# Sensor and Actuator Fault Isolation Using Parameter Interval based Method for Nonlinear Dynamic Systems

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## ABSTRACT

Fault isolation problem for sensor and actuator in nonlinear dynamic systems is studied. Parameter interval based fault isolation method has fast isolation speed and fits many kinds of nonlinear dynamic systems. This method is extended to the isolation of sensor fault and actuator fault. The example shows good performance.

## 1 INTRODUCTION

Fault detection and isolation (FDI) problem for nonlinear dynamic systems has received more and more attentions recently. The significant results mainly belong to three kinds of methods: The method based on nonlinear geometrical theory (1; 2; 3), based on parity space theory (4; 5) and based on adaptive observers (6; 7). The application of the first one is limited because there is not always suitable framework of decoupling for a general nonlinear dynamic system. The method based on parity space theory has only been studied for some special systems. The speed of adaptive observers based method is not ideal due the parameter identification which lasts a long time. In our previous work, we have put forward a parameter interval based method for nonlinear dynamic systems with respect to the faults of the dynamic part of the system (8; 9; 10). This method fits many kinds of nonlinear dynamic systems, its isolation speed is fast. In this paper, this method is extended to sensor and actuator fault isolation problems for nonlinear dynamic systems. The example shows good performance of this method for sensor and actuator fault isolations for nonlinear dynamic systems.

## 2 PARAMETER INTERVAL BASED FAULT ISOLATION METHOD

In the parameter interval based method, the practical range of each parameter is divided into a certain number of intervals. After occurrence of a fault, the value of the faulty parameter must be in one of the parameter intervals. After checking each interval whether or not it contains the faulty parameter value, the faulty parameter value is found, the fault is therefore isolated. In this section, we recall quickly the main points of the parameter interval based fault isolation method, for more details, the reader is referred to (8; 9; 10).

### 2.1 Nonlinear Dynamic System and Fault

In the original version of the method, the considered nonlinear dynamic system is as follows:

$$\begin{aligned} \dot{x} &= f(x, \theta, u) \\ y &= cx \end{aligned} \tag{1}$$

where: $x \in R^n$ is the system state vector. $\theta \in R^p$ is the system parameter vector, its nominal value is denoted by $\theta^0$. $u \in R^m$ is the system input vector and $y \in R^l$ the system output vector. $c \in R^{l\times n}$ is the system output coefficient matrix. $f(x, \theta, u)$ and its first partial derivatives on $x$ and $\theta$ are continuous, bounded in $x$ and $\theta$.

**Definition 1** *There is a fault in the dynamic system (1), if the dynamic difference*

$$\Delta f(x, \theta, \theta^0, u) = f(x, \theta, u) - f(x, \theta^0, u) \tag{2}$$

*between the system (1) and its nominal model $\dot{x} = f(x, \theta^0, u)$ caused by the difference of parameter vectors $\Delta\theta = \theta - \theta^0$ is great.* •

The system without fault is called fault-free system, its parameter vector will be close to $\theta^0$ and is regarded as $\theta^0$ for convenience. While a system with fault is called post-fault system, its parameter vector is denoted by $\theta^f$. $\theta$ represents $\theta^0$ or $\theta^f$ according to context. One notes the time of fault occurrence as $t_f$.

### 2.2 Fault Isolation

After the fault occurrence, the fault isolation task is triggered by the fault detection procedure. For fault detection, an existing method (6) is used. We assume that the fault detection is very fast, the time of the fault occurrence and the time when it is detected are considered as the same which is noted as $t_f$.

We assume that the considered faults are caused by the

change of single parameter.
For $p$ parameters $\theta_1, \theta_2, \ldots, \theta_j, \ldots, \theta_p$ of the system parameter vector one partitions the possible domain of each parameter into a certain number of intervals. For example, the parameter $\theta_j$ is partitioned as $w$ intervals, their bounds are denoted by $\theta_j^{(0)}, \theta_j^{(1)}, \ldots, \theta_j^{(i)}, \ldots, \theta_j^{(w)}$. The bounds of $i$th interval are $\theta_j^{(i-1)}$ and $\theta_j^{(i)}$, are also noted as $\theta_j^{b(ij)}(t)$ and $\theta_j^{a(ij)}(t)$. After fault occurrence, the faulty parameter value must be in one of the parameter intervals.
To verify if an interval contains the faulty parameter value of the post-fault system, a parameter filter is built for this interval. A parameter filter consists of two isolation observers which correspond to two bounds of the interval. The parameter filter for $i$th interval of $j$th parameter is given below. The isolation observers are:

$$\begin{aligned}
\dot{\hat{x}}^{a(ij)} &= f(\hat{x}^{a(ij)}, \theta^{oba(ij)}(t), u) + k(y - \hat{y}^{a(ij)}) \\
\hat{y}^{a(ij)} &= c\hat{x}^{a(ij)} \\
e^{a(ij)} &= x - \hat{x}^{a(ij)}, \varepsilon^{a(ij)} = y_h - \hat{y}_h^{a(ij)}
\end{aligned} \quad (3)$$

$$\begin{aligned}
\dot{\hat{x}}^{b(ij)} &= f(\hat{x}^{b(ij)}, \theta^{obb(ij)}(t), u) + k(y - \hat{y}^{b(ij)}) \\
\hat{y}^{b(ij)} &= c\hat{x}^{b(ij)} \\
e^{b(ij)} &= x - \hat{x}^{b(ij)}, \varepsilon^{b(ij)} = y_h - \hat{y}_h^{b(ij)}
\end{aligned} \quad (4)$$

Where: $\theta^{oba(ij)} \in R^p$, $\theta^{obb(ij)} \in R^p$ are the parameter vectors of the observers. $\varepsilon^{a(ij)} \in R$, $\varepsilon^{b(ij)} \in R$. $y_h$ is the $h$th component of $y$, $\hat{y}_h^{a(ij)}$ is the $h$th component of $\hat{y}^{a(ij)}$ and $\hat{y}_h^{b(ij)}$ is the $h$th component of $\hat{y}^{b(ij)}$.
It is assumed that before occurrence of fault, the observers states $\hat{x}^{a(ij)}$, $\hat{x}^{b(ij)}$ have converged to the system state $x$, so:

$$\begin{aligned}
e^{a(ij)}(t_f) = 0, e^{b(ij)}(t_f) = 0 \\
\varepsilon^{a(ij)}(t_f) = 0, \varepsilon^{b(ij)}(t_f) = 0
\end{aligned}$$

At the time $t_f$, the $s$th system parameter changes due to the fault occurrence:

$$\left\{ \begin{array}{l} \theta_s^f = \theta_s^0 + \Delta^f \\ \theta_l^f = \theta_l^0, l \neq s \end{array} \right. \quad t \geq t_f,$$

and the $j$th parameters of the observers change in order to isolate the fault:

$$\theta_j^{oba(ij)}(t) = \left\{ \begin{array}{l} \theta_j^0, t < t_f \\ \theta_j^{a(ij)}, t \geq t_f \end{array} \right.$$

$$\theta_l^{oba(ij)}(t) = \theta_l^0, \forall t, l \neq j$$

$$\theta_j^{obb(ij)}(t) = \left\{ \begin{array}{l} \theta_j^0, t < t_f \\ \theta_j^{b(ij)}, t \geq t_f \end{array} \right.$$

$$\theta_l^{obb(ij)}(t) = \theta_l^0, \forall t, l \neq j$$

The isolation index is:

$$v^{(ij)}(t) = sgn(\varepsilon^{a(ij)}(t)) sgn(\varepsilon^{b(ij)}(t)) \quad (5)$$

It is assumed that the function $f(x, \theta, u)$ of the system satisfies following Assumption 1 and Assumption 2:

**Assumption 1** *At any point $(x, u)$, the function $f(x, \theta, u)$ in the equation (1) satisfies that:*
*1) any component $f_i(x, \theta, u)$, $i \in \{1, \ldots, n\}$ which is an explicit function of the considered parameter $\theta_j$ is a monotonous function of parameter $\theta_j$.*
*2) $y_h$ is a monotonous function of the considered parameter $\theta_j$.* •

**Assumption 2** *If $s \neq j$, no matter what value of the change of the isolation observer parameter is, the dynamic difference between the isolation observer and the post-fault system at point $\hat{x} = x$ is great. That is to say:*

$$\Delta f(x, \theta^f, \theta^{ob}, u) = f(x, \theta^f, u) - f(x, \theta^{ob}, u) \quad (6)$$

*is great.* •

where $\theta^{ob}$ denotes $\theta^{oba(ij)}$ or $\theta^{obb(ij)}$ according to context.
Using Assumption 1, it can be proven that for the case where $s = j$, the estimation error $\varepsilon^{a(ij)}(t)$ of the observer is a monotonous function of the parameter difference $\delta\theta_j^{a(ij)} = \theta_j^{oba(ij)} - \theta_j^f$, and $\varepsilon^{b(ij)}(t)$ is a monotonous function of $\delta\theta_j^{b(ij)} = \theta_j^{obb(ij)} - \theta_j^f$, and no matter $s = j$ or not, the difference of the estimation error $\varepsilon^{ab(ij)}(t) = \varepsilon^{a(ij)}(t) - \varepsilon^{b(ij)}(t)$ is a monotonous function of parameter difference $\theta_j^{a(ij)} - \theta_j^{b(ij)}$ between the two interval bounds. Using Assumption 2, the monotonicities of $\varepsilon^{a(ij)}(t)$, of $\varepsilon^{b(ij)}(t)$ and of $\varepsilon^{ab(ij)}(t)$, the following rule of interval verification can be obtained:

**Rule 1** *After fault occurrence, if $i$th interval of $j$th parameter contains the faulty parameter value, then we have: $v^{(ij)}(t) = -1, \forall t$. As soon as $v^{(ij)}(t) = 1$, then it can be decided that this interval does not contain the faulty parameter value, in spite of that $v^{(ij)}(t)$ will be '-1' afterward or not.* •

After fault occurrence, if all the intervals of a parameter are excluded from containing the faulty parameter value, the fault is excluded from this parameter. If all the parameters except one are excluded from fault, the fault is isolated. The parameter which is not excluded corresponds to fault.
It is proven in (10) that the fault isolation of this approach is fast.

## 3 SENSOR AND ACTUATOR FAULT ISOLATION

### 3.1 Sensor Fault Isolation

In the nonlinear dynamic system model (1), if we consider sensor fault, the model should be modified as:

$$\begin{aligned}
\dot{x} &= f(x, \theta, u) \\
y &= c(x, \theta^c)
\end{aligned} \quad (7)$$

where: $\theta^c$ is the parameter vector of the sensor with proper dimension, $\theta^{c0}$ is its nominal value, $\theta^{cf}$ is used to denote $\theta^c$ with fault. $c(x, \theta^c)$ is a nonlinear function of the state vector $x$ and the parameter vector $\theta^c$, $c(x, \theta^c)$ and its first partial derivatives on $x$ and $\theta^c$ are continuous, bounded in $x$ and $\theta^c$. Similar to the discussion for dynamic fault, for sensor fault isolation each component of the parameter vector $\theta^c$ is divided into certain number of intervals.

**Assumption 3** *At any considered point $x$ in the state space, any component $c_k(x,\theta^c)$ of the function $c(x,\theta^c)$ which is an explicit function of the considered parameter $\theta_j^c$ is a monotonous function of the parameter $\theta_j^c$.* •

For the model (7), the parameter filter with respect to sensor fault can be built with the correspondent isolation observers:

$$\dot{\hat{x}}^{a(ij)} = f(\hat{x}^{a(ij)},\theta^0(t),u) + k(y - c(\hat{x}^{a(ij)},\theta^{coba(ij)})) \quad (8)$$

$$\dot{\hat{x}}^{b(ij)} = f(\hat{x}^{b(ij)},\theta^0(t),u) + k(y - c(\hat{x}^{b(ij)},\theta^{cobb(ij)})) \quad (9)$$

the observation error are:

$$\begin{aligned}\varepsilon^{a(ij)} &= c_h(x,\theta^{cf}) - c_h(\hat{x}^{a(ij)},\theta^{coba(ij)}) \\ &= y_h - c_h(\hat{x}^{a(ij)},\theta^{coba(ij)})\end{aligned} \quad (10)$$

$$\begin{aligned}\varepsilon^{b(ij)} &= c_h(x,\theta^{cf}) - c_h(\hat{x}^{b(ij)},\theta^{cobb(ij)}) \\ &= y_h - c_h(\hat{x}^{b(ij)},\theta^{cobb(ij)})\end{aligned} \quad (11)$$

Where: $\theta^{coba(ij)}$, $\theta^{cobb(ij)}$ are the parameter vectors of the observers corresponding to sensor parameter vector, their dimensions are same as the dimension of $\theta^c$. $\varepsilon^{a(ij)} \in R$, $\varepsilon^{b(ij)} \in R$. $c_h(x,\theta^c)$ is the $h$th component of $c(x,\theta^c)$.
We also assume that the fault of the system is caused by the change of a single parameter in the vector $\theta^c$, so the vector $\theta$ maintains as its nominal value when the sensor fault occurs.
At the time $t_f$, the $s$th sensor parameter changes due to the fault occurrence:

$$\begin{cases} \theta_s^{cf} = \theta_s^{c0} + \Delta^{cf} \\ \theta_l^{cf} = \theta_l^{c0}, l \neq s \end{cases} \quad t \geq t_f,$$

and the $j$th parameters of the observers change in order to isolate the fault:

$$\theta_j^{coba(ij)}(t) = \begin{cases} \theta_j^{c0}, t < t_f \\ \theta_j^{ca(ij)}, t \geq t_f \end{cases}$$

$$\theta_l^{coba(ij)}(t) = \theta_l^{c0}, \forall t, l \neq j$$

$$\theta_j^{cobb(ij)}(t) = \begin{cases} \theta_j^{c0}, t < t_f \\ \theta_j^{cb(ij)}, t \geq t_f \end{cases}$$

$$\theta_l^{cobb(ij)}(t) = \theta_l^{c0}, \forall t, l \neq j$$

where: $\theta_j^{ca(ij)}$ and $\theta_j^{cb(ij)}$ are the bounds of the $i$th interval of $j$ parameter of $\theta^c$.

$s = j$
Because $c(x,\theta^c)$ is the monotonous function of single parameter in $\theta^c$, according to the equations (8)-(11), for the case $s = j$, at the point:

$$(x,\hat{x},\theta,\theta^{cf},\delta\theta_j^{ca(ij)}) = (x,x,\theta^0,\theta^{cf},0)$$

$\varepsilon^{a(ij)}$ will be a monotonous function of the single parameter difference $\delta\theta_j^{ca(ij)} = \theta_j^{cf} - \theta_j^{coba(ij)}$. By the same way as in (8; 10), it can be proven that in the neighborhood of this point, $\varepsilon^{a(ij)}$ is a monotonous function of the single parameter difference $\theta_j^{cf} - \theta_j^{coba(ij)}$. Similarly, it can be proven that $\varepsilon^{b(ij)}$ is a monotonous function of the single parameter difference $\theta_j^{cf} - \theta_j^{cobb(ij)}$. Because $\varepsilon^{a(ij)}(t)|_{\delta\theta_j^{ca(ij)}=0} = 0$, $\varepsilon^{b(ij)}(t)|_{\delta\theta_j^{cb(ij)}=0} = 0$, according to the monotonicities of $\varepsilon^{a(ij)}$ and of $\varepsilon^{b(ij)}$, for the case where the interval contains the faulty parameter value, i.e. $\theta_j^{cf} \in [\theta_j^{cb(ij)},\theta_j^{ca(ij)}]$, it will be:

$$sgn(\varepsilon^{a(ij)}(t)) = -sgn(\varepsilon^{b(ij)}(t))$$

and for the case were the interval does not contain the faulty parameter value, it will be:

$$sgn(\varepsilon^{a(ij)}(t)) = sgn(\varepsilon^{b(ij)}(t))$$

$s \neq j$
Similar to the case of dynamic fault (8; 10), it can be also proven that, no matter $s = j$ or not, and whatever the value of the sensor parameter vector change is, the difference of the estimation error given by:

$$\varepsilon^{ab(ij)}(t) = \varepsilon^{a(ij)}(t) - \varepsilon^{b(ij)}(t)$$

is a monotonous function of parameter difference $\theta_j^{ca(ij)} - \theta_j^{cb(ij)}$ between the two interval bounds. Similar to Assumption 2, it is assumed:

**Assumption 4** *If $s \neq j$, no matter what value of the change of the isolation observer parameter $\theta_j^{cob}$ is, the sensor function difference between the isolation observer and the post-fault system at point $\hat{x} = x$ is great. That is to say:*

$$\Delta c(x,\theta^{cf},\theta^{cob}) = c(x,\theta^{cf}) - c(x,\theta^{cob}) \quad (12)$$

*is great.* •

where $\theta^{cob}$ denotes $\theta^{coba(ij)}$ or $\theta^{cobb(ij)}$ according to context. Using Assumption 4, along the same way in (8; 10), it can be proven that, in the case where $s \neq j$, the amplitudes of the estimation errors $\varepsilon^{a(ij)}(t)$ and $\varepsilon^{b(ij)}(t)$ of the observers are great. Therefore two curves $\varepsilon^{a(ij)}(t)$ and $\varepsilon^{b(ij)}(t)$ will be far from the axis of abscissa at many time. On the other hand, according to the monotonicity of $\varepsilon^{ab(ij)}$, for a small interval, $\varepsilon^{ab(ij)}$ is small, therefore two curves $\varepsilon^{a(ij)}(t)$ and $\varepsilon^{b(ij)}(t)$ will be very close. So, there must exist a time $t_e$, that:

$$sgn(\varepsilon^{a(ij)}(t_e)) = sgn(\varepsilon^{b(ij)}(t_e))$$

By combining the case of $s = j$ and the case of $s \neq j$, an interval verification rule can be obtained. The obtained rule is same as Rule 1. Therefore by merging the sensor parameter vector $\theta^c$ into the system parameter vector $\theta$, the parameter interval based isolation method in (8; 10) can be used directly for sensor fault isolation.

### 3.2 Actuator Fault Isolation

In the nonlinear dynamic system model (1), the parameter vector $\theta$ is associated with the state vector $x$ of the system dynamic. However, the relation of input vector $u$ with the system is also parameterized by the components of $\theta$. Therefore the actuator faults can be described by the changes of the parameter vector $\theta$, and the parameter interval based isolation method in (8; 10) can be used directly for actuator fault isolation.

## 4 SIMULATION

### 4.1 Plant Model

The single-link robotic arm model (11) is as follows:

$$\begin{aligned} J_l\ddot{q}_1 + F_l\dot{q}_1 + k(q_1 - q_2) + mgl\sin q_1 = 0 \\ J_m\ddot{q}_2 + F_m\dot{q}_2 - k(q_1 - q_2) = u \end{aligned} \quad (13)$$

where: $q_1$ is the link position angle, $q_2$ the rotor position, $J_l$ the link moment of inertia, $J_m$ the moment of inertia of the motor rotor, $k$ the coefficient of elasticity, $m$ the mass of the link, $g$ the acceleration due to gravity, $l$ the length of the link, $F_l$ and $F_m$ the viscous friction coefficients, $u$ the input torque. The parameters of model are: $k = 2$, $F_m = 1$, $F_l = 0.5$, $J_m = 1$, $J_l = 2$, $m = 4$, $g = 9.8$, $l = 0.5$. The input torque is selected as $u = 8\sin(t/3)$. Let $x_1 = q_1$, $x_2 = \dot{q}_1$, $x_3 = q_2$ and $x_4 = \dot{q}_2$, the model can be rewritten as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k}{J_l} & \frac{-F_l}{J_l} & \frac{k}{J_l} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k}{J_m} & 0 & \frac{-k}{J_m} & \frac{-F_m}{J_m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ a_1\frac{-mgl}{J_l}\sin x_1 \\ 0 \\ a_2\frac{u}{J_m} \end{bmatrix} \quad (14)$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ a_3 \end{bmatrix} \quad (15)$$

where: $x_1$, $x_3$ and $x_4$ are assumed measurable, parameters $a_1$, $a_2$ and $a_3$ are used to simulate dynamic fault, actuator fault and sensor fault respectively. Their nominal values are: $a_1^0 = 1$, $a_2^0 = 1$ and $a_3^0 = 0$, their possible domain are assumed as: $a_1 \in [0.6, 1]$, $a_2 \in [0, 1]$ and $a_3 \in [0, 0.2]$. $y_3$ is selected as $y_h$ for fault isolation.

### 4.2 Parameter Filters

For parameter $a_1$, its domain is partitioned into 8 intervals, the values of parameter filters are shown in Table 1.
For parameter $a_2$, its domain is partitioned into 6 intervals, the values of parameter filters are shown in Table 2.
For parameter $a_3$, its domain is partitioned into 3 intervals, the values of parameter filters are shown in Table 3.

Table 1: The values of parameter filters of $a_1$

| N | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $a_1^b$ | 0.60 | 0.65 | 0.70 | 0.75 |
| $a_1^a$ | 0.65 | 0.70 | 0.75 | 0.80 |
| N | 5 | 6 | 7 | 8 |
| $a_1^b$ | 0.80 | 0.85 | 0.90 | 0.95 |
| $a_1^a$ | 0.85 | 0.90 | 0.95 | 1.00 |

Table 2: The values of parameter filters of $a_2$

| N | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $a_2^b$ | 0.00 | 0.16 | 0.32 | 0.48 | 0.64 | 0.80 |
| $a_2^a$ | 0.16 | 0.32 | 0.48 | 0.64 | 0.80 | 1.00 |

### 4.3 Simulation Results

It is assumed that the fault is caused by a single parameter change, and the fault occurs at time $t = 10s$.

**The fault is sensor fault**

The fault is caused by the deviation of $a_3$. The value of $a_3$ changes from 0 to 0.1, while the parameter $a_1$ maintains as its nominal value 1 and $a_2$ maintains as its nominal value 1.

Figure 1 shows the results of the 2nd parameter filter of $a_3$, This is the case where $s = j$ and the interval contains the faulty parameter value. The figure shows that after $t_f = 10s$, the signals of two observer estimation errors are always different, so this interval cannot be excluded from "containing faulty parameter value", and the parameter $a_3$ can not be excluded from fault.

Figure 2 shows the results of the first parameter filter of $a_3$. This is the case where $s = j$ and the interval does not contain the faulty parameter value. The figure shows that after $t_f = 10s$, the signals of two observer estimation errors are the same, it means that this interval does not contain the faulty parameter value.

Figure 3 shows the results of the 2nd parameter filter of the parameter $a_2$ (actuator parameter). This is the case where $s \neq j$. The figure shows that after $t_f = 10s$, the signals of two observer estimation errors are the same, it means that this interval does not contain the faulty parameter value. The figure also shows that, the case where the signals of two observer estimation errors are the same occurs very close to the time $t_f$ (in this example, it occurs at the time $t_f$.), therefore we can very quickly deicide that the interval does not contain the faulty parameter value, it guarantees the speediness of the isolation. Similarly, the simulation shows that all the intervals of the parameter $a_2$ can be very quickly excluded from containing the faulty parameter value (limited by the length of the paper, the simulation curves of other intervals are not presented in the paper.), the parameter $a_2$ is very quickly excluded from fault.

Figure 4 shows the results of the 3rd parameter filter of the parameter $a_1$ (dynamic parameter). This is the case where $s \neq j$. The figure shows that after $t_f = 10s$, the signals of two observer estimation errors are the same, it means that this interval does not contain the faulty parameter value, the figure also shows

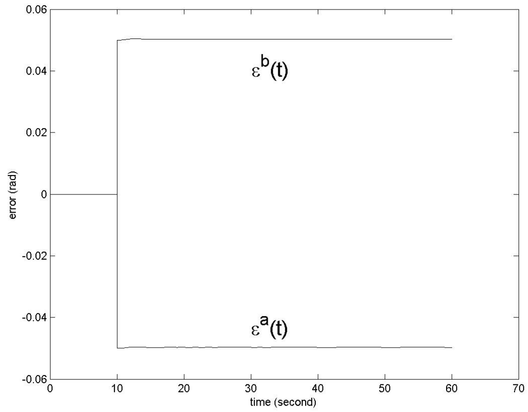


Figure 1:

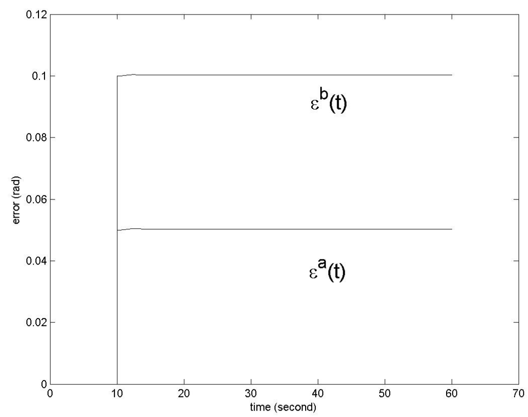


Figure 2:

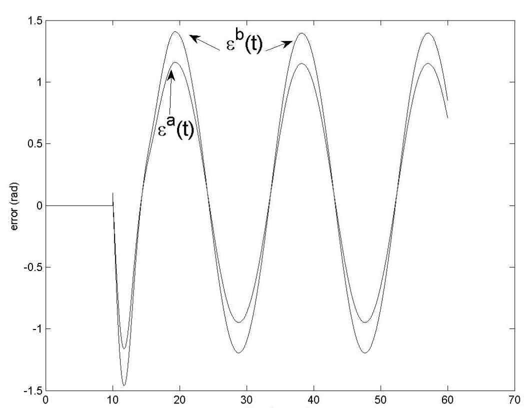


Figure 3:

Table 3: The values of parameter filters of $a_3$

| N | 1 | 2 | 3 |
|---|---|---|---|
| $a_3^b$ | 0.00 | 0.05 | 0.15 |
| $a_3^a$ | 0.05 | 0.15 | 0.20 |

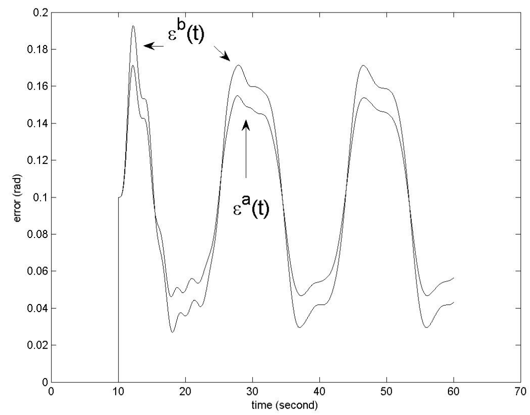


Figure 4:

that this result can be decided very quickly. Similarly, the simulation shows that all the intervals of $a_1$ do not contain the faulty parameter value, the parameter $a_1$ is very quickly excluded from fault.
Because the parameter $a_1$ and the parameter $a_2$ are excluded from fault, while the parameter $a_3$ cannot, so the fault is in $a_3$. Because the exclusions of $a_1$ and $a_2$ are very quick, therefore the isolation is very quick.

**The fault is actuator fault**

The fault is caused by the deviation of $a_2$. The value of $a_2$ changes from 1 to 0.1, while the parameter $a_1$ maintains as its nominal value 1 and $a_3$ maintains as its nominal value 0.

Figure 5 shows the results of the first parameter filter of $a_2$. This is the case where $s = j$ and the interval contains the faulty parameter value. The figure shows that after $t_f = 10s$, the signals of two observer estimation errors are always different, so this interval cannot be excluded from "containing faulty parameter value", and the parameter $a_2$ can not be excluded from fault.

Figure 6 shows the results of the 2nd parameter filter of the parameter $a_3$ (sensor parameter). This is the case where $s \neq j$. The figure shows that after $t_f = 10s$, the signals of two observer estimation errors are the same, it means that this interval does not contain the faulty parameter value, the figure also shows that this result can be decided very quickly. Similarly, the simulation shows that all the intervals of $a_3$ do not contain the faulty parameter value, the parameter $a_3$ is very quickly excluded from fault.

Figure 7 shows the results of the 4th parameter filter of the parameter $a_1$ (dynamic parameter). This is the case where $s \neq j$. The figure shows that after $t_f = 10s$, two curves of the two observer estimations almost superimpose together, the signals of two observer esti-

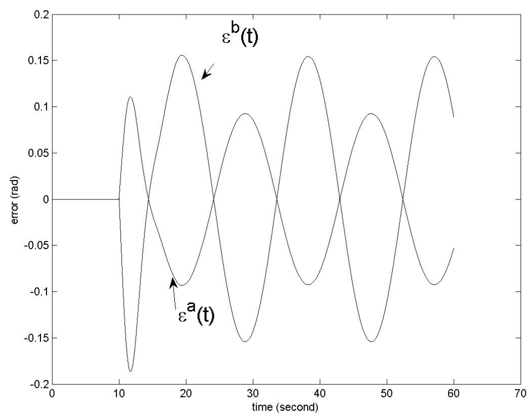


Figure 5:

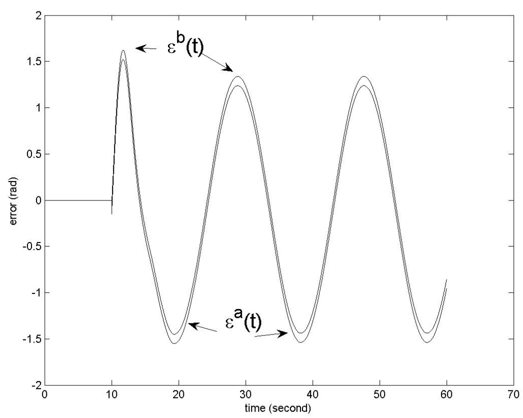


Figure 6:

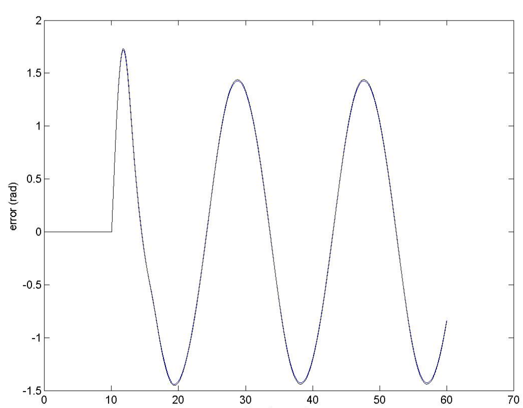


Figure 7:

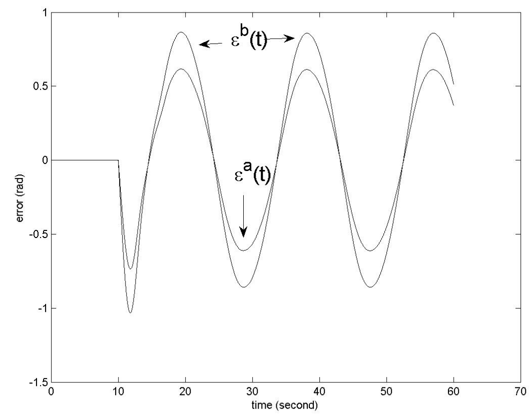


Figure 8:

mation errors are the same, it means that this interval does not contain the faulty parameter value. Similarly, the simulation shows that all the intervals of $a_1$ do not contain the faulty parameter value, the parameter $a_1$ is very quickly excluded from fault.
As $a_1$ and $a_3$ are very quickly excluded from the fault, so the fault is very quickly located at the parameter $a_2$.

**The fault is dynamic fault**

The fault is caused by the deviation of $a_1$. The value of $a_1$ changes from 1 to 0.73, while the parameters $a_2$ and $a_3$ maintains as their nominal value 1 and 0.
For the case in the parameter filters of $a_1$, the simulation shows that $a_1$ cannot be excluded from fault, this case has been discussed in the articles of our previous work (8; 10).
Figure 8 shows the results of the 4th parameter filter of the parameter $a_2$ (actuator parameter). This is the case where $s \neq j$. The figure shows that after $t_f = 10s$, the signals of two observer estimation errors are the same, it means that this interval does not contain the faulty parameter value. Similarly, the simulation shows that all the intervals of $a_2$ do not contain the faulty parameter value, the parameter $a_2$ is very quickly excluded from fault.
For the parameter filters of $a_3$ (sensor parameter), the simulation also shows that all the intervals of $a_3$ do not contain the faulty parameter value, the parameter $a_3$ is very quickly excluded from fault.
As $a_2$ and $a_3$ are very quickly excluded from the fault, so the fault is very quickly located at the parameter $a_1$.

## 5 CONCLUSION

In this paper, the parameter interval based fault isolation method is extended to sensor fault and actuator fault isolation problems for nonlinear dynamic systems. The parameter interval based fault isolation method can be used directly for sensor fault and actuator fault without need of any modifications. The example shows good performance of this method for sensor and actuator fault isolations.

**ACKNOWLEDGMENTS**

This work was supported by National Natural Science Foundation of P.R.China (60864003).

**REFERENCES**

(1) Hammouri, H., Kinnaert, M., and EI Yaagoubi, E. H. (1999). Observer based approach to fault detection and isolation for nonlinear systems. *IEEE Transactions on Automatic Control*, 44, 10, pp 1879-1884.

(2) De Persis, C., and Isidori, A. (2001). A geometric approach to nonlinear fault detection and isolation. *IEEE Transactions on Automatic Control*, 46(6), pp 853-865.

(3) Hammouri, H., Kabore, P., Othman, S., and Biston, J. (2002). Failure diagnosis and nonlinear observer application to a hydraulic process. *Journal of the Franklin Institute*, 339, pp 455-478.

(4) Comtet-Varga, G., Cassar, J. Ph., and Staroswiecki, M. (1997). Analytic redundancy relations for state affine systems. Proceedings of *the fourth European Control Conference (ECC'97, Brussels, Belgium)*.

(5) Staroswiecki, M., and Comtet-Varga, G. (2001). Analytical redundancy relations for fault detection and isolation in algebraic dynamic systems. *Automatica*, 37, pp 687-699.

(6) Zhang, Q. (2000). A new residual generation and evaluation method for detection and isolation of faults in non-linear systems. *International Journal of Adaptive Control and Signal Processing*, 14 (2000) 759-773.

(7) Xu, A., and Zhang, Q. (2004). Nonlinear system fault diagnosis based on adaptive estimation. *Automatica*, 40, pp 1181-1193.

(8) Zetao Li and B.Dahhou (2006). Parameter Intervals used for Fault Isolation in Nonlinear Dynamic Systems. *International Journal of Modelling, Identification and Control*, Vol. 1, No. 3, 2006.

(9) Zetao Li and B.Dahhou (2007). Fault Isolation for Nonlinear Dynamic Systems Based on Parameter Intervals. *International Journal of Systems Science*, Vol.38,Issue7,2007,pp.531-547.

(10) Zetao Li and B.Dahhou (2008). A New Fault Isolation and Identification Method for Nonlinear Dynamic Systems: Application to a Fermentation Process. *Applied Mathematical Modelling*, 32 (2008) 2806-2830.

(11) Zhang, X. Parisini, T. Polycarpou, M.M. (2005). Sensor Bias Fault Isolation in a Class of Nonlinear Systems. *IEEE Transactions on Automatic Control*, March 2005 Volume: 50, Issue: 3, On page(s): 370- 376.