Cost Comparison of Maintenance Policies

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ABSTRACT

Maintenance is crucial to all repairable engineering systems as they will degrade and fail. The cost of maintenance for a manufacturing plant can occupy up to 30% of the total operating cost. If maintenance is not scheduled properly, unexpected equipment failure can induce significant cost due to reduced productivity and sub-standard products produced, both of which may result in customer penalty.

Various maintenance policies have been proposed in the past. Among the various policies, age-dependent and periodic maintenances are the common policies employed in industries. Recently, predictive maintenance or condition based maintenance policies are also proposed owing to the advancement in the sensor technology. In this work, we compare the age-dependent and periodic maintenance policies as well as the predictive maintenance policies from the perspective of cost using Markov multi-state maintenance modeling and Monte Carlo simulation. To be realistic, imperfect maintenance is included, and both the sequential and continuous inspections are considered and compared.

1. INTRODUCTION

All industrial systems suffer from deterioration due to usage and age, which may leads to system failures. To some industry, system failures cause serious consequences, especially in industries such as transportation, construction, or energy sectors. These deterioration and failure can be controlled through a proper maintenance plan.

The cost of maintenance as a fraction of the total operating budget varies across industry sectors. In the mining industry, it can be as high as 50% and in transportation industry it varies in the range of 20-30 % (Murthy, Atrens, & Eccleston, 2002), which accounts only for the actions to keep the system in operating state. The consequential cost of failure could be much higher. Hence, it is vital to have a good maintenance policy so as to reduce the possibility of failure to the least while preserves a low maintenance cost.

Maintenance problems have been extensively investigated in the literature, and a number of maintenance policies have been proposed. These policies span from the most basic one as corrective maintenance (CM) to more advanced policy as preventive maintenance (PM). CM is carried out only when a system fails. PM is performed when the system is still operating, in attempt to preserve the system in its good condition, and the most popular PM policy is age-dependent PM policy (Barlow, Proschan, & Hunter, 1996). Under this maintenance policy, the system is preventively replaced at its age of T or at failure, whichever occurs first, where T is a constant. The extension of this maintenance policy includes considering the effect of imperfect maintenance or minimal repair at failure (Kijima, 1989; Nakagawa, 1984; SHEU, KUO, & NAGAGAWA, 1993). Another common maintenance policy is periodic PM (Barlow, et al., 1996). Under this maintenance policy, a system is preventively maintained at fixed time interval T regardless of the failure history of the system and at intervening failures. This policy is often applied to a group of units where the failure's history of one unit is often neglected. There are several modifications of this periodic PM policy. In (Nakagawa, 1986), minimal repair is performed at failure and the system is replaced at planned time kT if the number of failure exceeds n. Age-dependent PM and periodic PM can be combined as in (Berg & Epstein, 1976), in which the system is periodically replaced only if its age exceeds T_0. Although being common and popular, age-dependent PM and periodic PM do not account for the actual condition of the system, thus these policies may result in unnecessary replacement of good units and cost expenditure.

Recently, condition-based maintenance (CBM), which is a subset of Predictive Maintenance (PdM), is proposed in order to improve the cost effectiveness of existing PM policies. CBM is to make maintenance decisions based on the actual system's health condition (Lu, Tu, & Lu, 2007; Ming Tan & Raghavan, 2008). CBM is often applied to system with degradable performance, which can be represented by different states. A CBM policy assigns a maintenance action to each system state. By its definition, CBM must be carried out based on the observation of the system's health, which is obtained using either sequential or continuous inspection. With the advancement of sensor technology, the system's health condition can be observed continuously. In (Moustafa, Maksoud, & Sadek, 2004), a CBM policy is developed for continuous inspection, and two maintenance options are considered, namely replacement and minimal repair. Although continuous inspection is commonly used in detecting system's degradation, it usually swarms with unnecessary and excessive data. Also, the inspection process can be costly, especially with complex systems which requires huge number of monitoring devices. Hence, there are several works on maintenance policies in which the system is inspected only at specific time (sequential inspection) and replaced with a new identical one only when the degradation reaches a predefined threshold (Grall, Dieulle, Bérenguer, & Roussignol, 2002; Lam & Yeh, 1994; Ohnishi, Kawai, & Mine, 1986). In the formulation of such polices, they considered the cost of operation in different states of degradation, cost of inspection, and maintenance. Tomasevicz (Tomasevicz & Asgarpoor, 2009) extended their works by considering the effect of imperfect maintenance and by introducing maintenance states, from which the system can be recovered to a better operating state. Their comprehensive cost analysis showed that an optimal choice of inspection date and replacement threshold can improve the cost effectiveness of the maintenance policy.

It is widely assumed that the imperfect maintenance restores a system to a state between as good as new (replacement) and as bad as old (minimal repair). The two extreme cases are investigated thoroughly in early works. In general, these assumptions are not true in many applications. In practice, imperfection can arise due to the maintenance engineering skills, quality of the replaced parts and complexity of the degraded systems. Several theoretical models are developed that taking into account the imperfect maintenance (Nakagawa & Yasui, 1987; Pham & Wang, 1996). They can be broadly classified into four classes, namely the *probabilistic approach* (Nakagawa & Yasui, 1987), *improvement factor* (Chan & Shaw, 1993; Malik, 1979), *virtual age* (Kijima, 1989; Kijima, Morimura, & Suzuki, 1988), and the final class which is based on the *cumulative system degradation* model (Martorell, Sanchez, & Serradell, 1999). For detailed discussion on the various maintenance models, one can refer to (Brown & Proschan, 1983; Levitin & Lisnianski, 2000; Wang & Pham, 1996).

In this work, we will compare the age-dependent and periodic maintenance policies as well as the predictive maintenance policies from the perspective of cost using Markov multi-state system modeling and Monte Carlo simulation. To be realistic, imperfect maintenance is included, and both the sequential and continuous inspections are considered and compared. The novelty of this work lies in the introduction of imperfect maintenance in the optimization of CBM policy for Markov multi-state system. A clear comparison between age-dependent PM, periodic PM and condition-based maintenance under different cost-related conditions will be shown, and the advantages and disadvantages of each maintenance policy will be discussed.

2. MAINTENANCE POLICIES

2.1. System Description

The system under study is a multi-state system, and each state represents a system's health condition. These states can be defined by either a degradation index such as vibration's intensity, temperature, etc, or simply the system's performance. The system is assumed to be in a finite number of states $1,2,3 \dots N$ where state 1 is the as-good-as-new state and state N is the completely failed state. The states are in ascending deteriorating order.

The degradation process is represented by the transition from one state to another state. In normal operation, the failures of a complex system have been shown (Drenick, 1960) to follow the exponential distribution despite the fact that the individual components in the system may follow different distributions. Hence, the system's deterioration process can be modeled as a continuous-time Markov process. From state $i,\ (1 \leq i \leq N-1)$ the system can only transit to the more degraded state $j, (\ i \leq j \leq N)$ with a transition rate of λ_{ij}. In this work, for the sake of simplicity, we assume that the transition rates are constant for a given i and j. From the values of λ_{ij}, the probability $P_{ij}\,(t)$ that the system is at state j after a time t given that the system is originally at state i can be calculated. In actual cases, the transition rate can be changed after the system is maintained.

The state of the system is not known unless it is inspected. In the case of sequential inspection, the cost for each successive inspection is fixed at C^{SI}. During the time of inspection, the system state is unchanged. In the case of continuous inspection, since the system is continuously monitored, the state can be instantly detected and the cost is represented as a cost per unit time c^{CI}. The system's failure (system at state N) is detected without inspection and not recoverable by maintenance. The system upon replacement is recovered to the initial state 1 with a cost of C^{R}. However, the failure also results in a secondary consequential damage such as unplanned delay in production, lost of physical assets etc, which is represented by a cost of C^f. The value of C^f depends on the nature of the failures.

Upon maintenance, the system's state is improved to a better state $j, i \geq j \geq 1$, with probability P_{ij}^M. The probability that the system is recovered to as-good-as-new state is getting smaller as the system state is approaching N, and the maintenance cost and time varies with different states. C_{ij}^M is denoted as the cost of maintenance to repair the system from state i to a less degraded state j. The maintenance cost includes the cost due to system unavailability.

The illustration of all the different quantities is as shown in Figure 1.

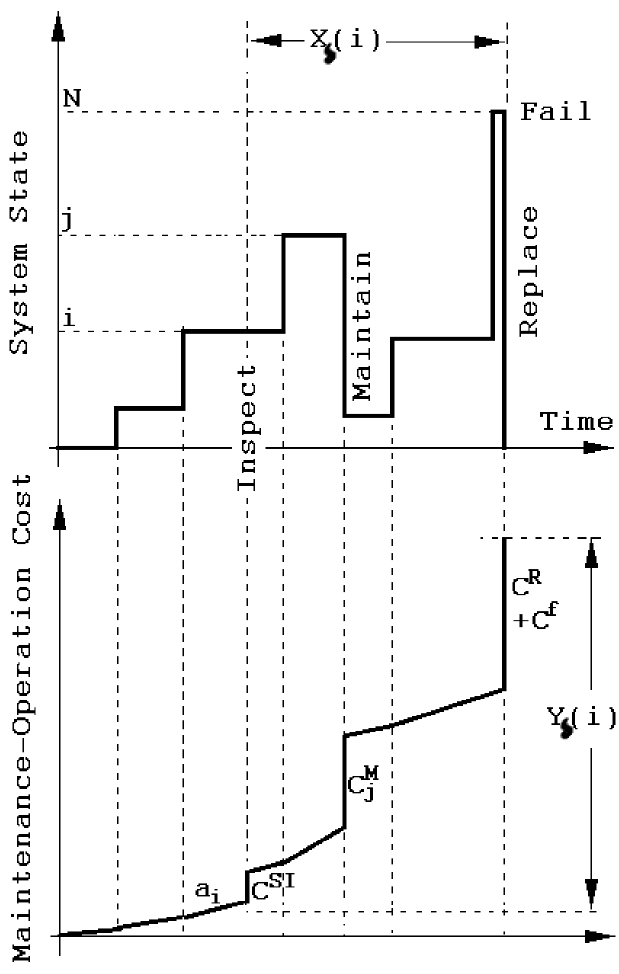

Figure 1.Schematic view of the system state degradation and its maintenance-operation cost

To proceed to the determination of the optimal maintenance policies, let us define the following terms:

δ: A policy which determines the action at each state, either replacement, maintenance or continue the inspection.

$D(i)$: Decision at state i. They can be either to inspect the system after time interval t_i ($I(t_i)$), maintain (M), replace (R) or keep monitoring in the case of employing continuous inspection (C).

$X_\delta(i)$: Mean operating time from the moment the system is detected to be at state i to the time where the system is replaced (at state N) for a given policy δ. Hence, $X_\delta(1)$ is the mean time from a new/newly replaced system till it is replaced.

$Y_\delta(i)$: Mean cost from the moment the system is detected to be at state i to the time where the system is replaced (at state N), for a given policy δ. Hence, $Y_\delta(1)$ is the mean cost from a new/newly replaced system till it is replaced.

$F_i(t)$: Probability that the system will fail in the interval $(0, t)$ given that the system is at state i.

a_i : operating cost at state i. The cost of operation is increasing with the degradation in order to accounts for the loss in profit due to the degradation in the system's performance.

The mean operating cost, given that the system initially at state i, after a time t is (Ohnishi, et al., 1986):

$$A_i(t) \triangleq \sum_{j=i}^{N} \int_0^t P_{ij}(u) a_j \, du \quad (1)$$

$P_{ij}(t)$: Probability that the system will is at state j after a time t given that the system is at state i.

2.2. Maintenance Policies

In this work, we compare four different maintenance policies for a multi-state system, namely age-dependent PM, periodic PM, sequential and continuous inspection CBM. The optimal maintenance policy refers to minimum overall operation cost rate $g^* \equiv \min_\delta Y_\delta(1)/X_\delta(1)$. Let us now look at the formulation of the optimization for each maintenance policy.

a. Age-dependent PM:

In this study, we only consider the most basic Age-dependent PM, which does not utilize maintenance. The system is preventively replaced at its age of T_a or at failure, whichever occurs first. T_a is chosen so that the cost rate is minimized.

The mean cost and operating time until system replacement can be expressed as:

$$Y_A = A_1(T_a) + C^R + F_1(T_a)C^f \quad (2)$$

$$X_A = \int_0^{T^A} \bar{F}_1(u)du \quad (3)$$

In (2), the terms $A_1(T_a), C^R$ and $F_1(T_a)C^f$ represent the mean operation cost in the interval $(0, T_a)$, replacement cost and mean failure-induced cost respectively. In (3), $\int_0^{T_a} \bar{F}_1(u)du$ is the expected operating time in the interval $(0, T_a)$. This can be derived by considering two possibility of the system's operation, i.e. the system can either work up to t_a with the expected operating time of $t_1 = t_a \bar{F}_1(t_a)$, or the system fails at u within the interval $(0, t_a)$ with the expected operating time of $t_2 = \int_0^{t_a} udF(u)$. We thus have $X_A = t_1 + t_2$.

b. Periodic PM:

The system is preventively replaced at fixed time interval T_b or at intervening failures regardless of the failure history of the system. Here T_b is a constant, and it is chosen so that the cost rate is minimized.

The mean cost until system replacement can be expressed as (4).

$$Y(T_b) = C^R + (C^R + C^f)M(T_b) + C_{ope}(T_b) \quad (4)$$

In (4), $M(t)$ is the mean number of failure and is given in (5) (Barlow, et al., 1996).

$$M(t) = \int_0^t \big(1 + M(t-x)\big)dF_1(x) = \sum_{n=1}^{\infty} F_1^n(t) \quad (5)$$

$$F_1^{n+1}(t) = \int_0^t F_1^n(t-x)dF_1(x), \; F_1^1(t) = F_1(t)$$

$C_{ope}(t)$ is the mean operation cost in the duration $(0, t)$, and they are given in (6). The term $C_k(t)$ represents the mean operation cost given that exactly k failures occur and can be calculated recursively as shown in (7). In (7), x is the time of the first failure occurs in the interval $(0, t)$. Thus, the $C_k(t)$ can be computed by integrating the summation of the operation cost before and after x for all x in $(0, t)$.

$$C_{ope}(t) = \sum_{k=1}^{\infty} C_k(t) \quad (6)$$

$$C_n(t) = \int_0^t \big(C_0(x) + C_{n-1}(t-x)\big)dF_1(x) \quad (7)$$

$$C_0(t) = A_1(t)$$

The mean operating time until system replacement can be expressed as (8).

$$X(T_b) = T_b \quad (8)$$

c. Sequential Inspection CBM (SI-CBM):

The system is inspected at a planned time. The decision depends on the indicated system state i, which is either preventively replaced $D(i) = R$, maintained $D(i) = M$, or to leave the system operating until the next planned inspection time $D(i) = I(t_i)$. Maintenance is considered to be imperfect. If $i = N$, the system fails and needs to be replaced. In that case, we have $X_\delta(N) = T^R, Y_\delta(N) = C^R + C^f$. The decision $D(i)$ at each state is chosen so that the cost rate is minimized.

1. If $D(i) = I(t_i)$

Under this decision, the system is left to degrade until the next inspection after an interval t_i. If the system fails at $u < t_i$, it is replaced. If the system passes the time interval t_i without failure, the time to replacement will be t_i plus the mean time to replacement of the arrived state j. Once the planned inspection time t_i is reached, the system is inspected. The mean cost and operating time until renewal under the decision $D(i) = I(t_i)$ can be expressed as:

$$Y_\delta(i) = A_i(t_i) + C^{SI}\bar{F}_i(t_i) + \sum_{j=i}^{N} P_{ij}(t_i)Y_\delta(j) \quad (9)$$

$$X_\delta(i) = \int_0^{t_i} \bar{F}_i(u)du + \sum_{j=i}^{N} P_{ij}(t_i)X_\delta(j) \quad (10)$$

In (9), $A_i(t_i)$ is the mean operating time in the interval $(0, t_i)$, $C^{SI}\bar{F}_i(t_i)$ is the mean inspection cost and $\sum_{j=i}^{N} P_{ij}(t_i)Y_\delta(j)$ is the expected cost until replacement given that the system is in the degraded state j.

In (10), $\int_0^{t_i} \bar{F}_i(u)du$ is the expected time to replacement in the interval $(0, t_i)$ and $\sum_{j=i}^{N} P_{ij}(t_i)X_\delta(j)$ is the expected operating time given that the system is in the degraded state j.

2. If $D(i) = M$

The system is maintained with a maintenance cost C_{ij}^M, and the system is thus improved from the current state i to a less degraded state j with an improvement probability of P_{ij}^M. The mean cost and operating time until replacement can be expressed as

$$Y_\delta(i) = \sum_{j=1}^{N} P_{ij}^M \left(C_{ij}^M + Y_\delta(j)\right) \quad (11)$$

$$X_\delta(i) = \sum_{j=1}^{N} P_{ij}^M X_\delta(j) \quad (12)$$

3. If $D(i) = R, \; i \neq N$

Except the failure case $i = N$, the system can be preventively replaced. Thus, the mean operating time and cost until replacement can be expressed as

$$Y_\delta(i) = C^R \quad (13)$$

$$X_\delta(i) = 0 \quad (14)$$

Overall, we have

$$Y_\delta(i) = \begin{cases} A_i(t_i) + C^{SI}\bar{F}_i(t_i) + \sum_{j=i}^{N} P_{ij}(t_i) Y_\delta(j), if\ D(i) = I(t) \\ \sum_{j=1}^{N} P_{ij}^M \left(C_{ij}^M + Y_\delta(j)\right), if\ D(i) = M \\ C^R, if\ D(i) = R \end{cases} \quad (15)$$

$$X_\delta(i) = \begin{cases} \int_0^{t_i} \bar{F}_i(u)du + \sum_{j=i}^{N} P_{ij}(t_i) X_\delta(j), if\ D(i) = I(t_i) \\ \sum_{j=1}^{N} P_{ij}^M X_\delta(j), if\ D(i) = M \\ 0, if\ D(i) = R \end{cases} \quad (16)$$

d. Continuous Inspection CBM (CI-CBM):

The system is inspected continuously. The decision depends on the indicated system state, which is either preventively replaced $D(i) = R$, maintained $D(i) = M$, or to leave the system operating while keep monitoring the system's condition $D(i) = C$. Maintenance is considered to be imperfect. The decision $D(i)$ at each state is chosen so that the cost rate is minimized.

For the first two decisions, the analysis is the same with SI-CBM case. Under the decision of continuous inspection, the system is operating at state i until it changes its state to a more degraded state j. The mean cost and operating time until renewal under the decision $D(i) = C$ can be expressed as:

$$Y_\delta(i) = (c^{CI} + a_i)\int_0^\infty P_{ii}(u)d\,u + \sum_{i=1}^{N} \frac{\lambda_{ij}}{\sum_{k=1}^{N}\lambda_{ik}} Y_\delta(j) \quad (17)$$

$$X_\delta(i) = \int_0^\infty P_{ii}(u)d\,u + \sum_{i=1}^{N} \frac{\lambda_{ij}}{\sum_{k=1}^{N}\lambda_{ik}} X_\delta(j) \quad (18)$$

In (17) and (18), $\int_0^\infty P_{ii}(u)d\,u$ is the mean time the system operate at state i, $\frac{\lambda_{ij}}{\sum_{k=1}^{N}\lambda_{ik}}$ is the probability that the system transit from state i to state j at any instant given that the system has to change its state. Thus, $(c^{CI} + a_i)\int_0^\infty P_{ii}(u)d\,u$ is the mean operation plus inspection cost when the system is at state i, $\sum_{i=1}^{N} \frac{\lambda_{ij}}{\sum_{k=1}^{N}\lambda_{ik}} Y_\delta(j)$ and $\sum_{i=1}^{N} \frac{\lambda_{ij}}{\sum_{k=1}^{N}\lambda_{ik}} X_\delta(j)$ are the mean cost and operating time until replacement averaging on the degraded state j.

Overall, we have

$$Y_\delta(i) = \begin{cases} (c^{CI} + a_i)\int_0^\infty P_{ii}(u)d\,u + \sum_{i=1}^{N} \frac{\lambda_{ij}}{\sum_{k=1}^{N}\lambda_{ik}} Y_\delta(j), if\ D(i) = C \\ \sum_{j=1}^{N} P_{ij}^M \left(C_{ij}^M + Y_\delta(j)\right), if\ D(i) = M \\ C^R, if\ D(i) = R \end{cases} \quad (19)$$

$$X_\delta(i) = \begin{cases} \int_0^\infty P_{ii}(u)d\,u + \sum_{i=1}^{N} \frac{\lambda_{ij}}{\sum_{k=1}^{N}\lambda_{ik}} X_\delta(j), if\ D(i) = C \\ \sum_{j=1}^{i} P_{ij}^M X_\delta(j), if\ D(i) = M \\ 0, if\ D(i) = R \end{cases} \quad (20)$$

3. EXAMPLE THROUGH HYPOTHETIC SYSTEM

In this section, a hypothetical system is studied to illustrate the impact of different policies on the system‘s total cost and number of maintenance. The system consists of twenty one states (1-21), which represents the system degradation levels in ascending order. State 1 is the state of no degradation (best performance) and state 21 is the state of total failure (worst performance). For simplicity, we only consider degradation in the sense that at any moment the system only degrades to the next degraded state (with a fixed degradation rate $\lambda_{i,i+1}$) or experienced a shock so that it fails immediately (with a failure rate λ_{iN}). From the assumption that the state transition is a continuous time Markov process, we have the set of Kolmogorov forward equations as shown in Eqn (21):

$$\frac{dP_{ij}(t)}{dt} = \sum_{k=i}^{j-1} \lambda_{kj} P_{ik}(t) - \sum_{k=j+1}^{N} \lambda_{jk} P_{ij}(t) \quad (21)$$

Here the first term on the right of the equation refers to the degradation process from state i, and the second term on the right refers to the further degradation process from state j. Eqn (21) can be re-written as follows:

$$\frac{d}{dt}\begin{bmatrix} P_{ii}(t) \\ P_{i,i+1}(t) \\ \dots \\ P_{iN}(t) \end{bmatrix} = Q_i \begin{bmatrix} P_{ii}(t) \\ P_{i,i+1}(t) \\ \dots \\ P_{iN}(t) \end{bmatrix} \quad (22)$$

where $Q_i = \begin{bmatrix} -\sum_{k=i+1}^{N}\lambda_{ik} & 0 & \cdots 0 \\ \lambda_{i,i+1} & -\sum_{k=j+1}^{N}\lambda_{jk} & \dots 0 \\ \vdots & \vdots & \ddots 0 \end{bmatrix}$

With Eqn (22), $P_{ij}(t)$ can be calculated numerically given that the initial probability $t=0$ is $\left[P_{ii}, P_{i,i+1}, \ldots P_{iN}\right]^T = [1,0,\ldots 0]^T$. The system state is then randomly generated in Monte Carlo simulation based on the probability $P_{ij}(t)$. A numerical example of Markov process with detail derivation can be found in (Ming Tan & Raghavan, 2008).

Due to the loss caused by degradation, namely lower productivity and higher recourses consumption, the cost of operation is increasing with the degradation levels. The degradation and failure rate, operation and maintenance cost for different states are hypothetically assumed and given in TABLE.I. In this study, we want to investigate the impact of failure-induced cost C^f on the optimal maintenance policies. When failure occurs, it will induce a further cost such as production delay, human and asset lost, etc. The total cost of the system maintenance at failure is the summation of system's replacement cost and the failure-induced cost.

For illustration purpose on the computation of the conditional probability of the post-maintained j state given pre-maintained state i, we further considered a special system in this case study. The system is assumed to consist of n identical sub-systems in parallel, in which a system state i represents the condition that $(i-1)$ subsystems are operating. With this special system, an analytical form of the maintenance probability P_{ij}^M can be derived.

The imperfect maintenance is characterized using the maintenance quality represented by a parameter p_m, which is the probability that a subsystem can be recovered to as new by maintenance actions. The value of p_m of a subsystem can be estimated using the method of determining the restoration factor RF described in (Ming Tan & Raghavan, 2008). Since our system consists of n identical sub-systems in parallel and all the failed sub-systems has equal probability p_m to be recovered at each maintenance, the probability of post-maintained state j is the probability that $i-j$ sub-systems are recovered and thus one can use the binomial distribution to compute the P_{ij}^M as follows.

$$P_{ij}^M = \binom{i-1}{j-1}(1-p_m)^{j-1} p_m^{i-j} \quad (12)$$

It follows that the expected post-maintained state and its variance are both linearly increasing with the pre-maintained state i, i.e. $E(j) = p_m + (1-p_m)i$ and $var(j) = (i-1)p_m(1-p_m)$. These indices indicate that as the system is more degraded, it is more difficult to maintain the system to the initial condition and the consistence of the maintenance quality decreases. For a general system, the optimization algorithm is still applicable as long as the maintenance probability P_{ij}^M is given. The investigation for such a general system is beyond the scope of the present work.

SS	DR	FR	OC	MC
1	0.4966	0.0082	2.7183	31.4942
2	0.5016	0.0091	2.8577	31.6111
3	0.5066	0.0099	3.0042	31.7371
4	0.5117	0.0108	3.1582	31.8736
5	0.5169	0.0118	3.3201	32.0216
6	0.5221	0.0129	3.4903	32.1826
7	0.5273	0.0141	3.6693	32.3587
8	0.5326	0.0155	3.8574	32.5531
9	0.5379	0.0169	4.0552	32.7705
10	0.5434	0.0185	4.2631	33.0183
11	0.5488	0.0202	4.4817	33.3091
12	0.5543	0.0221	4.4715	33.6631
13	0.5599	0.0242	4.9531	34.1154
14	0.5655	0.0265	5.207	34.7253
15	0.5712	0.0291	5.4739	35.59511
16	0.5769	0.0317	5.4746	36.9004
17	0.5827	0.0347	6.0496	38.945
18	0.5886	0.0381	6.3598	42.2541
19	0.5945	0.0416	6.6859	47.7363
20	0.6005	0.0455	7.0287	56.9556
21	0.6065	0.0498	0	72.6683

SS : System State
DR : Degradation Rate (per month)
FR : Failure Rate (per month)
OC : Operation Cost Rate ($ .000/month)
MC : Maintenance Cost ($ .000)

Table 1.System State's Maintenance Cost & Degradation Rate

4. MONTE CARLO SIMULATION RESULT & DISCUSSION

Using different values of the failure-induced cost C^f and maintenance quality p_m, the optimal maintenance plans are derived for each of the above-mentioned maintenance policy.

Monte Carlo simulation is run for each derived maintenance policy so as to investigate the impact of failure-induced cost and maintenance quality on the system's total operation-maintenance cost, number of maintenance and number of failure. For each value of C^f and p_m, the simulation is repeated for 500 random samples. The total system runtime is assumed to be 120 months.

4.1. Impact of Failure-Induced Cost

The failure-induced cost is assumed to range from 20 to 1000 ($ .000). For SI-CBM and CI-CBM, the maintenance quality is kept at $p_m = 0.8$.

Figure 2 shows the changes of mean value of the total maintenance-operation cost of different maintenance policies vs. failure-induced cost. The total maintenance-operation cost is the summation of all operation, maintenance and replacement cost:

$$replacement\ cost = \#replacement \times C^R$$

$$maintain\ cost = \sum_{i=1}^{N-1} \#(maintain\ at\ state\ i) \times C_i^M$$

$$operation\ cost = \sum_{i=1}^{N-1} (duration\ at\ state\ i) \times a_i$$

It appears that the mean value of the total cost has a linear relation with respect to the failure-induced cost.

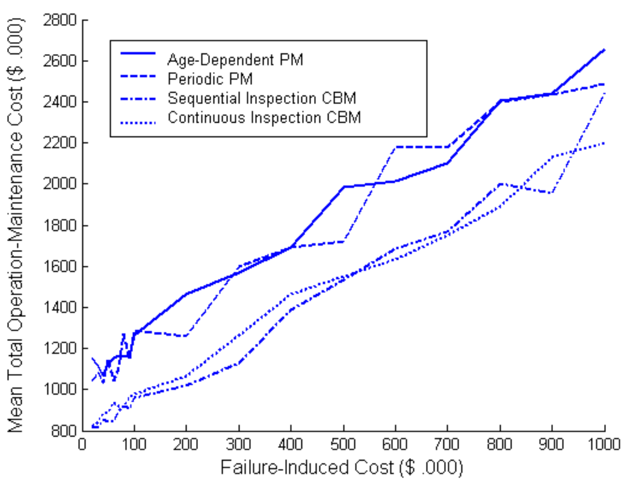

Figure 2.Mean value of total Operation-Maintenance Cost of various maintenance policies vs. failure-induced cost

One can also see that CBM policies have a clear advantage over traditional PM policies in term of cost reduction. At low failure-induced cost, utilizing CBM policies can save up to $(1100 - 800)/1100 = 27\%$ of the total cost under PM policies.

Figure 3 shows the normalized standard deviation (NSTD) curve of the total cost under different maintenance policies. This NSTD is the standard deviation of the total cost from 500 samples of Monte Carlo run divided by its mean value $\bar{\sigma} = \sigma/\mu$. The standard deviation appears to have a linear relation to the failure-induced cost. At low C^f, the NSTD values under CBM policies are approximately equal to the NSTD under PM policies as 10%. However, as C^f increase, the NSTD under CBM policies increases dramatically up to 70% for SI-CBM and 62% for CI-CBM while it is less than 50% for PM policies cases. This is due to the increase number of imperfect maintenance under CBM as the failure-induced cost increases. As a result of rising failure-induced cost, the optimal CBM policies have to increase the number of maintenance in order to reduce the number of failure.

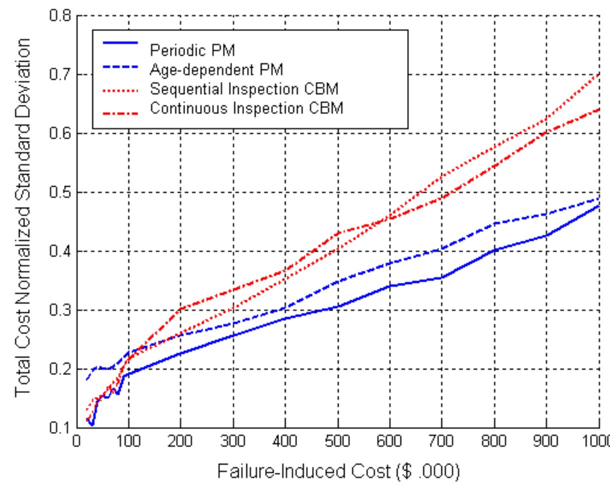

Figure 3.Cost normalized standard deviation under different maintenance policies vs. Failure-induced cost

Figure 4 shows that the mean number of failure decreases exponentially as the failure-induced cost rise. It is the effect of optimal maintenance policy, which tends to reduce the number of failure as the failure-induced cost increase. However, the mean number of failure only decreases to a certain value for each maintenance policy. This lower bound value is lower for CBM policies than PM policies by 25%, which proves that CBM is more advantageous than PM in preventing system failure.

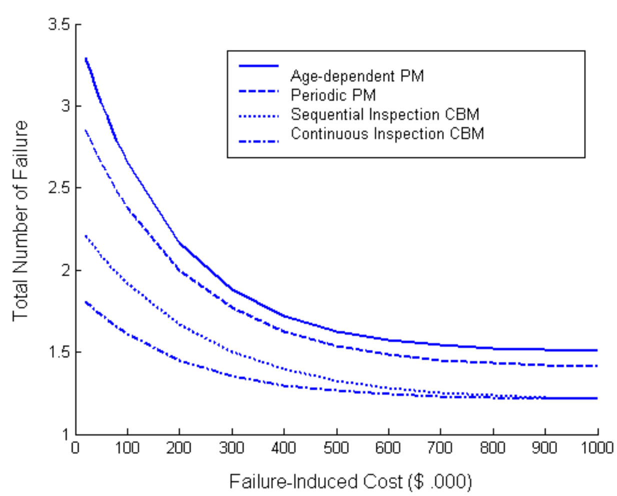

Figure 4.Mean number of failure vs. Failure-induced cost

In summary, one can see that SI-CBM and CI-CBM have a clear advantage in term of cost and failure reduction over Age-dependent and periodic PM. However, imperfect maintenance causes the total cost of CBM to vary significantly, especially at high failure-induced cost due to higher number of maintenance needed for the optimal policy. This large variation in cost may render the financial budgeting for using CBM difficult.

4.2. Impact of maintenance quality

In this case, the failure-induced cost is kept at 100 ($ .000) while the maintenance quality is ranging from $p_m = 1$ (perfect maintenance) to $p_m = 0.6$.

SI-CBM and CI-CBM policies are investigated to study the impact of maintenance quality. Figure 5 shows the total cost of SI-CBM and CI-CBM under two schemes: optimal policies and the policies assuming perfect maintenance ($p_m = 1$). Under the CBM policies that assume perfect maintenance while the maintenance is actually imperfect, the total cost increase dramatically as p_m decrease. As p_m close to 1, the difference between optimal CBM and the one assuming perfect maintenance is negligible as expected. However, the difference increase significantly when $p_m = 0.6$ and beyond. At $p_m = 0.6$, the optimal CI-CBM can save up to $((1500 - 1000))/1500 = 33\%$ of the total cost comparing to the policy assuming perfect maintenance. The cost under CBM policies assuming perfect maintenance eventually rise up to infinity as p_m approaches zero since at $p_m = 0$, maintenance take no effect. The total cost under both optimal CI-CBM and SI-CBM also tend to saturate as p_m decreases. This is due to the fact that maintenance is gradually ruled out due to its poor quality (referring to figure 6). Thus the saturated value is corresponding to the CBM policy that does not utilize maintenance.

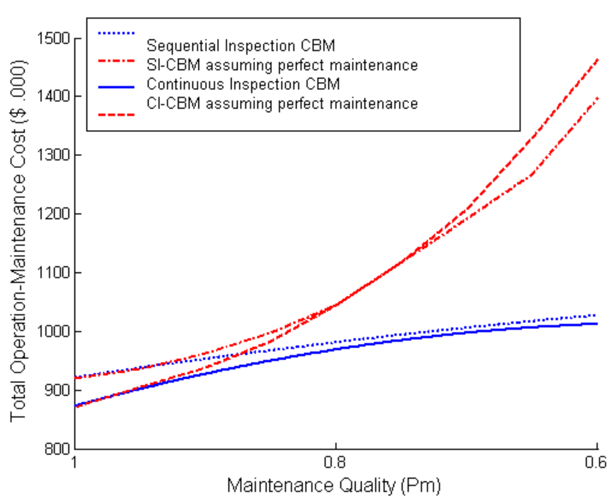

Figure 5.Total operation-Maintenance Cost vs. Maintenance Quality with $C^f = 100$

Figure 6 shows the mean number of maintenance changes with respect to the maintenance quality. The plots under SI-CBM and CI-CBM follow the same trend. When the maintenance quality gets worse, the mean number of maintenance increases to cover for the imperfection. However, at low values of p_m, the number of maintenance drops dramatically to zero as maintenance is too ineffective, and hence our optimization process for maintenance will try to reduce the number of maintenances.

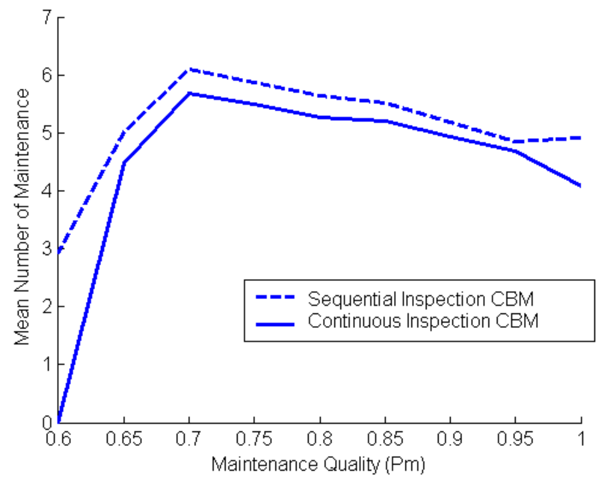

Figure 6.Mean number of maintenance vs. Maintenance Quality, $C^f = 100$

From this study, we can see that there is a threshold for maintenance quality, under which, the maintenance is no longer effective and should be changed to preventive replacement. The quality of maintenance must be carefully taken into account when making a maintenance policy since a poor maintenance quality can lead to a large portion in overall cost.

5. CONCLUSION

In this work, we study different maintenance policies for a multistate system. Four maintenance policies are investigated, namely age-dependent and periodic preventive maintenance, sequential and continuous inspection condition-based maintenance (CBM). The system has a state dependent degradation rate during its operation, and it also suffers shock failure which makes it fails immediately with a state dependent failure rate. The failure is assumed to induce further cost and maintenance is assumed to be imperfect. The maintenance policies are optimized correspondingly.

Monte Carlo simulation shows that CBM is more advantageous in term of cost and failure reduction than Age-dependent and periodic PM. On the other hand, the maintenance cost under CBM is less consistent than under PM, which renders the budgeting difficult. We also illustrate the important of maintenance quality since a poor maintenance quality can lead to a large waste in maintenance cost. It can be proven that the maintenance quality must be higher than a threshold to be worth carrying out.

One issue for CBM to be effectively applied is to have accurate inspection. Besides, CBM also need a dynamics logistic supply of spare parts, which may further cause some time delay between inspection and maintenance. Hence, for the future work, we will consider the inspection quality and time delay due to supply limit in our model.

In our paper, the Monte-Carlo simulation is run for two parameters, but varying only one parameter at a time. A matrix of multi-variables will be studied for a future work to better understand the trade-offs between different quantities. These will permit understanding strategies based on which one can practice non-CBM methods on some components versus CBM on others.

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