# Integrated Robust Fault Detection, Diagnosis and Reconfigurable Control System with Actuator Saturation

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## ABSTRACT

An integrated fault detection, diagnosis and reconfigurable control design method is studied in this paper with explicit consideration of control input constraints. The actuator fault to be treated is modeled as a control effectiveness loss, which is diagnosed by an adaptive algorithm. For fault detection, an observer is designed to generate the output residual and a minimum threshold is set by an $H_\infty$ index. To design the reconfigurable controller, an auxiliary matrix is introduced and a linear parameter varying (LPV) system is constructed by convex combination. Linear matrix inequality (LMI) conditions are presented to compute the design parameters of controllers and related performance index. The system performances are measured by the ellipsoidal sets regarding the domain of attraction and disturbance rejection respectively. For illustration, the proposed design techniques are applied to the flight control of a flying wing aircraft under large effectiveness loss of actuators.

## 1. INTRODUCTION

The reconfigurable fault-tolerant control design methods have been studied widely in the literature to meet increased requirements for reliability and safety in modern control systems (Zhang & Jiang, 2008). One key component in fault-tolerant control systems is the fault detection and diagnosis (FDD) module, which has been studied extensively in the past decades (Isermann, 2006). With information provided by FDD, the controller is adjusted according to some reconfiguration mechanism to maintain desirable performances. One challenging problem in designing reconfigurable fault-tolerant control system is how to integrate the FDD with the controller effectively to guarantee the system performance, such as stability, etc. Another practical consideration is to take the control input constraints into control system design procedure, since almost all practical applications involve actuators constrained by limited power, for example, the deflection of control surfaces in aircraft is constrained by amplitude and rate limitation. Hence, it is very significant to provide some design methods for the reconfigurable control problem with explicit consideration of control input constraints.

Currently, the constrained control systems are widely studied in the literature (Tarbouriech & Turner, 2009). Although there are still many open problems remained to be investigated, many useful results have been obtained due to efforts of past decades. Based on the fact that system performance can be improved if the controller can be designed to allow actuator saturation compared with that obtained within control limits. Along with this idea, many researchers have made their efforts in this direction of research. For example, a saturated system is represented by a polytopic model to solve the output tracking problem (Tarbouriech, Pittet & Burgat, 2000). An improved set invariance condition is given in (Hu, Lin & Chen, 2002) to obtain a less conservative estimation of domain of attraction. As will be shown in this paper, these results provide a tool to solve the reconfigurable control problem.

The reconfigurable control problem with actuator saturation is still not well addressed in the literature, and only a few results available in recent years. Generally speaking, there are two types of approaches to deal with such issues: one using the command management techniques (Bodson & Pohlchuck, 1998; Zhang & Jiang, 2003; Zhang, Jiang & Theilliol, 2008), and the other relating to controller design (Pachter, Chandler & Mears, 1995; Guan & Yang,

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2009). For example, the actuator rate saturation problem is solved by a linear programming algorithm in (Pachter et al., 1995). An adaptive output-feedback controller is designed with online estimation of actuator faults by Guan et al. (2009). However, only the stability problem is studied in that paper.

In this paper, we aim to solve the reconfigurable output tracking control problem of linear systems subject to both actuator saturation and disturbances. The actuator saturation is tackled with by using the set invariance condition given by Hu et al. (2002). The controller can be adjusted automatically with estimated fault amplitude provided by an adaptive diagnostic algorithm, after a fault occurs and is detected by an observer-based detector.

The paper is organized as follows: The problem to be treated is stated in Section 2. An integrated design of the reconfigurable controller with fault diagnosis is presented in Section 3. To detect a fault, an observer is designed in Section 4. Then, a nonlinear model of an aircraft is used to test the proposed design techniques in Section 5. Finally, some concluding remarks are given in Section 6.

## 2. PRELIMINARIES AND PROBLEM STATEMENT

To illustrate the basic ideas in this paper, a scalar control system with state-feedback controller is taken as an example:

$$\begin{aligned} \dot{x}(t) &= ax(t) + bu(t) \\ u(t) &= f(x) \end{aligned} \tag{1}$$

The fault under consideration is the loss of control effectiveness such that

$$u_f(t) = \lambda(t)u(t) \tag{2}$$

where $u_f(t)$ represents the output of the impaired actuator, and $\lambda(t) \in [0,1]$ is the control effectiveness factor. $\lambda(t) = 0$ means the total outage of the actuator, while 1 denotes a healthy actuator. Partial loss of control effectiveness is given by a value between 0 and 1. It is assumed that $\lambda(t) \neq 0$ in this paper.

To compensate the control effectiveness loss, the following control law can be adopted:

$$u(t) = \lambda^{-1}(t) f(x) \tag{3}$$

From Eqs. (2) and (3), it follows that

$$u_f(t) = \lambda(t)\lambda^{-1}(t) f(x) = f(x) \tag{4}$$

Obviously, the system performance is not impaired in the presence of actuator fault while the control law shown in Eq. (3) is in action. However, the fault cannot be known a prior, and only its estimation is available. In this case, Eq. (3) should be replaced by

$$u(t) = \tilde{\lambda}^{-1}(t) f(x) \tag{5}$$

where $\tilde{\lambda}(t)$ is an estimation of $\lambda(t)$.

If the estimation process can be carried out accurately and quickly enough, then the performance loss can be reduced to its minimum. For a constant fault $\lambda(t \geq t_f) = \lambda_0$ occurring at $t_f$, the performance can be recovered completely when $\tilde{\lambda}(t)$ converges to $\lambda_0$. The controller structure for compensation of effectiveness loss is shown in Fig. 1.

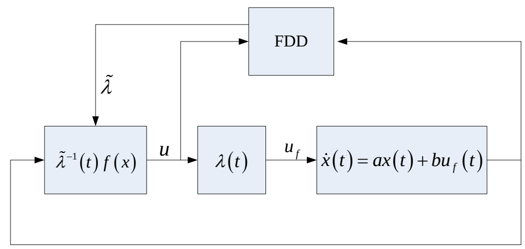


Fig. 1 Compensation principle for effectiveness loss

Above discussions can be extended readily to the multivariable systems. From practical point of view, since the control power is limited and the disturbance exists, then the plant to be controlled in this paper is given by:

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{M}(t)\mathrm{sat}\left[\boldsymbol{u}(t)\right] + \boldsymbol{E}\boldsymbol{\omega}(t) \\ \boldsymbol{y}(t) &= \boldsymbol{C}\boldsymbol{x}(t) \\ \boldsymbol{e}(t) &= \boldsymbol{r}(t) - \boldsymbol{y}(t) \end{aligned} \tag{6}$$

where $\boldsymbol{x}(t) \in \mathbb{R}^n$, $\boldsymbol{u}(t) \in \mathbb{R}^m$, $\boldsymbol{y}(t) \in \mathbb{R}^p$ are the state, input, and output vectors respectively. $\boldsymbol{\omega}(t) \in \mathbb{R}^q$ is an immeasurable disturbance vector bounded by $\|\boldsymbol{\omega}(t)\| \leq \omega_0$. $\boldsymbol{r}(t) \in \mathbb{R}^p$ is the reference signal vector bounded by $\|\boldsymbol{r}(t)\| \leq r_0$. $\boldsymbol{e}(t)$ is the tracking error vector. $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{E}$ are known parameter matrices of appropriate dimensions. It is assumed that $(\boldsymbol{A},\boldsymbol{C})$ is detectable. $\mathrm{sat}(\cdot)$ is a standard vector-valued saturation function with its elements given by:

$$\mathrm{sat}(\boldsymbol{u}_i) = \mathrm{sign}(\boldsymbol{u}_i) \cdot \min\{1, |\boldsymbol{u}_i|\},\ i = 1, 2, \cdots, m \tag{7}$$

where $\mathrm{sign}(\cdot)$ represents the signum function.

$\boldsymbol{M}(t) \in \mathbb{R}^{m \times m}$ is a diagonal matrix representing the effectiveness factors of actuators, and denoted by:

$$\begin{aligned} &\boldsymbol{M}(t) = \mathrm{diag}\{\lambda_1(t), \lambda_2(t), \cdots, \lambda_m(t)\} \\ &\lambda_i(t) \in \left[\underline{\lambda}_i, \bar{\lambda}_i\right],\ 0 < \underline{\lambda}_i \leq 1,\ \ \bar{\lambda}_i \geq 1,\ \ i = 1, 2, \cdots, m \end{aligned} \tag{8}$$

where $\mathrm{diag}\{\cdot\}$ represents a diagonal matrix. $\lambda_i(t),\ i=1,2,\cdots,m$ are unknown stepwise fault signals. $\underline{\lambda}_i$ and $\bar{\lambda}_i$ represent the known lower and upper bound of $\lambda_i(t)$ respectively.

The control objective in this paper is to realize stable tracking of a reference signal in the presence of faults and amplitude constraints of actuators. The overall control system configuration is shown in Fig. 2. The fault detection and diagnosis (FDD) module is used to detect a fault and provide an estimation of fault amplitude denoted by $\tilde{\boldsymbol{M}}(t)=\mathrm{diag}\{\tilde{\lambda}_1(t),\tilde{\lambda}_2(t),\cdots,\tilde{\lambda}_m(t)\}$.

With estimated control effectiveness factors from FDD, the reconfigurable controller adjusts automatically its parameter to recover the performance of the closed-loop system. In this paper, an observer is used to detect a fault, and an adaptive algorithm is designed to estimate the fault amplitude. After a fault is detected by the observer, the adaptive diagnostic algorithm is activated automatically. Otherwise, a unitary matrix is passed to $\tilde{\boldsymbol{M}}(t)$. In summary, the observer is used to determine when a fault occurs, and the adaptive diagnostic algorithm is used to estimate its amplitude.

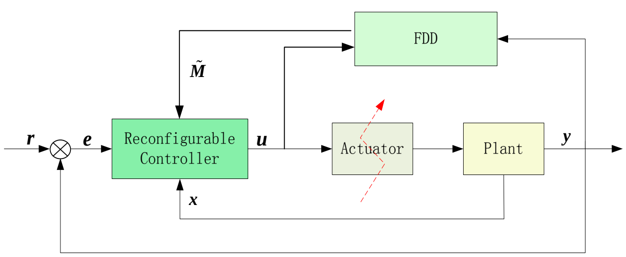


Fig. 2 System configuration

It is well known that the tracking error integral action of a controller can effectively eliminate the steady-state tracking error (Zhang & Jiang, 2001). Denote $\eta(t)=\int_0^t \boldsymbol{e}(\tau)d\tau$, $\zeta(t)=\left[\boldsymbol{x}(t)^{\mathrm{T}} \quad \eta(t)^{\mathrm{T}}\right]^{\mathrm{T}}$, then the following augmented system can be obtained from Eq. (6) such that

$$\dot{\zeta}(t)=\bar{\boldsymbol{A}}\zeta(t)+\bar{\boldsymbol{B}}\boldsymbol{M}(t)\mathrm{sat}\left[\boldsymbol{u}(t)\right]+\bar{\boldsymbol{E}}\boldsymbol{d}(t) \tag{9}$$

where $\bar{\boldsymbol{A}}=\begin{bmatrix}\boldsymbol{A} & \mathbf{0}_{n\times p}\\ -\boldsymbol{C} & \mathbf{0}_{p\times p}\end{bmatrix}$, $\bar{\boldsymbol{B}}=\begin{bmatrix}\boldsymbol{B}\\ \mathbf{0}_{p\times m}\end{bmatrix}$, $\bar{\boldsymbol{E}}=\begin{bmatrix}\boldsymbol{E} & \mathbf{0}_{n\times p}\\ \mathbf{0}_{p\times q} & \mathbf{I}_p\end{bmatrix}$, $\boldsymbol{d}(t)=\begin{bmatrix}\boldsymbol{\omega}(t)\\ \boldsymbol{r}(t)\end{bmatrix}$.

To eliminate the fault effect, the reconfigurable controller is realized by

$$\boldsymbol{u}(t)=\tilde{\boldsymbol{M}}^{-1}(t)\boldsymbol{K}\zeta(t) \tag{10}$$

Substituting Eq. (10) into Eq. (9), it is obtained that

$$\dot{\zeta}(t)=\bar{\boldsymbol{A}}\zeta(t)+\bar{\boldsymbol{B}}\boldsymbol{M}(t)\mathrm{sat}\left[\tilde{\boldsymbol{M}}^{-1}(t)\boldsymbol{K}\zeta(t)\right]+\bar{\boldsymbol{E}}\boldsymbol{d}(t) \tag{11}$$

## 3. INTEGRATION OF RECONFIGURABLE CONTROLLER WITH FAULT DIAGNOSIS

LMI conditions will be presented in this section to design the controller gain $\boldsymbol{K}$, while the estimated fault amplitude $\tilde{\boldsymbol{M}}(t)$ is obtained by an adaptive algorithm.

**Lemma 1** (Hu & Lin, 2001) Let $\boldsymbol{u},\boldsymbol{v}\in\mathbb{R}^m$ and suppose that $|\boldsymbol{v}_i|\le 1,\ i=1,2,\cdots,m$, then

$$\mathrm{sat}(\boldsymbol{u})\in\mathrm{Co}\left\{\Delta_j\boldsymbol{u}+\Delta_j^-\boldsymbol{v},\ j=1,2,\ldots,2^m\right\} \tag{12}$$

where $\mathrm{Co}\{\cdot\}$ denotes the convex hull. $\Delta_j\in\mathbb{R}^{m\times m}$ is a diagonal matrix whose elements are either 0 or 1, and $\Delta_j^-=\mathbf{I}_m-\Delta_j$. For brevity, $|\boldsymbol{v}_i|\le 1,\ i=1,2,\cdots,m$ is written as $|\boldsymbol{v}|\le 1$ in the following.

From Lemma 1, if there exists an auxiliary matrix $\boldsymbol{H}$ satisfying

$$\left|\tilde{\boldsymbol{M}}^{-1}(t)\boldsymbol{H}\zeta(t)\right|\le 1 \tag{13}$$

then there always exist $\alpha_j\ge 0,\ \sum_{j=1}^{2^m}\alpha_j=1$ such that

$$\dot{\zeta}(t)=\sum_{j=1}^{2^m}\alpha_j\left(\bar{\boldsymbol{A}}+\bar{\boldsymbol{B}}\boldsymbol{M}(t)\tilde{\boldsymbol{M}}^{-1}(t)\left[\Delta_j\boldsymbol{K}+\Delta_j^-\boldsymbol{H}\right]\right)\zeta(t)+\bar{\boldsymbol{E}}\boldsymbol{d}(t) \tag{14}$$

If $\alpha_j,\ j=1,2,\ldots,2^m$ are taken as the scheduled parameters and can be obtained online, then Eq. (14) is actually an LPV system. Define

$$\boldsymbol{F}(t)=\tilde{\boldsymbol{M}}^{-1}(t)\boldsymbol{H} \tag{15}$$

then it is not difficult to find out that (13) imposes a polyhedral set constraint on system states of Eq. (14) as follows:

$$\mathcal{L}\left(\boldsymbol{F}(t)\right)=\left\{\zeta(t)\,\middle|\,\left|\boldsymbol{F}_i(t)\zeta(t)\right|\le 1,\ i=1,2,\cdots,m\right\} \tag{16}$$

For estimation of domain of attraction, an ellipsoidal set is defined as follows:

$$\Omega(\boldsymbol{P})=\left\{\zeta(t)\,|\,\zeta^{\mathrm{T}}(t)\boldsymbol{P}\zeta(t)\le 1,\ \boldsymbol{P}>0\right\} \tag{17}$$

**Theorem 1** If there exist matrices $\boldsymbol{Y}_k\in\mathbb{R}^{m\times(n+p)}$, $\boldsymbol{Y}_h\in\mathbb{R}^{m\times(n+p)}$, a positive definite matrix $\boldsymbol{Q}\in\mathbb{R}^{(n+p)\times(n+p)}$, and a positive scalar $\mu$ such that

$$\begin{bmatrix} 1 & \phi_i \boldsymbol{k}_i^1 \boldsymbol{Y}_h \\ * & \boldsymbol{Q} \end{bmatrix} \geq 0, \quad i = 1, 2, \cdots, m \tag{18}$$

$$\bar{\boldsymbol{A}}\boldsymbol{Q} + \boldsymbol{Q}\bar{\boldsymbol{A}}^{\mathrm{T}} + \frac{1}{\mu}\bar{\boldsymbol{E}}\bar{\boldsymbol{E}}^{\mathrm{T}} + \mu\left(r_0^2 + \omega_0^2\right)\boldsymbol{Q} + 2\bar{\boldsymbol{B}}\left(\Delta_j \boldsymbol{Y}_k + \Delta_j^{-} \boldsymbol{Y}_h\right) < 0 \\ j = 1, 2, \cdots, 2^m \tag{19}$$

then $\Omega(\boldsymbol{P})$ is an invariant set with $\boldsymbol{P} = \boldsymbol{Q}^{-1}$, $\boldsymbol{K} = \boldsymbol{Y}_k \boldsymbol{P}$, $\boldsymbol{H} = \boldsymbol{Y}_h \boldsymbol{P}$, and with the fault diagnostic algorithm being realized by:

$$\begin{aligned} &\dot{\hat{\lambda}}_i\left(0 \leq t < t_f\right) = 0, \quad i = 1, 2, \cdots, m \\ &\dot{\hat{\lambda}}_i\left(t \geq t_f\right) = \mathrm{Proj}_{\left[\underline{\lambda}_i,\, \bar{\lambda}_i\right]}\left\{\gamma_i \tilde{\lambda}_i^{-1}(t)\zeta(t)^{\mathrm{T}} \boldsymbol{P}\bar{\boldsymbol{b}}_i \boldsymbol{k}_i^1 \sum_{j=1}^{2^m} \alpha_j \left[\Delta_j \boldsymbol{K}_r + \Delta_j^{-} \boldsymbol{H}_r\right]\zeta(t)\right\} \end{aligned} \tag{20}$$

where $\phi_i \in \left\{\underline{\lambda}_i, \bar{\lambda}_i\right\}$. $\gamma_i > 0$ is pre-specified positive scalar. $\bar{\boldsymbol{b}}_i$ is the $\boldsymbol{i}$-th column of $\bar{\boldsymbol{B}}$. $\boldsymbol{k}_i^1$ is a row vector with its $\boldsymbol{i}$-th element being 1 and the other elements being 0. $\mathrm{Proj}_{\left[\underline{\lambda}_i,\, \bar{\lambda}_i\right]}\{\bullet\}$ is a projection operator defined as follows:

$$\mathrm{Proj}_{\left[\underline{\lambda}_i,\, \bar{\lambda}_i\right]}\{X\} = \begin{cases} 0, & \hat{\lambda}_i \geq \bar{\lambda}_i \text{ and } X > 0 \\ & \text{or} \\ & \hat{\lambda}_i \leq \underline{\lambda}_i \text{ and } X < 0 \\ X, & \text{else} \end{cases} \tag{21}$$

**Proof:** Denote

$$\boldsymbol{E}_\lambda(t) = \tilde{\boldsymbol{M}}(t) - \boldsymbol{M}(t) = \mathrm{diag}\left\{e_{\lambda 1}(t), e_{\lambda 2}(t), \cdots, e_{\lambda m}(t)\right\} \tag{22}$$

Define a Lyapunov function

$$V(t) = \zeta(t)^{\mathrm{T}} \boldsymbol{P}\zeta(t) + \sum_{i=1}^{m} \gamma_i^{-1} e_{\lambda i}^2(t) \tag{23}$$

Its derivative with respect to time is given by:

$$\begin{aligned} \dot{V}(t) = {} & 2\zeta(t)^{\mathrm{T}} \boldsymbol{P} \sum_{j=1}^{2^m} \alpha_j \left(\bar{\boldsymbol{A}} + \bar{\boldsymbol{B}}\boldsymbol{M}\tilde{\boldsymbol{M}}^{-1}(t)\left[\Delta_j \boldsymbol{K} + \Delta_j^{-} \boldsymbol{H}\right]\right)\zeta(t) \\ & + 2\zeta(t)^{\mathrm{T}} \boldsymbol{P}\bar{\boldsymbol{E}}\boldsymbol{d}(t) + 2\sum_{i=1}^{m} \gamma_i^{-1} e_{\lambda i}(t)\dot{e}_{\lambda i}(t) \end{aligned} \tag{24}$$

Since

$$\begin{aligned} & 2\zeta(t)^{\mathrm{T}} \boldsymbol{P}\bar{\boldsymbol{E}}\boldsymbol{d}(t) \leq \frac{1}{\mu}\zeta(t)^{\mathrm{T}} \boldsymbol{P}\bar{\boldsymbol{E}}\bar{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\zeta(t) + \mu\boldsymbol{d}(t)^{\mathrm{T}}\boldsymbol{d}(t) \\ & \leq \frac{1}{\mu}\zeta(t)^{\mathrm{T}} \boldsymbol{P}\bar{\boldsymbol{E}}\bar{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P}\zeta(t) + \mu\left(r_0^2 + \omega_0^2\right) \end{aligned} \tag{25}$$

then it follows that:

$$\dot{V}(t) \leq \zeta(t)^{\mathrm{T}} \mathsf{M}\, \zeta(t) + 2\sum_{i=1}^{m} \gamma_i^{-1} e_{\lambda i}(t)\dot{e}_{\lambda i}(t) + \mu\left(r_0^2 + \omega_0^2\right) \tag{26}$$

where

$$\mathsf{M} \ddagger 2\boldsymbol{P}\sum_{j=1}^{2^m} \alpha_j \left(\bar{\boldsymbol{A}} + \bar{\boldsymbol{B}}\boldsymbol{M}\tilde{\boldsymbol{M}}^{-1}(t)\left[\Delta_j \boldsymbol{K} + \Delta_j^{-} \boldsymbol{H}\right]\right) + \frac{1}{\mu}\boldsymbol{P}\bar{\boldsymbol{E}}\bar{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P} \tag{27}$$

Since

$$\boldsymbol{M}\tilde{\boldsymbol{M}}^{-1}(t) = \mathbf{I}_m - \boldsymbol{E}_\lambda(t)\tilde{\boldsymbol{M}}^{-1}(t) \tag{28}$$

it follows that:

$$\mathsf{M} = \bar{\mathsf{M}} - 2\boldsymbol{P}\bar{\boldsymbol{B}}\boldsymbol{E}_\lambda(t)\tilde{\boldsymbol{M}}^{-1}(t)\sum_{j=1}^{2^m} \alpha_j \left[\Delta_j \boldsymbol{K} + \Delta_j^{-} \boldsymbol{H}\right] \tag{29}$$

where

$$\bar{\mathsf{M}} = \boldsymbol{P}\bar{\boldsymbol{A}} + \bar{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P} + \frac{1}{\mu}\boldsymbol{P}\bar{\boldsymbol{E}}\bar{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{P} + 2\boldsymbol{P}\bar{\boldsymbol{B}}\sum_{j=1}^{2^m} \alpha_j \left[\Delta_j \boldsymbol{K} + \Delta_j^{-} \boldsymbol{H}\right] \tag{30}$$

Since

$$\bar{\boldsymbol{B}}\boldsymbol{E}_\lambda(t)\tilde{\boldsymbol{M}}^{-1}(t) = \sum_{i=1}^{m} \bar{\boldsymbol{b}}_i e_{\lambda i}(t)\boldsymbol{k}_i^1 \tilde{\boldsymbol{M}}^{-1}(t) = \sum_{i=1}^{m} e_{\lambda i}(t)\tilde{\lambda}_i^{-1}(t)\bar{\boldsymbol{b}}_i \boldsymbol{k}_i^1 \tag{31}$$

then it can be obtained from Eqs. (20) and (26) that

$$\dot{V}(t) \leq \zeta(t)^{\mathrm{T}}\left[\bar{\mathsf{M}} + \mu\left(r_0^2 + \omega_0^2\right)\boldsymbol{P}\right]\zeta(t) + \mu\left(r_0^2 + \omega_0^2\right)\left[1 - \zeta(t)^{\mathrm{T}}\boldsymbol{P}\zeta(t)\right] \tag{32}$$

With Eqs. (23) and (32), it is not difficult to verify that $\Omega(\boldsymbol{P})$ is an invariant set by satisfying

$$\bar{\mathsf{M}} + \mu\left(r_0^2 + \omega_0^2\right)\boldsymbol{P} < 0 \tag{33}$$

which is equivalent to Eq. (19).

To complete the proof, it is still needed to guarantee that $\Omega(\boldsymbol{P}) \subset \mathsf{L}\left(\boldsymbol{F}(t)\right)$, of which an equivalent condition can be stated as follows:

$$\Theta_i = \begin{Bmatrix} \max\limits_{\zeta(t)} & \left|\boldsymbol{F}_i(t)\zeta(t)\right| \\ s.\,t. & \zeta^{\mathrm{T}}(t)\boldsymbol{P}\zeta(t) = 1 \end{Bmatrix} \leq 1 \tag{34}$$

By using the method of Lagrange multipliers, it is not difficult to obtain that

$$\Theta_i = \sqrt{\boldsymbol{F}_i(t)\boldsymbol{P}^{-1}\boldsymbol{F}_i^{\mathrm{T}}(t)} \tag{35}$$

Since

$$\boldsymbol{F}_i(t)\boldsymbol{P}^{-1}\boldsymbol{F}_i^{\mathrm{T}}(t) = \left[\tilde{\boldsymbol{M}}^{-1}(t)\right]_i \boldsymbol{Y}_h \boldsymbol{Q}^{-1}\boldsymbol{Y}_h^{\mathrm{T}}\left[\tilde{\boldsymbol{M}}^{-1}(t)\right]_i^{\mathrm{T}} \tag{36}$$

then by Schur complement, an equivalent condition for $\Omega(\boldsymbol{P}) \subset \mathsf{L}(\boldsymbol{F})$ is given by:

$$\begin{bmatrix} 1 & \left[\tilde{\boldsymbol{M}}^{-1}(t)\right]_i \boldsymbol{Y}_h \\ * & \boldsymbol{Q} \end{bmatrix} \geq 0, \quad i = 1, 2, \cdots, m \tag{37}$$

The extreme point set of $\tilde{\boldsymbol{M}}^{-1}(t)$ can be defined as follows:

$$\Sigma \triangleq \left\{\Psi^j \middle| \Psi^j = \mathrm{diag}\{\phi_1,\phi_2,\cdots,\phi_m\},\ \phi_i = \underline{\lambda}_i^{-1}\ or\ \bar{\lambda}_i^{-1},\quad i=1,2,\cdots,m;\ j=1,2,\cdots,2^m\right\} \tag{38}$$

then from the convexity of $\sum$, there always exist $\beta_j \geq 0$, $\sum_{j=1}^{2^m}\beta_j = 1$ such that

$$\tilde{\boldsymbol{M}}^{-1}(t) = \sum_{j=1}^{2^m}\beta_j \Psi^j \tag{39}$$

From Eq. (39), it gives

$$\left[\tilde{\boldsymbol{M}}^{-1}(t)\right]_i = \sum_{j=1}^{2^m}\beta_j\left(\Psi^j\right)_i = \sum_{j=1}^{2^m}\beta_j\phi_i\boldsymbol{k}_i^1 \tag{40}$$

then (37) can be written as:

$$0 \leq \begin{bmatrix} 1 & \sum_{j=1}^{2^m}\beta_j\phi_i\boldsymbol{k}_i^1\cdot\boldsymbol{Y} \\ * & \boldsymbol{Q}\end{bmatrix} = \sum_{j=1}^{2^m}\beta_j\begin{bmatrix} 1 & \phi_i\boldsymbol{k}_i^1\boldsymbol{Y}_h \\ * & \boldsymbol{Q}\end{bmatrix} \tag{41}$$
$$i=1,2,\cdots,m;\ j=1,2,\cdots,2^m$$

It is sufficient for (18) to guarantee that (41) holds true. This ends the proof. □

**Remark 1:** The values of $\alpha_j, j=1,2,\ldots,2^m$ are needed online in the adaptive diagnostic algorithm as shown in Eq. (20). One way to obtain them (Wu, Lin & Zheng, 2007) is shown as follows:

$$\alpha_j = \prod_{i=1}^{m}\left[z_i\left(1-\lambda_i\right)+\left(1-z_i\right)\lambda_i\right] \tag{42}$$

where $z_1 2^{m-1} + z_2 2^{m-2} + \cdots + z_m = j-1$, and

$$\lambda_i = \begin{cases} 1, & \boldsymbol{H}_r = \boldsymbol{K}_r \\ \dfrac{\mathrm{sat}\left(\tilde{m}_i^{-1}(t)\boldsymbol{k}_i^1\boldsymbol{K}_r\zeta(t)\right) - \tilde{m}_i^{-1}(t)\boldsymbol{k}_i^1\boldsymbol{H}_r\zeta(t)}{\tilde{m}_i^{-1}(t)\boldsymbol{k}_i^1\boldsymbol{K}_r\zeta(t) - \tilde{m}_i^{-1}(t)\boldsymbol{k}_i^1\boldsymbol{H}_r\zeta(t)}, & \text{else}\end{cases} \tag{43}$$

Since $\Omega(\boldsymbol{P})$ is an estimation of the domain of attraction, it is desirable to obtain the largest one. This is a volume maximization problem. In general, there are two ways to maximize $\Omega(\boldsymbol{P})$. Since the volume of $\Omega(\boldsymbol{P})$ is proportional to $\det(\boldsymbol{Q})$, one direct way is to construct an determinant maximization problem (Vandenberghe, Boyd & Wu, 1998) as follows:

$$\begin{aligned} &\sup_{\boldsymbol{Q}>0,\boldsymbol{Y}_k,\boldsymbol{Y}_h,\mu>0} && \log\det\boldsymbol{Q} \\ &\text{s.t.} && (18)\text{ and }(19)\end{aligned} \tag{44}$$

The other way is to use a prescribed bounded convex reference set $X_R$ to maximize $\Omega(\boldsymbol{P})$, which can take its shape into consideration. Two typical sets of $X_R$ are the ellipsoids and polyhedrons. By taking an ellipsoid $X_0 = \left\{\zeta(t)\in\mathbb{R}^{n+p} \mid \zeta(t)^{\mathrm{T}}\boldsymbol{R}\zeta(t)\leq 1,\ \boldsymbol{R}>0\right\}$ as the reference set, the following optimization problem can be formulated:

$$\begin{aligned} &\sup_{\boldsymbol{Q}>0,\boldsymbol{Y}_k,\boldsymbol{Y}_h,\mu>0} && \alpha \\ &\text{s.t.} && (a)\quad \alpha X_0 \subset \Omega(\boldsymbol{P}) \\ & && (b)\quad (18)\text{ and }(19)\end{aligned} \tag{45}$$

Let $\gamma = 1/\alpha^2$, since $\alpha X_0 = \Omega(\gamma\boldsymbol{R})$, then $\alpha X_0 \subset \Omega(\boldsymbol{P})$ is equivalent to $\gamma\boldsymbol{R} \geq \boldsymbol{P}$. By Schur complement, (45) can be written as:

$$\begin{aligned} &\inf_{\boldsymbol{Q}>0,\boldsymbol{Y}_k,\boldsymbol{Y}_h,\mu>0} && \gamma \\ &\text{s.t.} && (a)\quad \begin{bmatrix}\gamma\boldsymbol{R} & \mathbf{I} \\ \mathbf{I} & \boldsymbol{Q}\end{bmatrix} \geq 0 \\ & && (b)\quad (18)\text{ and }(19)\end{aligned} \tag{46}$$

For a reference set described by a polyhedron $X_0 = \mathrm{conv}\left\{x_1^0, x_2^0, \cdots, x_N^0\right\}$, $\alpha X_0 \subset \Omega(\boldsymbol{P})$ is equivalent to $\left(x_i^0\right)^{\mathrm{T}}\boldsymbol{P}x_i^0 \leq \gamma$. Then by Schur complement, the first LMI in Eq. (46) should be replaced by:

$$\begin{bmatrix}\gamma & \left(x_i^0\right)^{\mathrm{T}} \\ x_i^0 & \boldsymbol{Q}\end{bmatrix} \geq 0 \tag{47}$$

In another aspect, the system states cannot be guaranteed to converge to the origin due to the disturbances and actuator faults. Hence, a performance index is needed for the disturbance rejection problem, which can also be described by a prescribed bounded convex reference set. Assumed that this set is denoted by $X_\infty$, then an optimization problem can be formulated as follows:

$$\begin{aligned} &\inf_{\boldsymbol{Q}>0,\boldsymbol{Y}_k,\boldsymbol{Y}_h,\mu>0} && \beta \\ &\text{s.t.} && (a)\quad \Omega(\boldsymbol{P}) \subset \beta X_\infty \\ & && (b)\quad (18)\text{ and }(19)\end{aligned} \tag{48}$$

To address the disturbance rejection and domain of attraction simultaneously, a scaled version of $\Omega(\boldsymbol{P})$ is defined as follows:

$$\Omega(\boldsymbol{S}) = \left\{\zeta(t)\in\mathbb{R}^{n+p} \mid \zeta(t)^{\mathrm{T}}\boldsymbol{S}\zeta(t)\leq 1,\ \boldsymbol{S} = \rho^{-1}\boldsymbol{P},\ \boldsymbol{S}>0,\ 0<\rho\leq 1\right\} \tag{49}$$

From the convexity (Hu *et al*., 2002) of both (18) and (19), it is not difficult to verify that all the trajectories staring from within $\Omega(\boldsymbol{P})$ will enter $\Omega(\boldsymbol{S})$ and remain inside it if there exist $\boldsymbol{Q}>0, \boldsymbol{Y}_k, \boldsymbol{Y}_h, \boldsymbol{Y}_s, \mu>0$ satisfying (18), (19) and

$$\rho \bar{\boldsymbol{A}}\boldsymbol{Q} + \rho \boldsymbol{Q}\bar{\boldsymbol{A}}^{\mathrm{T}} + \frac{1}{\mu}\bar{\boldsymbol{E}}\bar{\boldsymbol{E}}^{\mathrm{T}} + \mu r_0^2 \rho \boldsymbol{Q} + 2\bar{\boldsymbol{B}}\left(\Delta_j \boldsymbol{Y}_k + \Delta_j^{-} \boldsymbol{Y}_s\right) < 0,\ j = 1,2,\cdots,2^m$$

$$\begin{bmatrix} 1 & \phi_i \boldsymbol{k}_i^1 \boldsymbol{Y}_s \\ * & \rho \boldsymbol{Q} \end{bmatrix} \geq 0,\quad i = 1,2,\cdots,m \tag{50}$$

Therefore, to solve the disturbance rejection problem with guaranteed domain of attraction, the following optimization problem can be formulated:

$$\begin{aligned} &\inf_{\boldsymbol{Q}>0,\boldsymbol{Y}_k,\boldsymbol{Y}_h,\boldsymbol{Y}_s,\mu>0} && \beta \\ &\text{s.t.}\quad (a) && \Omega(\boldsymbol{S}) \subset \beta X_\infty \\ &\qquad\ \ (b) && X_0 \subset \Omega(\boldsymbol{P}) \\ &\qquad\ \ (c) && (18),(19) \text{ and } (50) \end{aligned} \tag{51}$$

**Remark 2:** The controller gain $\boldsymbol{K}$ computed from (51) may be too high to be used in practice. To adjust the controller gain $\boldsymbol{K}$, since $\boldsymbol{K} = \boldsymbol{Y}_k \boldsymbol{P}$, then the following inequality can be added into the optimization problems:

$$\boldsymbol{Y}_k \boldsymbol{Y}_k^{\mathrm{T}} \leq \sigma \mathbf{I}_m,\ \sigma > 0 \tag{52}$$

By Schur complement, (52) is equivalent to

$$\begin{bmatrix} \sigma \mathbf{I}_m & \boldsymbol{Y}_k \\ * & \mathbf{I}_n \end{bmatrix} \geq 0 \tag{53}$$

## 4. Observer-based Fault Detection

To activate the adaptive diagnostic algorithm as shown in Eq. (20), the time $t_f$ when a fault occurs is needed to be known. It is the responsibility of fault detection. In this paper, the fault detection is carried out by comparing the output residual with the threshold to be set.

To detect the fault, an observer is defined as follows:

$$\begin{aligned} \dot{\tilde{\boldsymbol{x}}}(t) &= \boldsymbol{A}\tilde{\boldsymbol{x}}(t) + \boldsymbol{B}\mathrm{sat}\left[\boldsymbol{u}(t)\right] + \boldsymbol{L}\left(\boldsymbol{y}(t) - \tilde{\boldsymbol{y}}(t)\right) \\ \tilde{\boldsymbol{y}}(t) &= \boldsymbol{C}\tilde{\boldsymbol{x}}(t) \end{aligned} \tag{54}$$

where $\tilde{\boldsymbol{x}}(t)$ and $\tilde{\boldsymbol{y}}(t)$ are estimation of $\boldsymbol{x}(t)$ and $\boldsymbol{y}(t)$ respectively.

Denote $\boldsymbol{e}_x(t) = \tilde{\boldsymbol{x}}(t) - \boldsymbol{x}(t)$, $\boldsymbol{e}_y(t) = \tilde{\boldsymbol{y}}(t) - \boldsymbol{y}(t)$, then an observer error equation can be obtained without incorporating $\boldsymbol{\omega}(t)$

$$\begin{aligned} \dot{\boldsymbol{e}}_x(t) &= (\boldsymbol{A} - \boldsymbol{L}\boldsymbol{C})\boldsymbol{e}_x(t) + \boldsymbol{B}\left(\mathbf{I}_m - \boldsymbol{M}(t)\right)\mathrm{sat}\left[\boldsymbol{u}(t)\right] \\ \boldsymbol{e}_y(t) &= \boldsymbol{C}\boldsymbol{e}_x(t) \end{aligned} \tag{55}$$

With $(\boldsymbol{A},\boldsymbol{C})$ being detectable, it is not difficult to obtain the observer gain $\boldsymbol{L}$ such that $\boldsymbol{A} - \boldsymbol{L}\boldsymbol{C}$ is stable. Then a fault is detected if $\left\|\boldsymbol{e}_y(t)\right\| > \lambda_f$, where $\lambda_f$ is a pre-specified threshold. If $\tilde{\boldsymbol{x}}(0) = \boldsymbol{x}(0)$, then $\lambda_f = 0$ is sufficient to detect a fault.

However, when $\boldsymbol{\omega}(t)$ is presented, false alarm may be generated with above detector, even if $\tilde{\boldsymbol{x}}(0) = \boldsymbol{x}(0)$ is satisfied. Increasing $\lambda_f$ may prevent a false alarm, but it may lead to a detector which is insensitive to a fault of small amplitude. Hence, it is desirable to determine a minimum threshold.

In the presence of the disturbance $\boldsymbol{\omega}(t)$, the observer error equation becomes:

$$\begin{aligned} \dot{\boldsymbol{e}}_x(t) &= (\boldsymbol{A} - \boldsymbol{L}\boldsymbol{C})\boldsymbol{e}_x(t) + \boldsymbol{B}\left(\mathbf{I}_m - \boldsymbol{M}(t)\right)\mathrm{sat}\left[\boldsymbol{u}(t)\right] - \boldsymbol{E}\boldsymbol{\omega}(t) \\ \boldsymbol{e}_y(t) &= \boldsymbol{C}\boldsymbol{e}_x(t) \end{aligned} \tag{56}$$

Assumed that $\boldsymbol{e}_x(0) = 0$, then by Laplace transformation, it is obtained that

$$\boldsymbol{e}_y(s) = \boldsymbol{C}\left(s\mathbf{I}_n - \boldsymbol{A} + \boldsymbol{L}\boldsymbol{C}\right)^{-1}\boldsymbol{B}\left(\mathbf{I}_m - \boldsymbol{M}(t)\right)\mathrm{sat}\left[\boldsymbol{u}(s)\right] + G(s)\boldsymbol{\omega}(s) \tag{57}$$

where $G(s) = -\boldsymbol{C}\left(s\mathbf{I}_n - \boldsymbol{A} + \boldsymbol{L}\boldsymbol{C}\right)^{-1}\boldsymbol{E}$.

Since no fault occurs when $t < t_f$, that is $\boldsymbol{M}\left(t < t_f\right) = \mathbf{I}_m$, then Eq. (57) can be written as:

$$\boldsymbol{e}_y(s) = G(s)\boldsymbol{\omega}(s) \tag{58}$$

Since the disturbance $\boldsymbol{\omega}(t)$ is unknown, then the $\mathrm{H}_\infty$ norm of $G(s)$ can be used, which is denoted by:

$$\left\|G(s)\right\|_\infty = \sup_{w\in\mathbb{R}} \sigma_{\max}\left[G(jw)\right] \tag{59}$$

Where sup denotes the least upper bound, $\sigma_{\max}$ denotes the maximum singular value of a matrix, and $j = \sqrt{-1}$.

$\left\|G(s)\right\|_\infty$ actually gives out the peak gain of $G(s)$ across all frequencies. Hence, a minimum threshold for setting fault alarms can be given by:

$$\min\left(\lambda_f\right) = \left\|G(s)\right\|_\infty \omega_0 \tag{60}$$

With the minimum threshold, the fault detection can be carried out by

$$\left\{\begin{aligned} \left\|\boldsymbol{e}_y(t)\right\| &< \min\left(\lambda_f\right): && \text{No fault occurs} \\ \left\|\boldsymbol{e}_y(t)\right\| &\geq \min\left(\lambda_f\right): && \text{A fault has occurred} \end{aligned}\right\} \tag{61}$$

**Remark 3:** With the threshold given in (60), there still exist a possibility that the fault detector is insensitive to some kinds of fault which may result in small output residuals compared with the threshold. In this case, the adaptive diagnostic algorithm will not be activated after fault occurring, and the system performance can only be guaranteed by the robustness of the controller designed. Since our emphasis is put on avoiding the false alarm due to disturbance, other fault detection methods which may be more sensitive to the faults will not be discussed in detail in this paper. Actually, as will be shown in next section, the controller is robust enough to guarantee the tracking performance under serious faults while the adaptive diagnostic algorithm is not activated.

## 5. APPLICATION EXAMPLE

For illustration, the design techniques are applied to the flight control of a Zagi flying wing aircraft (Beard & McLain, 2011). In this example, the control objective is to track the pitch angular and roll angular commands. In the straight and level trim condition with airspeed 10 (m/s) and altitude 50 (m), a linearized model can be obtained as follows:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t)$$
$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t)$$

$$\boldsymbol{A} = \begin{bmatrix} 0 & 0 & 0 & 1 & -0.0001 \\ 0 & 0 & 1 & 0 & 0.1665 \\ 0 & 0 & -2.5369 & 0 & 1.3228 \\ 0 & 0 & -0.0000 & -5.6319 & 0 \\ 0 & 0 & 0.1817 & 0 & -3.4009 \end{bmatrix},$$

$$\boldsymbol{B} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 4.8744 & 6.3103 \\ -20.8139 & 0 & 0 \\ 0 & 3.6834 & -1.8480 \end{bmatrix}, \boldsymbol{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

where the states $\boldsymbol{x} = [\theta, \phi, p, q, r]^{\mathrm{T}}$ represent the pitch angle (rad), roll angle (rad), and roll rate (rad/s), pitch rate (rad/s), yaw rate (rad/s) in body frame. The controls $\boldsymbol{u} = [\delta_e, \delta_a, \delta_r]^{\mathrm{T}}$ represent the deflection angles (rad) of elevator, aileron and rudder respectively. The control effectiveness matrix $\boldsymbol{B}$ is a normalized control matrix such that the control inputs are constrained by the unitary limits.

To compute the controller gains with the design method in Section 4.1, it is assumed that $\boldsymbol{E} = \mathbf{I}_{5\times 1}$, $\boldsymbol{R}_0 = 10\mathbf{I}_7$, $\boldsymbol{R}_\infty = \mathbf{I}_7$, $r_0 = 1.5$, $\rho = 1$, $\omega_0 = 1$, $\underline{\lambda}_i = 0.2,\ \overline{\lambda}_i = 1,\ i = 1, 2, \cdots, m$, $\sigma = 10^5$. Then by solving the optimization problem (51) with (53), it is obtained that $\mu^* = 0.22$, $\beta^* = 10.1667$, and

$$\boldsymbol{P} = \begin{bmatrix} 0.5505 & -0.3489 & -0.1205 & 0.1068 & -0.0567 & -0.2250 & 0.1294 \\ -0.3489 & 0.4144 & 0.1617 & -0.0543 & 0.0704 & 0.1420 & -0.1615 \\ -0.1205 & 0.1617 & 0.0805 & -0.0169 & 0.0318 & 0.0468 & -0.0628 \\ 0.1068 & -0.0543 & -0.0169 & 0.0330 & -0.0082 & -0.0430 & 0.0186 \\ -0.0567 & 0.0704 & 0.0318 & -0.0082 & 0.0249 & 0.0220 & -0.0265 \\ -0.2250 & 0.1420 & 0.0468 & -0.0430 & 0.0220 & 0.1079 & -0.0549 \\ 0.1294 & -0.1615 & -0.0628 & 0.0186 & -0.0265 & -0.0549 & 0.0764 \end{bmatrix}$$

$$\boldsymbol{K} = \begin{bmatrix} 34.0816 & -17.4027 & -5.4353 & 10.1004 & -2.6318 & -13.7182 & 5.9938 \\ 33.3249 & -43.7326 & -20.7667 & 4.7188 & -10.6075 & -12.9678 & 16.7942 \\ 27.1748 & -36.9970 & -18.2546 & 3.7820 & -6.2969 & -10.5928 & 14.4132 \end{bmatrix}$$

For design of the fault detector, the desired poles for $\boldsymbol{A} - \boldsymbol{LC}$ are assumed to be $\{-1, -2, -3, -4, -5\}$. Then by pole placement, it is obtained that

$$\boldsymbol{L} = \begin{bmatrix} 1.3112 & -0.4096 & -0.0615 & 2.3566 & 0.0939 \\ -0.5300 & 2.1190 & -1.4849 & 0.4902 & 0.3049 \end{bmatrix}^{\mathrm{T}}$$

It follows from (60) that a minimum threshold for setting fault alarms can be given by

$$\min(\lambda_f) = 1.6424$$

To verify the tracking performance of the designed controller under fault situations, the nonlinear model with 6 degree of freedom is used, and it is assumed that the effectiveness factor of the elevator is reduced to be 0.2 at $t_f = 15$, and the effectiveness factors of both aileron and rudder are reduced to be 0.2 at $t_f = 55$. The learning rates for the adaptive diagnostic algorithm are specified by $\gamma_i = 100,\ i = 1, 2, \cdots, m$. The reference commands for the pitch angle and the roll angle are both given by the square signals with time period of 20 seconds each, and the amplitudes for both maneuvers are 10 degrees.

Then through simulation with the nonlinear model of the aircraft, the tracking results are given by Fig. 3. For comparison, the tracking results in normal case are also presented in this figure. It is obvious that good performance is achieved for both tracking of the pitch angle and roll angle commands. Though the effectiveness loss of elevator at $t_f = 15$ has impaired the tracking performance, it is recovered quickly. This is actually contributed by excellent function of our integrated fault detection, adaptive diagnosis and reconfiguration algorithm. After malfunction of the elevator, the output residuals exceed the threshold for fault alarm as shown in Fig. 4. Then the adaptive diagnostic algorithm presented in (20) is activated to start the fault estimation process, which is shown in Fig. 5. Due to fast estimation of the effectiveness factor of the elevator, according to the control law in (10), the effectiveness loss is compensated quickly as shown in Fig. 6, which results in good tracking performance under fault condition as shown in Fig. 3.

In addition, from Fig. 4, it can be found out that the residuals in normal case are not equal to zero, which results from the un-modeled dynamics of the aircraft. However, their values are smaller than thresholds. Hence, a false alarm has been avoided by using the fault detection method proposed in Section 4.

For effectiveness loss of both aileron and rudder at $t_f = 55$, the output residuals are smaller than the threshold, and these faults have not been detected. However, good tracking performance of the roll angle command can still be achieved as shown in Fig. 3 due to strong robustness of the controller designed. Since the effectiveness loss of aileron and rudder is not compensated, the responses of these two actuators are not approaching those in normal case as shown in Fig. 6.

For information, some other state variables from the nonlinear model are also given as in Fig. 7, which indicates that the aircraft has reached new equilibrium points under both the normal case and the fault case. These states are $X_e, Y_e, Z_e$ for aircraft position in inertial frame, $U, V, W$ for aircraft velocity in body frame, and $\psi$ for yaw angle. From Fig. 7, it can be seen that the main influence of effectiveness loss of elevator is on the pitch rate, while the effectiveness loss of aileron and rudder mainly affect the roll rate, yaw rate, and lateral-directional velocity in body frame. For an intuitionistic comparison, the 3D trajectories of the aircraft under both normal case and fault case are also presented in Fig. 8.

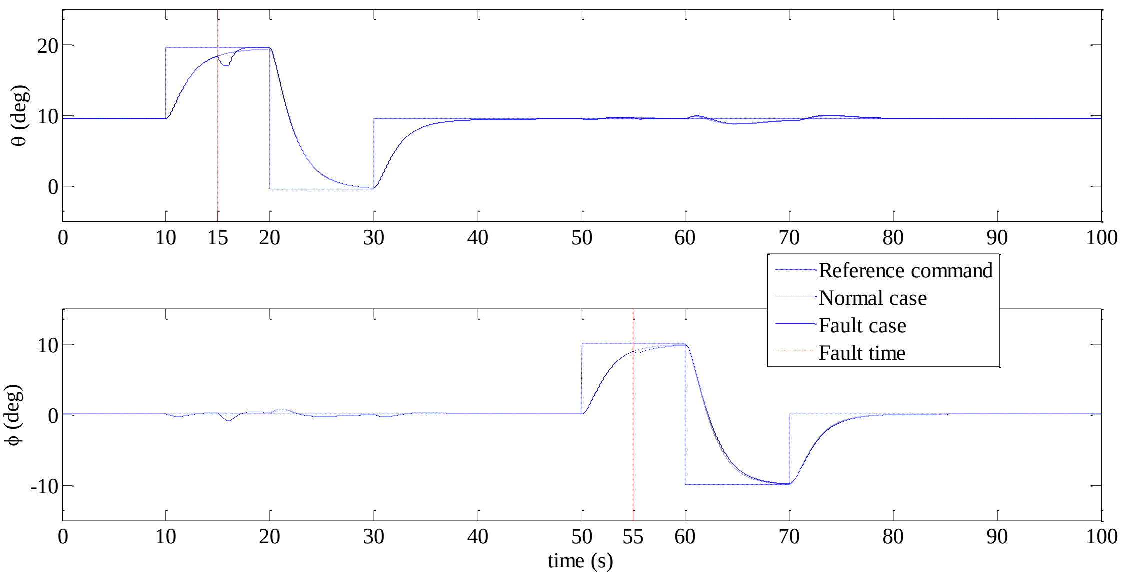


Fig. 3 Tracking of Pitch Angle and Roll Angle Command

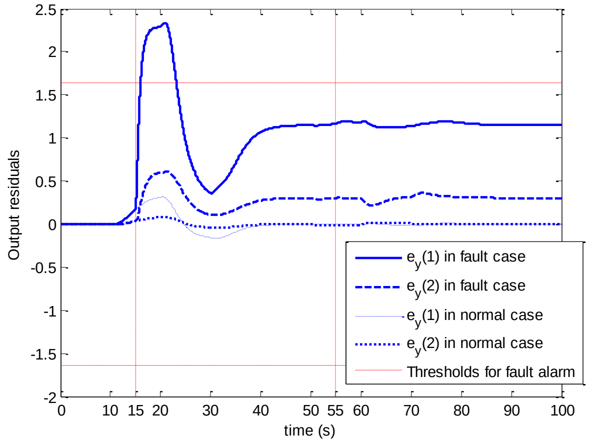


Fig. 4 Output residuals

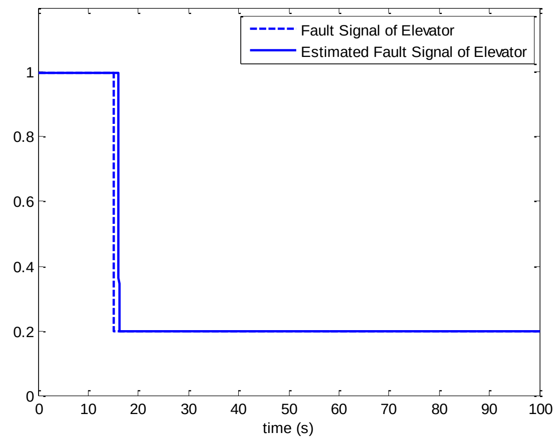


Fig. 5 Effectiveness factor estimation

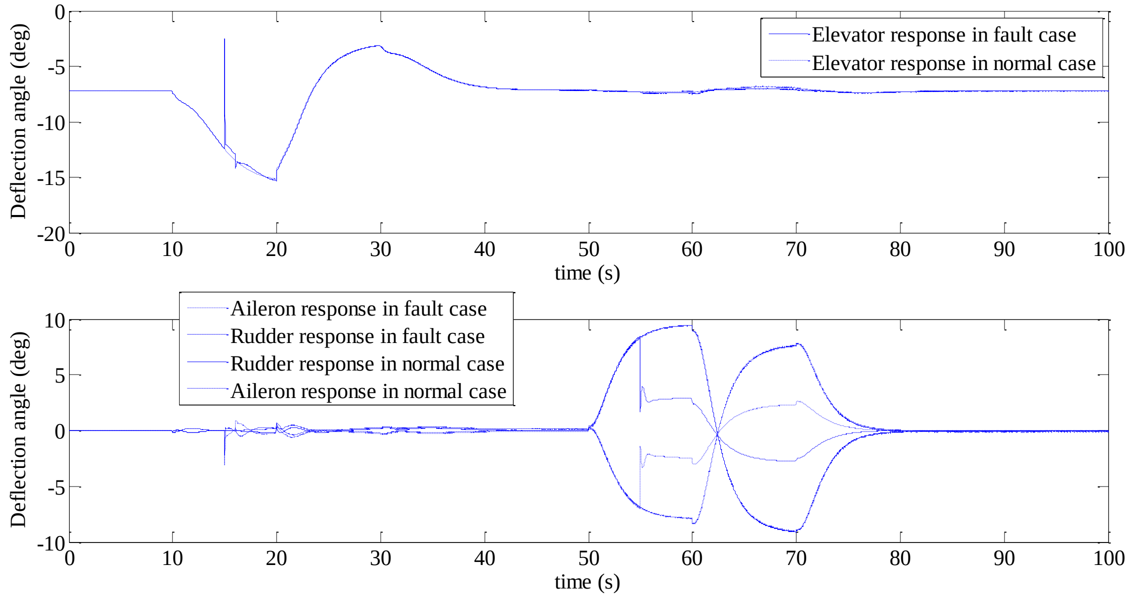


Fig. 6 Actuator outputs

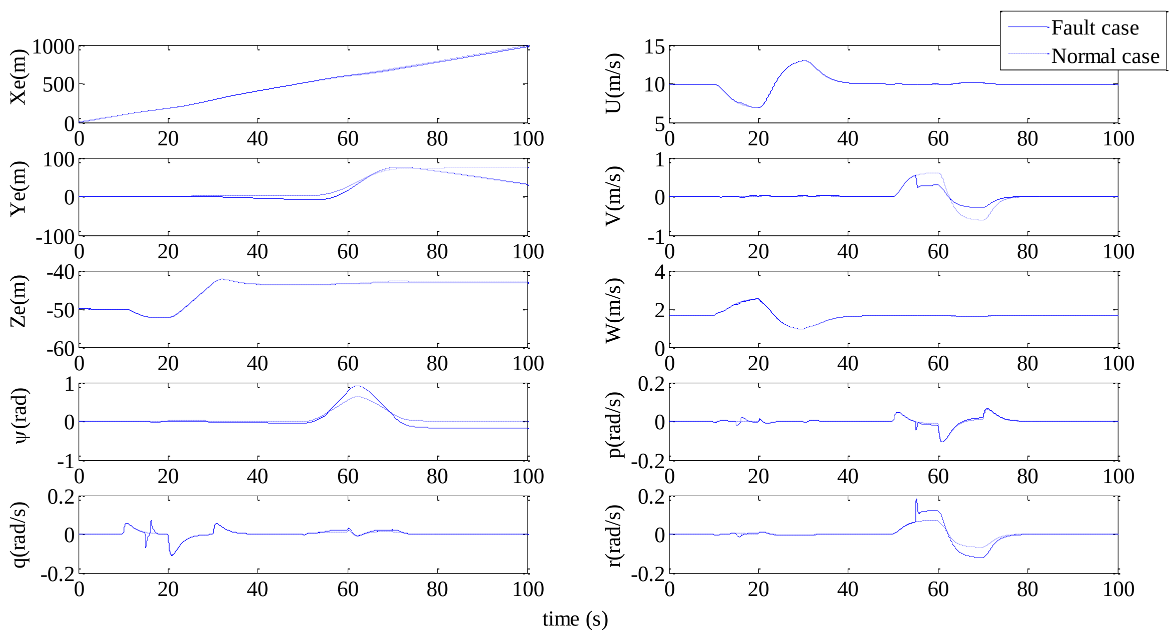


Fig. 7 Other states of aircraft

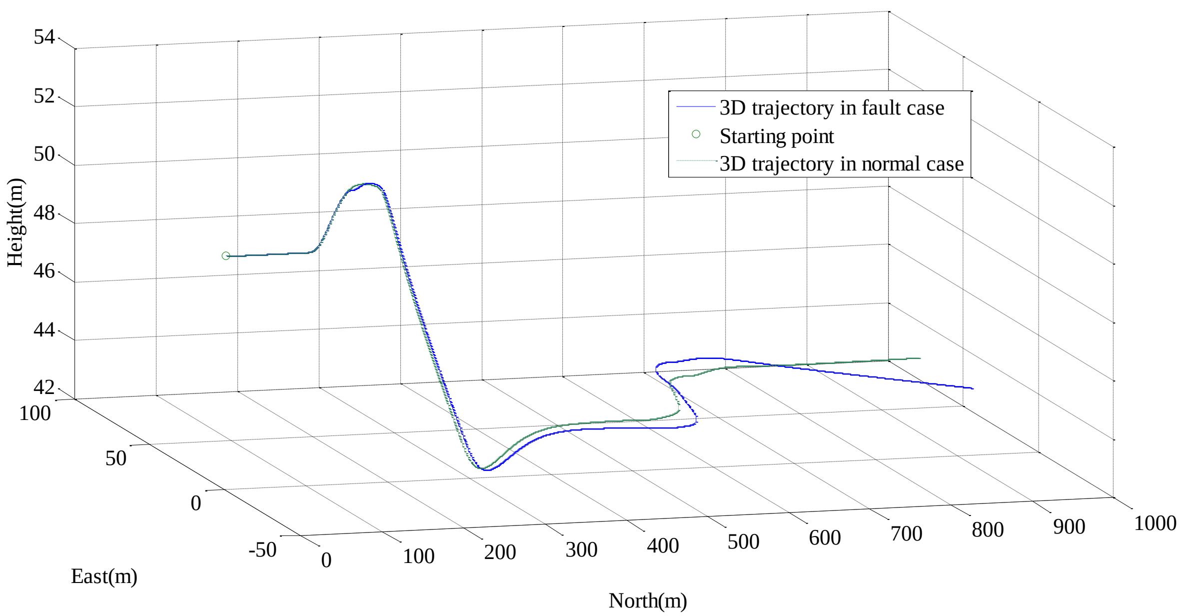


Fig. 8 3D trajectories of the aircraft

## 6. Conclusion

An integrated active fault-tolerant control method against partial loss of actuator effectiveness and saturation is proposed in this paper. LMI conditions are presented to compute the design parameters by integrated design of reconfigurable controller and fault diagnosis module. An observer is designed to detect a fault, and a minimum threshold is set to avoid the false alarm induced by disturbances. The system performance is described by two ellipsoidal sets regarding the domain of attraction and disturbance rejection respectively.

The proposed design techniques are applied to flight control of a flying wing aircraft under actuator faults. The nonlinear model of the aircraft is used for simulation, and satisfactory tracking performance can be obtained. The effectiveness loss of the elevator can be detected and compensated by the proposed integrated design method. However, the fault detector proposed in this paper is not sensitive to the faults of both aileron and rudder, though good tracking performance can still be achieved. This should be improved in our future work.

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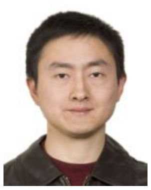

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