance contracts. Insurance companies maintain large databases with policy(holder) characteristics and claim histories and use these to build risk based pricing models. Pricing is challenged by new evolutions in data availability and an increasing focus on individual risk based pricing. The current state-of-the-art, see Denuit et al. (2007) and De Jong et al. (2008) for an overview, uses generalized linear models (GLMs), to include risk factors in the frequency as well as the severity model. This idea is new in the context of pricing full-service maintenance contracts and will be used to detect which risk factors indicate significant differences in the expected number of failures on the one hand and the expected cost incurred on the other hand. As such, a tariff plan

that differentiates prices between different customers results. A third element commonly used in actuarial pricing are premium principles (Kaas et al., 2008). These principles dictate the premium or price of the insurance with respect to the risk-averseness of the insurer. In short, a risk-neutral insurer will be satisfied if the premium is equal to the expected value of the costs incurred during the policy period of the insurance contract. However, a risk-averse insurer will want to add some extra safety margin. Premium principles will give the service provider strategies to determine these safety margins while accounting for the distributional properties of the incurred costs.

Whereas Huber & Spinler (2012) set out a stochastic model for the frequency of failures and associated costs to determine the price of a full-service repair contract, they do not consider regression procedures to fit their model on data. Motivated by the literature on warranty contracts and insurance pricing, our paper extends the current literature by introducing a data-driven pricing methodology, which accommodates for price differentiation based on the assessment of risks as pricing actuaries do. Hereby our focus is on a full-service contract involving both maintenance and repair. This is in line with Luo & Wu (2018b)'s call for data-driven methodologies for warranty contracts.

Since the regression relies on the availability of data from the service provider and the collection of such historical data is not obvious, we will test our methodology on simulated data first. A simulation engine is developed that is capable of simulating both time to failure data, and as such the number of failures during a certain period of time, and maintenance costs data which reflect as good as possible the real data a service provider could have available. We choose to simulate our data in the most granular way possible to maximise the applicability of our methodology. We then apply the regression models to our simulated data and a break-even price is determined. On the one hand we consider the frequency of failure and the maintenance costs to be statistically independent, in line with Huber & Spinler (2012), on the other hand our approach will also include predictive models. These predictive models will be inspired by the regression models used in actuarial science and medical statistics for frequency and severity modelling.

The rest of the paper has the following structure. In Section 2, we introduce our pricing methodology and associated notation. Section 3 deals with setting up our simulation. In Section 4, predictive regression models are introduced to estimate the break-even price. The conclusion follows in Section 5.

2. A FREQUENCY AND SEVERITY APPROACH FOR PRICING FULL-SERVICE MAINTENANCE CONTRACTS

We consider a full-service contract covering all maintenance and repair costs during a period $[t, t + \Delta t]$, where Δt is the

duration of the contract. During the duration of this contract, a number of (preventive) maintenance actions are planned, which are already scheduled at initiation of the contract. The preventive maintenance scheme can be comprised of different types of maintenance actions. The cost of such a preventive maintenance can be estimated quite well, although there may be some small fluctuations between the actual cost and the planned cost of the maintenance. In addition to these preventive maintenance visits, the contract also covers the costs of the corrective maintenance visits due to failures that are unplanned and due to malfunctioning of a component of the machine that requires repair or even replacement. We distinguish between different types of failures depending on the component that causes the machine to stop working. The costs associated with failures are highly uncertain.

The total, aggregated failure cost $F(\Delta t)$ covered by the contract is expressed as

$$F(\Delta t) = \sum_{i=1}^{n_f} \sum_{j=1}^{N_{f,i}(\Delta t)} X_{i,j}, \quad (1)$$

where n_f is the number of types of failures covered by the contract, $N_{f,i}(\Delta t)$ the number of failures of type i that occur during the contract period $[t, t + \Delta t]$ and $X_{i,j}$ the associated cost of failure j of type i .

The total, aggregated preventive maintenance cost $M(\Delta t)$ is expressed as

$$M(\Delta t) = \sum_{i=1}^{n_m} \sum_{j=1}^{n_{m,i}(\Delta t)} S_{i,j}, \quad (2)$$

where n_m is the number of types of maintenance. $n_{m,i}(\Delta t)$ is the number of maintenance actions of type i planned during the duration of the contract. Since these maintenance actions are planned, $n_{m,i}(\Delta t)$ is usually deterministic and that will be our assumption here. $S_{i,j}$ is the associated cost of the j th maintenance of type i . In general, $S_{i,j}$ can be considered a random variable, but this must not be the case. In the case of stochastic costs $X_{i,j}$ and $S_{i,j}$ we will assume their distribution to have a finite support (Huber & Spinler, 2012). In line with the literature (Huber & Spinler, 2012; Luo & Wu, 2018b,a; Henckaerts et al., 2018) we consider the arrival of failures and the associated costs to be statistically independent. The total aggregated cost $C(\Delta t)$ covered by the service contract is then

$$C(\Delta t) = F(\Delta t) + M(\Delta t). \quad (3)$$

We should however keep in mind that the preventive maintenance scheme will influence the occurrence of failures. Although the notation may lead to believe the terms $F(\Delta t)$ and $M(\Delta t)$ are statistically independent, this is not necessarily the case. An extra source of costs covered by the contract could come from downtime compensation, which is a compensation

paid by the service agent in case the corrective maintenance is not executed within a predetermined time window (Huber & Spinler, 2012). However, we do not consider these costs here.

In Figure 1 we illustrate the timeline of a contract during the coverage period $[t, t + \Delta t]$. On the horizontal axis, indicating the time dimension, we display the different events: the planned preventive maintenance actions and the failures of different types. The height of the vertical lines, for every action, represents the size of the associated cost. On the vertical axis we plot the probability density functions of the costs to illustrate their stochastic nature. $T_{i,j}$ denotes the inter-arrival time j between failures of type i , see Figure 1.

We introduce the price P , paid at initiation of the contract, and profit R at termination of the contract,

$$R = P - C(\Delta t). \quad (4)$$

Our main interest is the determination of the break-even price P^* for which the expected profit $E[R] = 0$. This leads to the following expression for the break-even price P^* ,

$$\begin{aligned} P^* &= E[C(\Delta t)] \\ &= E \left[\sum_{i=1}^{n_f} \sum_{j=1}^{N_{f,i}(\Delta t)} X_{i,j} + \sum_{i=1}^{n_m} \sum_{j=1}^{n_{m,i}(\Delta t)} S_{i,j} \right] \\ &= \sum_{i=1}^{n_f} E[N_{f,i}(\Delta t)] \cdot E[X_{i,j}] \\ &\quad + \sum_{i=1}^{n_m} n_{m,i}(\Delta t) \cdot E[S_{i,j}], \end{aligned} \quad (5)$$

where for the latter equality we assumed independence between frequency and severity. Equation (5) shows how the break-even price depends on the frequency of failure and the number of preventive maintenance actions on the one hand and the severity of failure and preventive maintenance on the other hand. We illustrate the risk drivers involved in our full-service contract for an example of a full-service maintenance contract in Figure 1.

The heterogeneity between customers in the portfolio of the service provider (Huber & Spinler, 2012) may justify a price list that distinguishes between good and bad risks, as an insurance company typically does. This heterogeneity can be reflected by introducing risk factors $z_{k,l}$ (Henckaerts et al., 2018), where k is the customer and l the machine. We will use observable and measurable risk factors to differentiate in prices charged to the clients. These risk factors can be both dynamic, e.g. temperature of the machine, as well as static, e.g. the country of residence. The (conditional) break-even price

$P^*|z_{k,l}$ for customer-machine risk profile $z_{k,l}$ is

$$\begin{aligned} P^*|z_{k,l} &= \sum_{i=1}^{N_f} E[N_{f,i}(\Delta t)|z_{k,l}] \cdot E[X_{i,j}|z_{k,l}] \\ &\quad + \sum_{i=1}^{N_m} n_{m,i}(\Delta t) \cdot E[S_{i,j}|z_{k,l}]. \end{aligned} \quad (6)$$

The goal is to estimate P^* for a portfolio of service contracts. A first step is the collection of covariate information, i.e. risk factors, and historical data on failures and costs. A second step is the cleaning and exploratory analysis of the data. Finally, predictive models can be calibrated for $E[N_{f,i}(\Delta t)|z_{k,l}]$ based on the historical failure data of the total portfolio of the service agent, for $E[X_{i,j}|z_{k,l}]$ from the historical severity data and $E[S_{i,j}|z_{k,l}]$ from the historical severity of maintenance data (Henckaerts et al., 2018).

3. A SIMULATION ENGINE FOR FAILURE EVENT AND COST DATA ON SERVICE CONTRACTS

3.1. Motivation

Ultimately, our pricing methodology should be applied on real company data. However, it is not obvious to collect detailed historical data on various contracts and characteristics of customers and machine. Setting up a simulation engine is then an obvious first step. This has the added benefit of flexibility because a lot of different scenarios can be generated. A simulation engine that generates failure times, failure types, associated costs and preventive maintenance costs will allow us to test performance and adequacy of statistical models (R. Bender et al., 2005; Metcalfe & Thompson, 2006; Montez-Rath et al., 2017) and to generate datasets reflecting real-world datasets (Hendry, 2014).

3.2. Data

Generating data in the context of full-service contracts can be a challenge since the simulation should include some complex features as illustrated on the example shown in Figure 1. Making abstraction of some features in the simulation process is not necessarily an option since the generated datasets need to reflect real-world data otherwise the results obtained will not be transferable to real-world datasets (Montez-Rath et al., 2017).

The first set of attributes of the dataset will have to deal with failures and maintenance actions during the duration of the contract. Regarding the preventive maintenance the dataset must contain all times stamps of planned maintenance visits and their associated costs as well as an indication for the type of maintenance that was executed. The latter is only necessary if there is a distinction between multiple types of preventive maintenance. For the repairs, all times, associated costs and indication of type of failure should be available. The timeline

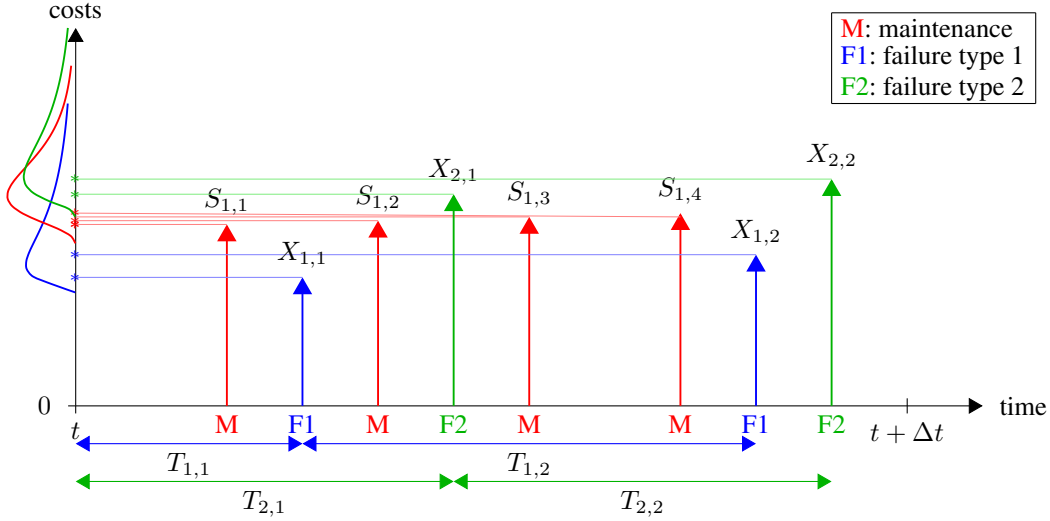


Figure 1. A full-service maintenance contract's timeline: example of failure events and maintenance actions, and their associated costs

from Figure 1 illustrates the occurrence of recurrent events (Cook & Lawless, 2007), because failures of a specific type may occur multiple times during the contract period. Metcalfe & Thompson (2006), Jahn-Eimermacher et al. (2015) and Pénichoux et al. (2015) describe recurrent event simulation in a biostatistical framework. Since different types of failures may occur, the simulation engine should also be able to generate competing risks data (Beyersmann et al., 2009).

A second attribute of the data is in reflecting characteristics of the customer and his machine. These so-called covariates can be both time-independent (Metcalfe & Thompson, 2006; Jahn-Eimermacher et al., 2015), e.g. the country where the machine is operating, or varying over time (Hendry, 2014; Pénichoux et al., 2015), e.g. temperature measurements of machines' most critical components. This part of the data will serve as a basis for determining significant risk factors (Henckaerts et al., 2018) and will consequently be used to categorize new customers and new machines.

Thirdly, some less explicit attributes can be added to the dataset. The data could contain particular subgroups which are more prone to failure events than others. This additional proneness to failure may not be explained by the observable and measurable covariates but can be considered a stochastic susceptibility or frailty (Pénichoux et al., 2015). We could also keep into account that machines are not at risk for a failure during the execution of a maintenance operation. Jahn-Eimermacher et al. (2015) demonstrate how to incorporate such risk-free intervals. Alternatively, we can consider maintenance operations as instantaneous, in other words the machines are at risk for a new failure immediately after a failure occurred.

3.3. Set up

Each simulated dataset considers n machines with a randomly assigned set of covariates, both fixed and time-varying. For each machine we simulate $N_{f,i}(\Delta t)$ failure times and their associated costs for each failure type i . This results, for a specific machine, in the following tuples for each event (i, T_k, X, I) with $k = 1, \dots, \sum_i N_{f,i}$ and where i indicates the failure type and T_k the event time, X the costs and I indicates if the event is censored or not. Censoring will indicate if the contract is observed until termination or if it is still ongoing. For each machine, a schedule with preventive maintenance actions is set up and the associated costs are simulated. The simulated data have a similar set-up as the time-line of the machine, see Figure 1.

4. LOSS ANALYTICS FOR PRICING OF SERVICE CONTRACTS

Having simulated data from the machine described in Section 3 available we will now discuss the data analytic tools at our disposal to estimate the expected values in equation (6) from data. The regression consists of two parts, one for the frequency of failure and one for the severity of failure.

We can distinguish two types of regression models for the frequency. On the one hand, we can make use of counting models that focus on the number of failures during the contract period. On the other hand, we can employ time-to-event models that model the time to failure. The latter has the advantage to make predictions about the remaining period the machine is still under contract. The main counting model is Poisson regression but many extensions are available (Denuit et al., 2007). For time-to-event models, the key model is the Cox model (Cox, 1975). Recent work from A. Bender et al. (2018) shows that a large class of time-to-event models can be represented by

generalized additive models.

To model the severity of the failures multiple distributions can be fitted. The distributions should however reflect the properties of the costs, i.e. costs are positive and they are bound from below and above (Huber & Spinler, 2012). Popular choices in insurance are the exponential, the lognormal, the gamma and the inverse Gaussian distribution (Frees, 2009). Although these distributions have infinite support they can be reduced to a finite interval. It is also possible to include covariate in the regression procedure for the severity.

5. CONCLUSION

In this paper we set out a data-driven methodology for pricing full-service maintenance contracts. Our methodology builds on the work of Huber & Spinler (2012). The extensions provided here mainly focus on calibration of predictive models for frequency of failures and the associated costs, which results in a break-even price driven by the analysis of relevant historical data. These models do not only take into account the failure times and the associated costs but also customer and machine characteristics. A benefit of this approach is discrimination of the price based on the customer's risk profile. We also develop a simulation engine to test our methodology in a variety of settings.

The pricing methodology in its current form is inspired by a priori ratemaking in insurance, where prices differ according to observable and measurable characteristics of the insured risk. Next to this, insurers take the remaining unobservable heterogeneity among risks into account with an a posteriori pricing correction. This correction is based on the history of claims reported by the policyholder. Credibility theory and a bonus malus scale are examples of such a posteriori corrections (Denuit et al., 2007). Adding a posteriori ratemaking to the current methodology is an interesting extension and would lead to a more advanced price discrimination scheme.

In its current state, our model does not incorporate condition-based maintenance (CBM). However, it could be an interesting avenue to explore the effects of CBM integration on our data-driven pricing methodology. This could be done by considering a failure type that can be detected using sensor information.

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