Structural Observability. Application to decompose a System with Possible Conflicts.

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ABSTRACT

Model based diagnosis of large continuous dynamic systems requiring quantitative simulation has a high computational cost, which can be reduced by distributing the computation. Distribution can be obtained partitioning the original diagnosis problem into the analysis of simpler subproblems. In this work, Possible Conflicts are used to partition a system because they provide a systematic way to decompose a system. However, a requirement of any decomposition method is that the resulting subsystems are observable.

This paper focuses on structural observability, a powerful concept because it allows analyzing the observability of a system in terms of its configuration, i.e., independently of system parameter values. However, the literature provides different definitions of structural observability, adapted to different modeling formalisms: equations, bipartite graphs and bond graphs. This paper shows that definitions for these formalisms are equivalent. The three tank system benchmark and a spring-mass system are used to illustrate the definitions and their equivalence. Then, it will be applied through Possible Conflicts to build independent subsystems that can be used for monitoring and diagnosis.

1 INTRODUCTION

Model-based diagnosis provides a formal framework for systematic design, analysis, and implementation of system diagnosers. However, model based diagnosis of large continuous dynamic systems using quantitative models has a high computational cost. A sensible approach to reduce this complexity is distributing the computation to achieve efficiency and to develop practical diagnosers. Distributing the computation requires partitioning of the diagnosis problem so that analysis of the original system can be performed by analysis of its subsystems. However, subsystems interact, therefore, the computational savings obtained by decomposition may be overwhelmed by the communication overhead needed to generate consistent global results. Therefore, the goal is to create partitions which minimize the communication between subsystems.

(Roychoudhury *et al.*, 2009) have proposed an approach to obtain factored Dynamic Bayesian Networks (DBNs) to avoid simulation of the DBN of the whole system. Another factoring proposal based on Possible Conflicts(PCs) (Pulido and Alonso-Gonzalez, 2004) is presented in (Alonso-Gonzalez *et al.*, 2010).

The idea of decomposing a system using conflicts like structures was first exploited by (Williams and Millar, 1998), which introduced the concept of dissent. A dissent is a minimal subsystem with analytical redundancy and it is, conceptually, equivalent to a PC. Dissents were originally proposed for parameter estimation using reduced models. PCs have also been used for system decomposition in the context of fault identification, to perform parameter estimation (Bregon *et al.*, 2009a) and to generate ensembles of classifiers based on system structure (Alonso-Gonzalez *et al.*, 2008). The advantage of using PCs for systems decomposition is that they provide a systematic way to create a unique decomposition of a dynamic system into independent components.

As discussed earlier, a first issue to ensure effectiveness of the decomposition is to establish the observability of each component and the independence between the subsystems to avoid computational overhead. A further advantage of independent components is that it can form the basis for independent diagnosers.

Given that the structural notion of observability is more general than the traditional definition (Ogata, 2001), we adopt this approach to establish the observability of the decomposed subsystems. Structural observability, has been defined for bi-partite graph representations by (Blanke *et al.*, 2006; Staroswiecki, 2007), and for Bond Graphs (BGs) (Samantaray and Bouamama, 2008), by (Sueur and Dauphin-Tanguy, 1991). Structural observability is more general because it is independent of parameter values, i.e, it holds for a configuration and not a particular set of parameter values. Our focus in structural observability is also due to the fact that we are interested in structural approaches to system decomposition, like PCs.

Additionally, (Sueur and Dauphin-Tanguy, 1991) compares definition of structural observability in bond graphs with the definition of observability in state space formulation (Ogata, 2001), showing their equivalence, so we want to check whether the definitions of (Blanke *et al.*, 2006; Staroswiecki, 2007) are equivalent to the Sueur and Dauphin-Tanguy definition. If this were the case, then we can show that different approaches used in different modeling formulations are equivalent. Moreover, the observability properties hold for different representations of a system model (e.g., equations, bi-partite graphs, and bond graphs).

This paper studies the notion of observability for dynamic systems, and then applies the observability principle through PCs to build independent subsystems that can be used for monitoring and diagnosis. The rest of the paper is organized as follows. Section 2 explains the general definition of structural observability (Blanke *et al.*, 2006; Staroswiecki, 2007). A running example, the standard three tank system benchmark, is also presented here to clarify and explain every definition used by the different approaches. In section 3 the definition of structural observability from the point of view of the BG modeling is presented. Section 4 discusses Possible Conflicts' (PCs) structural observability. In section 5 we explain the spring-mass system as a case study. Section 6 shows the discussion and the conclusions related with this piece of work.

2 STRUCTURAL OBSERVABILITY. GENERAL DEFINITION

A general method to check the structural observability for a dynamic system is to represent its model as a bi-partite graph, with two sets of vertices (C of constraints and Z of variables) and a set of edges that join nodes in C to nodes in Z. **Bi-partite graphs** are undirected graphs which can be interpreted as 'All the variables and parameters connected with a constraint vertex have to satisfy the equation or rule this vertex represents'.

A bi-partite graph can be expressed as an **incidence matrix**, also named in some cases biadjacency matrix, which rows and columns are the set of constraints and variables respectively. Every edge (c_i, z_j) is represented by a '1' in the intersection of row c_i and column z_j.

A **subsystem** is defined by a set of constraints together with the set of variables that occur in these constraints. The subgraph related with a subsystem is its structure.

The set of variables can be divided in two subsets: (1) **known variables** (their values are known or they are measured) and (2) **unknown variables** (they are not directly measured but there might exist ways in which their value can be computed from the values of other known variables.). Following the same division, we have two types of constraints: (1) constraints that only link known variables, and (2) constraints that include at least one unknown variable.

A matching in a bi-partite graph is a causal assignment which associates unknown system variables with the system constraint from which they can be calculated. Unknown variables that cannot be matched cannot be calculated and the ones which can be matched with several constraints can be determined in different ways (redundancy).

A **matching** is a subset of edges such that any of two edges have no common node (neither a constraint nor a variable).

A **matching** is called **maximal** when no edge can be added without violating the constraint of no common nodes (any of two edges have no common node).

A **matching** can also be **complete** respect to the variables, in case that each variable belongs to one edge, or respect to the constraints, in case that each constraint belongs to one edge. It is not guaranteed that a complete matching exists, either to the constraints or to the unknown variables.

A matching can be represented in the incidence matrix by selecting at most one '1' in each row and each column. We will show later how to check the structural observability of a system using the incidence matrix.

A graph is **over-constrained** if there is a complete matching on the variables but not on the constraints. It is called **just-constrained** if there is a complete matching on the variables and on the constraints. And finally, it is **under-constrained** if there is a complete matching on the constraints but not on the variables.

In this context a system is **structurally observable** (Blanke *et al.*, 2006; Staroswiecki, 2007) if there exists a complete matching on the unknown variables in the bi-partite graph which represents its structure. In a more formal way, it can be said that a necessary and sufficient condition for a system to be structurally observable is that, under derivative causality:

1. All the unknown variables are reachable from the known ones.
2. The over-constrained and the just-constrained subsystems are causal.
3. The under-constrained subsystem is empty.

Basically, the first point of the formal definition is the same as saying that the behaviour of the system is reflected on the behaviour of the known variables, if an unknown variable cannot be reached from the known ones, its value cannot be calculated, so the behaviour of the system will not be linked to the behaviour of the known variables. The other two conditions mean that all the subsystems we can get from the whole system have a causal relation between its variables and we can also match each variable with a different constraint or equation to get a complete matching on the variables.

2.1 Running example

To facilitate the comprehension of the previous definition we will provide a short example (see Figure 1) which will be also used further in this paper.

The equations to define the system in Figure 1 are Eq. 1 to Eq. 12:

$$\dot{h}_1(t) = F_{in}(t) - F_{12}(t) \tag{1}$$

$$\dot{h}_2(t) = F_{12}(t) - F_{23}(t) \tag{2}$$

$$\dot{h}_3(t) = F_{23}(t) - F_{out}(t) \tag{3}$$

$$F_{in}(t) = input\ flow \tag{4}$$

$$F_{12}(t) = R_{12} \cdot sign(h_1(t) - h_2(t)) \cdot \sqrt{|h_1(t) - h_2(t)|} \tag{5}$$

$$F_{23}(t) = R_{23} \cdot sign(h_2(t) - h_3(t)) \cdot \sqrt{|h_2(t) - h_3(t)|} \tag{6}$$

$$F_{out}(t) = R_{out} \cdot \sqrt{h_3(t)} \tag{7}$$

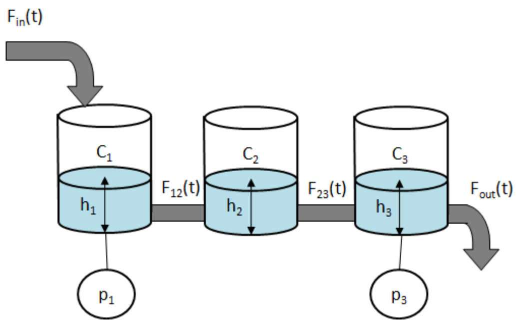

Figure 1: Three tanks system with two measurements (pressures).

$$p_1(t) = h_1(t) \cdot \rho \cdot g \tag{8}$$

$$p_3(t) = h_3(t) \cdot \rho \cdot g \tag{9}$$

$$\dot{h}_1(t) = dh_1(t)/dt \tag{10}$$

$$\dot{h}_2(t) = dh_2(t)/dt \tag{11}$$

$$\dot{h}_3(t) = dh_3(t)/dt \tag{12}$$

Equations 1, 2 and 3 are the mass balance equations (in the BG, they will be the state transition model). Equation 4 is the input flow, it is a known variable. Equations 5, 6 and 7 are the equations to get the values of the flows between tanks and the output flow. Equations 8 and 9 are the observational model. Equations 10, 11 and 12 are introduced to show that the variables $\dot{h}_1(t)$, $\dot{h}_2(t)$ and $\dot{h}_3(t)$ are the derivatives of $h_1(t)$, $h_2(t)$ and $h_3(t)$. So, the known variables in this system are the input flow and the two pressures.

Bond Graphs are a graphical method to model the energy transfer in the system using efforts and flows, which are associated with each bond. The system in Figure 1 can be modeled as the BG in Figure 2. From the BG, it can be derived automatically the Temporal Causal Graph (TCG) (Mosterman and Biswas, 1999) in Figure 3, which shows the relations between efforts and flows in the BG.

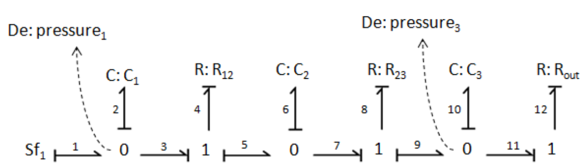

Figure 2: Bond graph for the system in Figure 1.

To apply the previous definition of structural observability to the system in Figure 1 we have to build the incidence matrix. In Table 1 there is that incidence matrix and it also contains a complete matching on the unknown variables. The matching has been found using one of the algorithms described in (Blanke *et al.*, 2006). Regarding to the definition explained previously in this section, the system in Figure 1 is structurally observable.

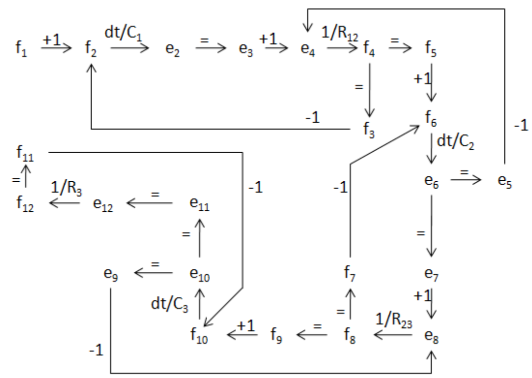

Figure 3: Temporal causal graph for the system in Figure 1 derived from the BG in Figure 2.

Table 1: Incidence matrix of the system in Figure 1 with a complete matching on the unknown variables.

	F_{in}	p_1	p_3	F_{12}	F_{23}	F_{out}	h_1	$\dot{h}_1$	h_2	$\dot{h}_2$	h_3	$\dot{h}_3$
1	1			**1**				1				
2				1	1					**1**		
3					**1**	1						1
5				1			1		**1**			
6					1				1		1	
7						**1**					1	
8		1					**1**					
9			1								**1**	
10							1	**1**				
11									1	1		
12											1	**1**

3 STRUCTURAL OBSERVABILITY IN BOND GRAPH MODELLING

The observability of a system using a state space approach can be checked with the rank of the observability matrix (Ogata, 2001) but it depends on the value of the parameters, which is not robust enough to measure the full-state observability. (Sueur and Dauphin-Tanguy, 1991; Samantaray and Bouamama, 2008) have proposed extensions to determine system observability analysing the system bond graph model. We discuss this approach in this section, and then demonstrate its applicability to the running example presented in Figure 1. We follow this up by a comparison of the two approaches.

A BG is structurally observable if it fulfills two conditions (Sueur and Dauphin-Tanguy, 1991; Samantaray and Bouamama, 2008):

- *Attainability/reachability condition:* every storage element (I and C) in the bond graph model with preferred integral causality must have at least one causal path to a sensor (De or Df element).
- *Inverted reachability condition:* every integrally causalled storage element (I and C) in the bond graph model can be assigned derivative causality and produce a valid causal assignment for the entire bond graph. If necessary, some sensor elements (De or Df) may be dualized to achieve the valid assignment.

So, to apply the previous definition of structural observability, we can look at the TCG in Figure 3 where f_1 is the input flow and the state variables are e_2, e_6 and e_{10}. Checking the BG in Figure 2, the dynamical elements are the three capacitances (C_1, C_2 and C_3).

We have to look for a causal path in the TCG from each state variables to a detector to check the *attainability/reachability condition*. e_2 and e_{10} are directly measured, so we need to find a path from e_6 to a detector. The path from e_6 to the detector e_2 is marked in bold in Figure 4.

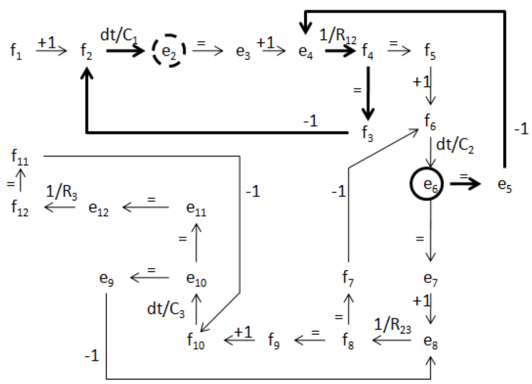

Figure 4: Temporal causal graph for the system in Figure 1 derived from the BG in Figure 2 with a bold path between the state variable e_6 and the detector in e_2.

The second condition, i.e., the *inverted reachability*, is illustrated in Figure 5. The BG with the new valid causal assignment using derivative causality in the storage elements, implies that the system in Figure 1 is structurally observable.

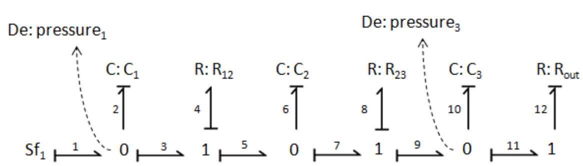

Figure 5: Bond graph for the system in Figure 1 with derivative causality in the integrally causalled storage elements. There is a valid causal assigment.

Using the running example we will show the equivalence between the two previous definitions. To do that, we will start looking for the structure of the system (Figure 1) represented as matrices equations:

$$\dot{X} = AX + BU$$

$$Y = CX + DU$$

The matrix

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

is the adjacency matrix of the system. With this matrix it can be checked the observability as well as other characteristics of the system.

When the system is represented as a bond graph (Figure 2), there is a block diagram of its junction structure (Figure 6) and also an equation (Eq. (13)) which represents the bond graph's block diagram (Sueur and Dauphin-Tanguy, 1991).

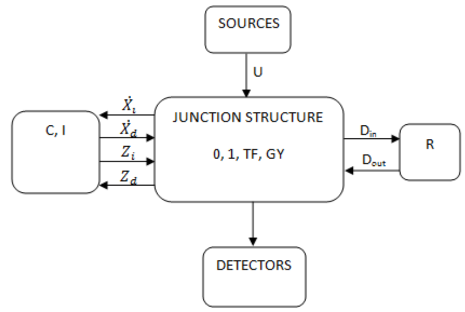

Figure 6: Block diagram of the junction structure of a linear bond graph model.

$$\begin{bmatrix} \dot{X}_i \\ D_{in} \\ Y \end{bmatrix} = \begin{bmatrix} S_{11} S_{12} S_{13} S_{14} \\ S_{21} \;\; 0 \;\; S_{23} S_{24} \\ S_{31} \;\; 0 \;\; S_{33} S_{34} \end{bmatrix} \begin{bmatrix} Z_i \\ \dot{X}_d \\ D_{out} \\ U \end{bmatrix} \tag{13}$$

Where the state vector $\dot{X}_i$ has the energy variables in integral causality, $\dot{X}_d$ is the vector for the elements in derivative causality and all the components follow the laws in Eqs. 14.

$$\left\{ \begin{array}{c} D_{out} = LD_{in} \\ Z_i = F_i X_i \\ Z_d = F_d X_d \end{array} \right\} \textit{with } L,\ F_i \textit{ and } F_d \textit{ diagonal matrices.} \tag{14}$$

Using the previous information (Eqs. (13) and (14)) we can derive the system equation and how we can calculate matrices A, B, C and D (Eq. (15))

$$\dot{X}_i = AX_i + BU$$

$$A = [I + S_{12}F_d^{-1}S_{12}^t F_i]^{-1}[S_{11} + S_{13}L(I - S_{23}L)^{-1}S_{21}]F_i$$
$$B = [I + S_{12}F_d^{-1}S_{12}^t F_i]^{-1}[S_{14} + S_{13}L(I - S_{23}L)^{-1}S_{24}]$$

$$Y = CX_i + DU$$

$$\begin{array}{c} C = [S_{31} + S_{33}L(I - S_{23}L)^{-1}S_{21}]F_i \\ D = [S_{34} + S_{33}L(I - S_{23}L)^{-1}S_{24}] \end{array} \tag{15}$$

The junction structure matrix for the system in Figure 2 is in Eq. (16).

$$\begin{bmatrix} \dot{e}_2 \\ \dot{e}_6 \\ \dot{e}_{10} \\ e_4 \\ e_8 \\ e_{12} \\ e_2 \\ e_{10} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_2 \\ e_6 \\ e_{10} \\ f_4 \\ f_8 \\ f_{12} \\ f_1 \end{bmatrix} \tag{16}$$

Matrices L and F_i are the ones in Eq. (17).

$$L = \begin{bmatrix} 1/R_{12} & 0 & 0 \\ 0 & 1/R_{23} & 0 \\ 0 & 0 & 1/R_3 \end{bmatrix}$$

$$F_i = \begin{bmatrix} 1/C_1 & 0 & 0 \\ 0 & 1/C_2 & 0 \\ 0 & 0 & 1/C_3 \end{bmatrix} \quad (17)$$

We can work out matrices A, B, C and D to get Eqs. (18) and (19).

$$\begin{bmatrix} \dot{e}_2 \\ \dot{e}_6 \\ \dot{e}_{10} \end{bmatrix} = \dots$$

$$= \begin{bmatrix} -\frac{1}{C_1 R_{12}} & \frac{1}{C_1 R_{12}} & 0 \\ \frac{1}{C_2 R_{12}} & -(\frac{1}{C_2 R_{12}} + \frac{1}{C_2 R_{23}}) & \frac{1}{C_2 R_{23}} \\ 0 & \frac{1}{C_3 R_{23}} & -(\frac{1}{C_3 R_{23}} + \frac{1}{C_3 R_3}) \end{bmatrix} \cdot$$

$$\cdot \begin{bmatrix} e_2 \\ e_6 \\ e_{10} \end{bmatrix} + \begin{bmatrix} \frac{1}{C_1} \\ 0 \\ 0 \end{bmatrix} f_1 \quad (18)$$

$$\begin{bmatrix} e_2 \\ e_{10} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e_2 \\ e_6 \\ e_{10} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} f_1 \quad (19)$$

We can write matrix in 16 with the same shape of an incidence matrix: $\dot{X}_i$ and Z_i are the states variables (the unknown), Y are the sensors (known variables) and U is the input (known variable). The incidence matrix is the one in Table 2.

Table 2: Incidence matrix derived from the structure matrix (16) of the bond graph in Figure 2.

	e_2	e_{10}	f_1	e_2	$\dot{e}_2$	e_6	$\dot{e}_6$	e_{10}	$\dot{e}_{10}$
1			1		1				
2							1		
3									1
4				1		1			
5						1		1	
6								1	
7	1			1					
8		1						1	

According to (Sueur and Dauphin-Tanguy, 1991) the system is structurally observable, because there is a causal path from each state variable to a detector. e_2 and e_{10} are measured, and it is represented in relations 7 and 8 from Table 2. e_6 has a path to e_2 and this is relation 4 on Table 2. Then the matrix reflects (rows 4, 7 and 8) the definition of structural observability in (Sueur and Dauphin-Tanguy, 1991), each causal path is reflected in a constraint. It has also a complete matching on the unknown variables (definition in (Blanke *et al.*, 2006; Staroswiecki, 2007)).

Looking at both definitions it can be said that the causal paths which might exist to have a structurally observable system are equations or relations which appears in the matrix representing the junction structure of the bond graph.

Taking this into account, the three conditions in section 2 ((Blanke *et al.*, 2006; Staroswiecki, 2007)) can be matched with the definition in section 3 ((Sueur and Dauphin-Tanguy, 1991)). The first one is the same as having a path from the known variables to the unknown ones under derivative causality. The second and the third conditions in (Blanke *et al.*, 2006; Staroswiecki, 2007) means that all the subsystems we can get from the whole system have a causal relation between its variables and we can match each variable with a different constraint or equation to get a complete matching on the variables. Talking about the definition in (Sueur and Dauphin-Tanguy, 1991), it demands a causal path, so the relation between variable must be causal and each path is a different constraint (as it has been seen above), so each unknown variable is matched with a different constraint.

4 POSSIBLE CONFLICTS AND STRUCTURAL OBSERVABILITY

Possible Conflicts (Pulido and Alonso-Gonzalez, 2004) is a structural based compilation technique which facilitates on line Consistency Based Diagnosis (Reiter, 1987) of dynamic systems.

PCs are minimal over-constrained sets of equations that have analytical redundancy. Moreover, they have associated a causal assignment, which we will exploit to show that PCs are structurally observable according to (Blanke *et al.*, 2006; Staroswiecki, 2007) definition.

In the state space formalism, computation of PCs can be realized in three steps.

The first one represents the system as an hypergraph, $H_{SD} = \{V, R\}$, where V is the set of variables of the system and $R = \{r_1, r_2, \dots, r_m\}$ is a family of sub-sets in V, where each r_k represents a constraint in the model.

The second step looks for minimal over-constrained subsystems, call *Minimal Evaluation Chains (MEC)*, $H_{ec} = \{V_{ec}, R_{ec}\}$, where $V_{ec} \subseteq V$, $R_{ec} \subseteq R$. Evaluation chains are necessary conditions for a PC to exits. Additionally, each MEC identifies, by definition, a subsystem of H_{SD}.

In the third step, extra knowledge is added to assure that a MEC, $H_{ec} = \{V_{ec}, R_{ec}\}$, may be solved using local propagation criteria. When this is possible, a *Minimal Evaluation Model (MEM)* is defined, $H_{mem} = \{V_{mem}, R_{mem}\}$, with $V_{mem} = V_{ec}$ and $R_{mem} = \{r_{1_{k1}}, r_{2_{k2}}, \dots, r_{m_{km}}\}$. $r_{i_{ki}}$ is a casual constraint obtained assigning a causality to $r_i \in R_{ec}$. The set of relations of a MEM is called *possible conflict*.

PCs method find every MEC that has a globally valid causal assignment, that is, a MEM. Hence PCs induce a unique decomposition of the system model and can be used to systematically decompose a system.

The three tanks system in Figure 1 has two possible conflicts, one of them is shown in Figure 7.

In textual notation, the MEM_1 in Figure 7 is given by $MEM_1 = \{V_{MEM}, R_{MEM}\}$ where V_{MEM} is the set of variables in the PC ($V_{MEM} = \left\{\dot{h}_1, \dot{h}_2, h_1, h_2, h_3, F_{in}, F_{12}, F_{23}, p_1, p_3\right\}$) and R_{MEM} is the set of relations in the PC ($R_{MEM} = \{ec1_1, ec2_1, ec5_1, ec6_1, ec8_2, ec9_2, ec10_2, ec11_2\}$). Each

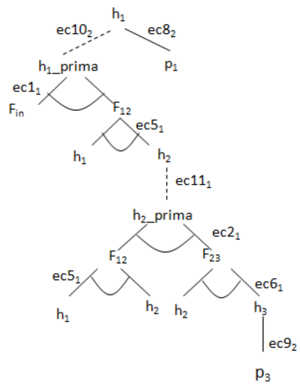

Figure 7: MEM_1 of PC 1 of system in Figure 1.

relation in R_{MEM} is defined as the variables appearing in the equation with a causal assignment, for instance:

$$ec1_1 =< \dot{h}_1 \leftarrow F_{in}, F_{12} >$$

$$ec9_2 =< h_3 \leftarrow p_3 >$$

Each relation of MEM_1 can be written as a row in an incidence matrix, in Table 3 there is the incidence matrix of PC 1 and the shaded cells represent the causal assignment. Those cells are a complete matching on the unknown variables, as it has been previously described in section 2. This complete matching on the unknown variables, obtained from MEM_1, fulfills the definition of structural observability in (Blanke *et al.*, 2006; Staroswiecki, 2007). Hence PC_1 is structurally observable. Given that the same procedure can be applied to every PC, we can conclude that Possible Conflicts are structurally observable.

Table 3: Incidence matrix of PC in Figure 7.

	F_{in}	p_1	p_3	F_{12}	F_{23}	h_1	$\dot{h}_1$	h_2	$\dot{h}_2$	h_3
1 (ec1)	1			1			1			
2 (ec2)				1	1				1	
5 (ec5)				1		1		1		
6 (ec6)					1			1		1
8 (ec8)		1				1				
9 (ec9)			1							1
10 (ec10)						1	1			
11 (ec11)								1	1	

Interesting, PCs can be derived from the TCG of the system (Bregon *et al.*, 2009b). In (Bregon, 2010) a hint of their observability is also included.

5 CASE STUDY

In this section we apply the observability concepts to the Spring-Mass system (Figure 8).

The system can be modeled as the BG in Figure 9 and we can automatically derive the TCG in Figure 10.

We are working with three sensors (see Figure 9): two effort detectors to measure the force of each mass and a flow detector to measure the velocity of the first spring. The state variables of the system are e_{11}, e_{12}, f_5 and f_9. The three sensors previously described correspond to variables e_{11}, e_{12} and f_5, so, to check the

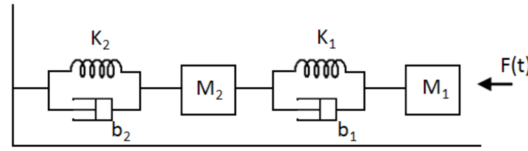

Figure 8: Spring mass system.

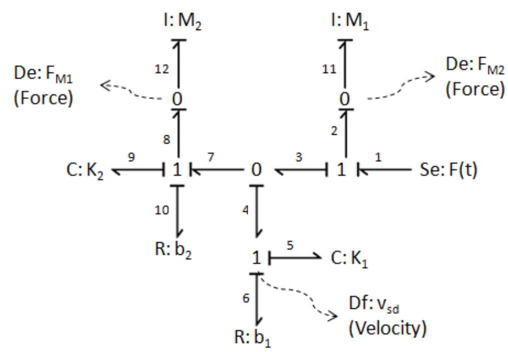

Figure 9: Bond graph for the system in Figure 8.

reachability condition we need to find a path from f_9 to a detector. Looking at the TCG in Figure 10 we can see a causal path between f_9 and e_2, which is a detector, so the first condition is fulfilled. To check the inverted reachability condition we have inverted the causality of the storage elements and we have gotten the causal assigment in Figure 11.

Working with the other definition of structural observability (Blanke *et al.*, 2006; Staroswiecki, 2007) we have obtained the junction structure matrix for the spring-mass system (Eq. (20)) where we can identify the vectors in Eq. (21). In Table 4 we show the incidence matrix derived from Eq. (20) with a complete matching on the unknown variables.

Looking at $\dot{X}_i$ in Eq. (21) can be seen that the state variables are $\dot{f}_{11}$, $\dot{e}_5$, $\dot{f}_{12}$ and $\dot{e}_9$ which is the same as e_{11}, f_5, e_{12} and f_9, the ones we have said previously.

$$\begin{bmatrix} \dot{f}_{11} \\ \dot{e}_5 \\ \dot{f}_{12} \\ \dot{e}_9 \\ f_6 \\ f_{10} \\ e_{11} \\ e_{12} \\ f_5 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 & -1 & -1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} f_{11} \\ e_5 \\ f_{12} \\ e_9 \\ e_6 \\ e_{10} \\ e_1 \end{bmatrix} \tag{20}$$

$$\dot{X}_i = \begin{bmatrix} \dot{f}_{11} \\ \dot{e}_5 \\ \dot{f}_{12} \\ \dot{e}_9 \end{bmatrix} D_{in} = \begin{bmatrix} f_6 \\ f_{10} \end{bmatrix} Y = \begin{bmatrix} e_{11} \\ e_{12} \\ f_5 \end{bmatrix}$$

$$Z_i = \begin{bmatrix} f_{11} \\ e_5 \\ f_{12} \\ e_9 \end{bmatrix} D_{out} = \begin{bmatrix} e_6 \\ e_{10} \end{bmatrix} U = [e_1] \tag{21}$$

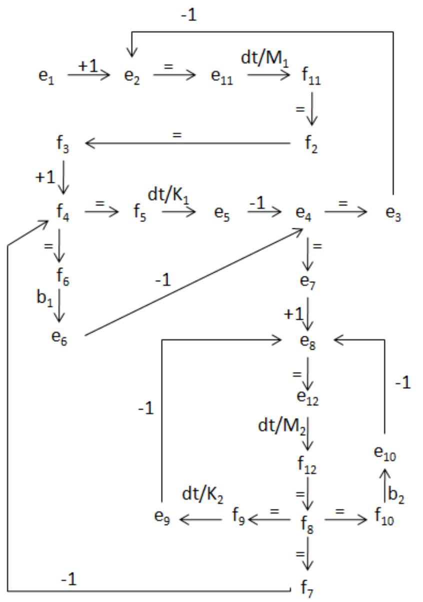

Figure 10: Temporal causal graph for the system in Figure 8 derived from the BG in Figure 9.

From the system in Figure 8 we can derived its possible conflicts using its TCG. One of the PCs can be derived starting in the observation e_{11} and going backwards in the TCG until reach measurements, inputs or variables which have already been visited. In Figure 10 there is the possible conflict 1 derived as it has been explained before.

The PC1 (Figure 12) can also be written as an incidence matrix, in this case, we have the matrix in Table 5 (it is a submatrix of the matrix in Table 4, the incidence matriz for the complete system). In shaded cells it is highlighted a complete matching on the unknown variables to proof the structural observability of the system.

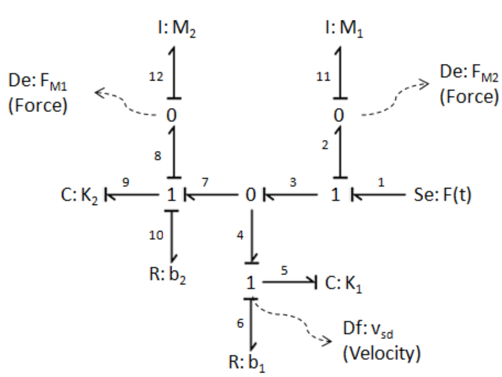

Figure 11: Bond graph for the system in Figure 8 with derivative causality in the integrally causalled storage elements (C and I). There is a valid causal assigment.

Table 4: Incidence matrix derived from the structure matrix (20) of the bond graph in Figure 9.

	e_{11}	e_{12}	f_5	e_1	f_{11}	$\dot{f}_{11}$	e_5	$\dot{e}_5$	f_{12}	$\dot{f}_{12}$	e_9	$\dot{e}_9$
1				1		1	1					
2					1			1	1			
3							1			1	1	
4									1			1
5					1				1			
6									1			
7	1			1			1					
8		1					1				1	
9			1		1				1			

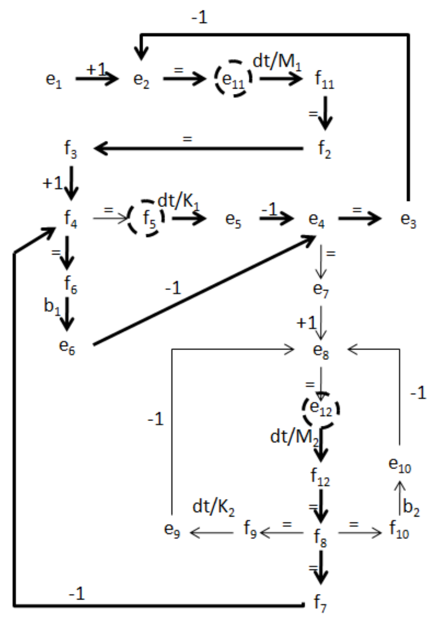

Figure 12: Possible Conflict 1 of the system in Figure 8 derived from the TCG in Figure 10.

6 DISCUSSION AND CONCLUSIONS

This work provides a deep analysis with different definitions of structural observability. These definitions has been applied to two systems, the standard three tank system benchmark that allows to easily illustrate the concepts, and a spring mass system, a more general system because it includes capacitance and inductance elements.

The equivalence of the definitions by (Blanke *et al.*, 2006; Staroswiecki, 2007) and the definition by (Sueur and Dauphin-Tanguy, 1991) has been proved showing that the structural requirements of each of them implies that the system satisfies the requirements of the other. This equivalence is important because it warranties that, except for parameter values of a concrete system, both definitions are equivalent to the standard definition of observability in the state space formulation. Consequently, we can use whatever definition adapts to our modeling formalism and we are sure that observability property holds when we change the modeling formalism. The study on structural observability was motivated because of the need to decompose a sys-

Table 5: Incidence matrix derived from the possible conflict in Figure 12.

	e_{11}	e_{12}	f_5	e_1	f_{11}	$\dot{f}_{11}$	e_5	f_{12}
1				1		1	1	
6								1
7	1			1			1	
9			1		1			1

tem into observable subsystems to facilitate diagnosis of complex systems. Decomposition looks for obtaining simpler subsystems, and observability allows an independent analysis of the subsystems. Possible Conflicts has been selected because they provide a systematic approach to system decomposition. The paper has shown that the subsystem associated to a possible conflict is always observable. Hence, PCs are an adequate tool to systematically decompose a system into observable subsystem, providing the existence of at least one possible conflict, which only depends of system structure and available observations.

This work has presented the theoretical basis to justified system decomposition with PCs. The companion work (Alonso-Gonzalez *et al.*, 2010) shows how PCs decomposition may be used to obtain factored DBNs from observable subsystems. There are other approaches to system decomposition for distribute diagnosis. In (Roychoudhury *et al.*, 2009) a method is proposed to obtain Factored DBNs based on conditional independence. But observability issues need to be considered once they have the initial factors, in case of having unobservable factors they merge two or more of them to get an observable one. Further work requires comparing both approaches on a complex system.

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