Fault Diagnosis in Non-Stationary Systems via Interval Observers

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ABSTRACT

The paper is devoted to the problem of fault diagnosis in technical systems described by non-stationary linear dynamic equations under disturbances and measurement noise via interval observers. The problem is to design the observer insensitive to the disturbances and having fewer dimension than that of the original system. Such an observer generates two residuals such that if zero is between these residuals, then the faults in the system are absent; if zero is out of these residuals, one concludes that a fault has occurred. The interval observer consists of two subsystems: the first one generates the lower residual, the second one the upper residual. The relations describing both subsystems are given. Theoretical results are illustrated by practical example of the electric servoactuator for which the fault detection problem is solved.

1. INTRODUCTION

The problem of system state vector estimation is very important for many practical applications. It is known that this problem in some cases can be solved by sliding mode observers (Edwards, Spurgeon, & Patton, 2000). Recently, an alternative approach has been developed to solve the last problem. It deals with the uncertainties and disturbances by determining upper and lower bounds for the system states. Such an approach is known as interval observer design used to evaluate the state of the dynamic system. An advantage of the interval observer is that it allows to take into account many types of uncertainties in the system.

During recent years, different kinds of interval observers have been presented for many types of the system models: for both linear and non-linear continuous-time (Chebotarev, Efimov, Raïssi, & Zolghadri, 2015; Degue, Efimov, & Richard, 2016; Dinh, Mazenc, & Niculescu, 2014; Mazenc & Bernard, 2011; Raïssi, Efimov, & Zolghadri, 2012), discrete-time (Alives et al., 2022; Efimov, Perruquetti, Raïssi, & Zolghadri, 2013; Mazenc, Dinh, & Niculescu, 2014), time delay systems (Efimov & Raïssi, 2015), switched systems (Marouani, Dinh, Raïssi, Wang, & H., 2021), and algebraic differential systems (Efimov, Polyakov, & Richard, 2015). They are also successfully applied to solve many real-time life problems (Blesa, Rotondo, & Puig, 2014; Rotondo, Fernandez-Canti, Tornil-Sin, Blesa, & Puig, 2016). Exhaustive reviews are in (Efimov & Raïssi, 2015; Khan, Xie, Zhang, & Liu, 2020, 2021).

It should be noted that all mentioned above papers solve the problem of full state vector interval estimation while only a specified function of the state vector may be necessary in practice. Such an approach based on functional interval observers was suggested in (Kravaris & Venkateswaran, 2021; Liu, Xie, Khan, & Zhang, 2021) enables estimating some linear function of the state vector.

Different methods are used to solve the problems of fault diagnosis: diagnostic observers, parity relations, Kalman filters, identification. In this paper, we accent on diagnostic observers. In (Rotondo et al., 2016; Zhang & Yang, 2017a, 2017b; Yi, Xie, Khan, & Xu, 2020) the interval observers are used to solve the fault diagnosis problem. In (Zhang & Yang, 2017a, 2017b; Yi et al., 2020) the observers are designed based on the original system that results in complex methods of minimization of the external disturbances influence the process of fault diagnosis. Some practical problems by Takagi-Sugeno interval observers are solved in (Rotondo et al., 2016).

The main contribution of this paper is that the interval observers are designed to solve the problem of fault diagnosis for dynamic systems described by linear non-stationary models. To increase the quality of diagnostic procedure, the observer is designed to be insensitive or minimum sensitive to the disturbances. Unlike (Zhang & Yang, 2017a, 2017b; Yi et al., 2020) where the observers are constructed based on the original system and are of full dimension, the designed

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observers are based on the reduced-order model of the original system. This allows to reduce the interval width and to decrease the number of erroneous diagnostic decisions about faults. Such a model is used in (Zhirabok, Zuev, & Shumsky, 2024) to solve the problem of fault diagnosis in linear stationary systems. Unlike (Zhirabok et al., 2024), systems described by linear non-stationary models are studied in this paper. The suggested approach assumes that the non-stationary system is transformed into the nonlinear stationary reduced-order model, and the method to design the interval observers for nonlinear systems is developed.

The rest of the paper is organized as follows. In section 2, the main models are introduced and the reduced-order model insensitive to the disturbance is designed. The nonlinear interval diagnostic observer is constructed in Section 3. In Section 4, the practical example is considered. Section 5 concludes the paper.

2. THE MAIN MODELS

Consider a system described by linear non-stationary model

$$\begin{array}{rcl} \dot{x}(t) & = & F(t)x(t) + Gu(t) + Dd(t) + L\rho(t), \\ y(t) & = & Hx(t) + w(t) \end{array} \qquad (1)$$

where $x \in X \subseteq R^n$, $u \in R^m$, $y \in R^l$ are vectors of state, control, and output; G, H, L, and D are constant matrices of the appropriate dimensions, $F(t)$ is the known matrix function; $\rho(t) \in R^q$ is the disturbance, it is assumed that $\rho(t)$ is unknown bounded function of time, $|\rho(t)| \leq \rho_*$ for known ρ_*; $w(t)$ is the measurement noise, $|w(t)| \leq w_*$ for known w_*. The function $d(t) \in R^p$ describes the faults; they are considered as a consequence of inadmissible changes of the system parameters. It is assumed that the case when $|d(t)| \leq d_*$ for known d_* is not considered as a fault; the case $d(t) \notin [-d_*, d_*]$, $t \geq t_0$ for some t_0, is qualified as a fault which must be detected.

Solution of the fault diagnosis problem is based on the model of system (1) of minimal dimension. To design such a model, unlike the stationary case, we assume that

$$\begin{array}{rcl} x_*(t) & = & \Psi(t)x(t), \\ y_*(t) & = & R_*(t)y(t) \end{array}$$

with the differentiable matrix functions $\Psi(t)$ and $R_*(t)$ where $x_*(t) \in R^k$, $k < n$ is the model dimension. We assume that the function $\dot{\Psi}(t)x(t)$ can be expressed via $x_*(t)$ and $y(t)$, that is

$$\dot{\Psi}(t)x(t) = \beta(x_*(t), Hx(t)) \qquad (2)$$

for some function β. Note that we use the term $Hx(t)$ instead of $y(t)$ in order to take into account measurement noise.

In this case, the model is described by the equations

$$\begin{array}{rcl} \dot{x_*}(t) & = & F_*x_*(t) + G_*(t)u(t) + J_*(t)Hx(t) \\ & & +\beta(x_*(t), Hx(t)) + D_*(t)d(t) + L_*(t)\rho(t), \\ y_*(t) & = & H_*x_*(t), \end{array} \qquad (3)$$

where $R_*(t)$, $G_*(t)$, $J_*(t)$, H_*, $D_*(t)$, and $L_*(t)$ are matrices to be determined.

Note that unlike the observer, the model (3) is a virtual object, actually it is a part of system (1). For this reason, by analogy with (2), we use the term $Hx(t)$ instead of $y(t)$.

The main advantage of the suggested approach is that the matrix F_* is constant and is of the Jordan canonical form

$$F_* = \begin{pmatrix} \lambda_1 & 0 & 0 & ... & 0 \\ 0 & \lambda_2 & 0 & ... & 0 \\ ... & ... & ... & ... & \\ 0 & 0 & 0 & ... & \lambda_k \end{pmatrix}$$

with different eigenvalues $\lambda_1, \lambda_2, ..., \lambda_k$. Note that if the purpose is to obtain stable matrix F_*, these eigenvalues should be negative.

It is known (Zhirabok, Shumsky, Solyanik, & Suvorov, 2017; Zhirabok, Zuev, & Kim, 2022) that matrices describing the model meet the conditions

$$\begin{array}{rcl} \Psi(t)F(t) & = & F_*\Psi(t) + J_*(t)H, \\ R_*(t)H & = & H_*\Psi(t), \\ \Psi(t)G & = & G_*(t), \\ \Psi(t)D & = & D_*(t), \\ \Psi(t)L & = & L_*(t). \end{array} \qquad (4)$$

To design the interval observer based on the model (3), the matrix F_* should be Metzler, i.e. its non-diagonal elements should be nonnegative, and stable (Efimov & Raïssi, 2015). Clearly, the presentation of the matrix F_* in the Jordan canonical form with $\lambda_i < 0$ has both properties.

The best observer is when the model (3) does not affected by the disturbances $\rho(t)$, i.e. $\Psi(t)L = 0$. Introduce the matrix L^0 with maximal rank that satisfies the condition $L^0L = 0$. It follows from $\Psi(t)L = 0$ that $\Psi(t) = N(t)L^0$ with some matrix $N(t)$.

When the model is designed in the Jordan canonical form, the following equation should be solved (Zhirabok et al., 2017, 2022)

$$(N_i(t) \;\; -J_{*i}(t)) \begin{pmatrix} D^0(F(t) - \lambda_i I_n) \\ H \end{pmatrix} = 0, \quad i = 1, ..., k, \qquad (5)$$

where I_n is $n \times n$ identical matrix. The matrices $R_*(t)$ and H_* are found from the equation

$$(R_*(t) \;\; -H_*) \begin{pmatrix} H \\ \Psi(t) \end{pmatrix} = 0. \qquad (6)$$

To construct the model (3), the minimal number of rows $J_{*i}(t)$ and $\Psi_i(t) = N_i(t)L^0$ satisfying the condition (2) for some function $\beta(\cdot)$ are found from (5), then the matrices $R_*(t)$ and H_* are calculated based on the equations (6); set $G_*(t) := \Phi(t)G$. As a result, the model has been designed. The interval observer is based on this model.

3. THE INTERVAL OBSERVER DESIGN

To design the interval observer, the following assumption similar to that suggested in (Efimov & Raïssi, 2015) is introduced.

Assumption. If $x_{*l} \leq x_* \leq x_{*u}$ for some $x_{*l},\, x_{*u} \in R^k$, then

$$\underline{\beta}(x_{*l}, x_{*u}, y_l, y_u) \leq \beta(x_*, Hx) \leq \overline{\beta}(x_{*l}, x_{*u}, y_l, y_u)$$

for some functions $\underline{\beta}(x_{*l}, x_{*u}, y_l, y_u)$ and $\overline{\beta}(x_{*l}, x_{*u}, y_l, y_u)$ where $y_l = y - v_*$ and $y_u = y + v_*$. For arbitrary vectors $q^{(1)}, q^{(2)}$, the inequality $q^{(1)} \leq q^{(2)}$ one understands componentwise.

According to (Efimov, Raïssi, Chebotarev, & Zolghadri, 2013), for a continuous $\beta(x_*, Hx)$, the functions $\underline{\beta}(x_{*l}, x_{*u}, y_l, y_u)$ and $\overline{\beta}(x_{*l}, x_{*u}, y_l, y_u)$ satisfying Assumption can be computed by the interval arithmetics.

Denote $Q^+ = max\{0, Q\}$ and $Q^- = Q^+ - Q$; clearly, $Q^+ \geq 0$ and $Q^- \geq 0$ for the arbitrary matrix Q.

The IO based on (3) is given by

$$\begin{array}{rcl} \dot{x}_{*l}(t) & = & F_* x_{*l}(t) + G_*(t)u(t) + J_*(t)y(t) \\ & & +\underline{\beta}(x_{*l}, x_{*u}, y_l, y_u) - D_*^+ d_* - D_*^- d_* \\ & & -J_*(t)^- w_* - J_*(t)^+ w_*, \\ \dot{x}_{*u}(t) & = & F_* x_{*u}(t) + G_*(t)u(t) + J_*(t)y(t) \\ & & +\overline{\beta}(x_{*l}, x_{*u}, y_l, y_u) + D_*^+ d_* + D_*^- d_* \\ & & +J_*(t)^- w_* + J_*(t)^+ w_*, \\ y_{*l}(t) & = & H_*^+ x_{*l}(t) - H_*^- x_{*u}(t), \\ y_{*u}(t) & = & H_*^+ x_{*u}(t) - H_*^- x_{*l}(t). \end{array} \tag{7}$$

Two residuals $r_l(t)$ and $r_u(t)$ are generated as follows:

$$\begin{array}{rcl} r_l(t) & = & R_* y(t) - y_{*u}(t), \\ r_u(t) & = & R_* y(t) - y_{*l}(t), \end{array} \tag{8}$$

respectively.

Theorem. If $x_{*u}(0) \geq x_*(0) \geq x_{*l}(0)$ and there no faults, then for all $t \geq 0$ one obtains $0 \in [r_l(t),\, r_u(t)]$.

Proof. Define the estimation errors $\underline{e}(t) = x_*(t) - x_{*l}(t)$ and $\overline{e}(t) = x_{*u}(t) - x_*(t)$ and obtain the equation for $\underline{e}(t)$:

$$\begin{array}{rcl} \dot{\underline{e}}(t) & = & \dot{x}_*(t) - \dot{x}_{*l}(t) \\ & = & F_* x_*(t) + G_*(t)u(t) + J_*(t)Hx(t) \\ & & +\beta(x_*(t), Hx(t)) - (F_* x_{*l}(t) + G_*(t)u(t) \\ & & +J_*(t)y(t) + \underline{\beta}(x_{*l}, x_{*u}, y_l, y_u) \\ & & -D_*^+(t)d_* - D_*^-(t)d_* \\ & & -J_*(t)^- w_* - J_*(t)^+ w_*) \\ & = & F_* \underline{e}(t) + \beta(x_*(t), Hx(t)) - \underline{\beta}(x_{*l}, x_{*u}, y_l, y_u) \\ & & +D_*(t)d(t) - (-D_*^+(t)d_* - D_*^-(t)d_*) \\ & & -J_*(t)w(t) - (-J_*(t)^- w_* - J_*(t)^+ w_*). \end{array} \tag{9}$$

If there are no faults, $\underline{d} \leq d(t) \leq \overline{d}$ for all $t \geq 0$, then it can be shown (Efimov & Raïssi, 2015) that

$$D_*(t)d(t) - (-D_*^+(t)d_* - D_*^-(t)d_*) \geq 0.$$

Since $|w(t)| \leq w_*$, then

$$J_*(t)^+ w_* + J_*(t)^- w_* - J_*(t)w(t) \geq 0 \tag{10}$$

as well. The matrix F_* is Metzler and $\underline{e}(0) \geq 0$; then due to Assumption and two relations above, the solution of (9) is nonnegative elementwise; this means that $\underline{e}(t) \geq 0$ for all $t \geq 0$. It can be shown analogously that $\overline{e}(t) \geq 0$ for all $t \geq 0$. Two last inequalities are equivalent to $x_{*l}(t) \leq x_*(t) \leq x_{*u}(t)$.

Calculate the residual $r_l(t)$ using the relation similar to (10):

$$\begin{array}{rcl} r_l(t) & = & R_* y(t) - y_{*u}(t) = R_* Hx(t) \\ & & -(H_*^+ x_{*ul}(t) - H_*^- x_{*l}(t)) \\ & = & H_* x_*(t) - (H_*^+ x_{*u}(t) - H_*^- x_{*l}(t)) \leq 0. \end{array}$$

By analogy one obtains $r_u(t) \geq 0$, i.e. $0 \in [r_l(t),\, r_u(t)]$. Theorem has been proved.

Remark. If (5) has no solutions for all $\lambda_i < 0$, one has to use the solution of (5) with $L^0 = I_n$. In this case $L_*(t) \neq 0$, and the interval observer (7) is supplemented by the terms describing the disturbances:

$$\begin{array}{rcl} \dot{x}_{*l}(t) & = & F_* x_{*l}(t) + G_* u(t) + J_* y(t) \\ & & -J_*^-(t)w_* - J_*^+(t)w_* \\ & & -L_*^+(t)\rho_* - L_*^-(t)\rho_*, \\ \dot{x}_{*u}(t) & = & F_* x_{*u}(t) + G_* u(t) + J_* y(t) \\ & & +J_*^-(t)w_* + J_*^+(t)w_* \\ & & +L_*^+(t)\rho_* + L_*^-(t)\rho_*, \\ y_{*l}(t) & = & H_*^+ x_{*l}(t) - H_*^- x_{*u}(t), \\ y_{*u}(t) & = & H_*^+ x_{*u}(t) - H_*^- x_{*l}(t). \end{array}$$

In this case, the equation for the error (9) becomes

$$\begin{aligned}\dot{\underline{e}}(t) &= F_* x_*(t) + G_*(t)u(t) + J_*(t)Hy(t) \\ &\quad + L_*(t)\rho(t) - (F_* x_{*l}(t) + G_*(t)u(t) \\ &\quad + J_*(t)y(t) - J_*^-(t)w_* - J_*^+(t)w_* \\ &\quad - L_*^+(t)\rho_* - L_*^-(t)\rho_*) \\ &= F_*\underline{e}(t) - J_*(t)w(t) + (J_*^+(t)w_* + J_*^-(t)w_*) \\ &\quad + L_*(t)\rho(t) - (-L_*^+(t)\rho_* - L_*^-(t)\rho_*).\end{aligned}$$

As above, $L_*(t)\rho(t) - (-L_*^+(t)\rho_* - L_*^-(t)\rho_*) \geq 0$ for all $t \geq 0$ that provides $\underline{e}(t) \geq 0$. It can be shown analogously $\overline{e}(t) \geq 0$. Clearly, the interval width becomes greater.

As a result, probability of the erroneous decisions becomes greater as well. This difficulty can be resolved by estimating the disturbances via sliding mode observer (if it is possible) and using the obtained estimate $\hat{\rho}(t)$ in (3) for compensation.

4. PRACTICAL EXAMPLE

Consider the linear model of the electric servoactuator:

$$\begin{aligned}\dot{x}_1(t) &= \tfrac{1}{i_r} x_2(t), \\ \dot{x}_2(t) &= -\tfrac{k_m + p^*(t)}{J_d + P^*(t)} x_2(t) + \tfrac{k_M}{J_m + P_*(t)} x_3(t), \\ \dot{x}_3(t) &= -\tfrac{k_\omega}{L_m} x_2(t) - \tfrac{R_m}{L_m} x_3(t) + \tfrac{k_u}{L_m} u(t) + d(t), \\ y(t) &= x_3(t) + w(t)\end{aligned} \tag{11}$$

Here, x_1 is the angle of rotation of the reducer output shaft; x_2 is the output rotation velocity; x_3 is the current through the servoactuator windings; i_r is the reducer rate; k_m is the coefficients of viscous friction in the motor; J_m is the moment of inertia; $P^*(t)$ and $p^*(t)$ are the interference components from the rest of the manipulator links; k_M is the moment coefficient; k_ω is the coefficient of counter-emf; L_m and R_m are the inductance and resistance of the motor windings, respectively; k_u is the power gain; $u(t)$ is the motor voltage. The fault $d(t)$ is due to the unexpected change of the resistance: if $\Delta R_m(t)$ is the unexpected change of R_m, then $d(t) = \Delta R_m(t)/L_m$. Many technical systems contain the servoactuators, in particular, they are essential parts of different robots.

The matrices describing the servoactuator are as follows:

$$F = \begin{pmatrix} 0 & \frac{1}{i_r} & 0 \\ 0 & -\frac{k_m + p^*(t)}{J_m + P^*(t)} & \frac{k_M}{J_m + P^*(t)} \\ 0 & -\frac{k_\omega}{L_m} & -\frac{R_m}{L_m} \end{pmatrix},$$

$$B = \begin{pmatrix} 0 \\ 0 \\ \frac{k_u}{L_m} \end{pmatrix}, \quad H = (0\ \ 0\ \ 1).$$

To simplify the presentations, introduce the notations:

$$\begin{aligned}\gamma_1 &= \tfrac{1}{i_r},\ \gamma_2 = \tfrac{k_m + p^*(t)}{J_m + P^*(t)},\ \gamma_3 = \tfrac{k_M}{J_m + P^*(t)}, \\ \gamma_4 &= \tfrac{k_\omega}{L_m},\ \gamma_5 = \tfrac{R_m}{L_m},\ \gamma_6 = \tfrac{k_u}{L_m}.\end{aligned}$$

The problem is to design the interval diagnostic observer to detect the fault.

Since the disturbances are absent, set $L^0 = I_3$. The equation (5) with $\lambda_1 = -1$ has a solution

$$\begin{aligned}J_{*1}(t) &= \gamma_3\gamma_4 - (\gamma_5 + 1)(\gamma_2 + 1), \\ \Psi_1(t) &= (\ 0 \quad \gamma_4 \quad -(\gamma_2 + 1)\ ), \\ G_{*1}(t) &= -\gamma_6(\gamma_2 - \gamma_5).\end{aligned}$$

The value $\lambda_2 = \gamma_5$ in (5) provides a solution

$$\begin{aligned}J_{*2}(t) &= \gamma_3\gamma_4, \\ \Psi_2(t) &= (\ 0 \quad \gamma_4 \quad -(\gamma_2 - \gamma_5)\ ), \\ G_{*2}(t) &= -\gamma_6(\gamma_2 - \gamma_5).\end{aligned}$$

It can be shown that (6) has a solution

$$R_* = -1/(1\gamma_5), \quad H_* = (1 \quad -1).$$

Obviously, $H_*^+ = (1\ \ 0)$ and $H_*^- = (0\ \ 1)$. It can be shown that $J_{*1}(t) \leq 0$ and $J_{*2}(t) \leq 0$, therefore, $J_{*1}^+(t) = 0$, $J_{*1}^-(t) \neq 0$, $J_{*2}^+(t) = 0$, and $J_{*2}^-(t) \neq 0$.

It is known that $\dot{P}^*(t) = p^*(t)$, then one obtains

$$\beta_j(x_*(t), Hx(t)) = \dot{\Psi}_j(t)x(t) = \beta_*(t)Hx(t),$$

$j = 1, 2$, where

$$\beta_*(t) = \frac{\dot{p}_*(t)(J_m + P^*(t)) - p^*(t)(k_m + p^*(t))}{(J_m + P^*(t))^2} Hx(t).$$

It can be show that $\beta_*(t)$ is known function and $\beta_*(t) > 0$ for all $t \geq 0$.

The model is given by

$$\begin{aligned}\dot{x}_{*1}(t) &= -x_{*1}(t) + J_{*1}Hx(t) + G_{*1}u(t) \\ &\quad + \beta_*(t)Hx(t), \\ \dot{x}_{*2}(t) &= \gamma_5 x_{*2}(t) + J_{*2}Hx(t) + G_{*2}u(t) \\ &\quad + \beta_*(t)Hx(t), \\ y_*(t) &= x_{*1}(t) - x_{*2}(t).\end{aligned}$$

The interval observer is described as follows:

$$\begin{aligned}\dot{x}_{*l1}(t) &= -x_{*l1}(t) + J_{*1}(t)y(t) + G_{*1}(t)u(t) \\ &\quad + \beta_*(t)y_l(t) - J_{*1}^-(t)w_*, \\ \dot{x}_{*l2}(t) &= \gamma_5 x_{*l2}(t) + J_{*2}y(t) + G_{*2}(t)u(t) \\ &\quad + \beta_*(t)y_l(t) - J_{*2}^-(t)w_*, \\ \dot{x}_{*u1}(t) &= -x_{*u1}(t) + J_{*1}(t)y(t) + G_{*1}(t)u(t) \\ &\quad + \beta_*(t)y_u(t) + J_{*1}^-(t)w_*, \\ \dot{x}_{*u2}(t) &= \gamma_5 x_{*u2}(t) + J_{*2}y(t) + G_{*2}(t)u(t) \\ &\quad + \beta_*(t)y_u(t) + J_{*2}^-(t)w_*, \\ y_{*l}(t) &= x_{*l1}(t) - x_{*u2}(t), \\ y_{*u}(t) &= x_{*u1}(t) - x_{*l2}(t).\end{aligned}$$

The residuals $r_u(t)$ and $r_l(t)$ are given by (8).

For simulation, consider the system (11) under the control $u(t) = 0.1 + 0.1sin(t)$. Assume for simplicity that $\gamma_1 =$

$\gamma_3 = \gamma_6 = 1$ and $\gamma_2 = -1.5$, $\gamma_4 = -1$, $\gamma_5 = -2$. The measurement noise $w(t)$ is modeled by random processes evenly distributed on the interval $[-0.01;\ 0.01]$, $w_* = 0.01$. The admissible interval for $d(t)$ is $[-0.1; 0.1]$, $d_* = 0.1$. The last interval can be calculated based on the relation above $d(t) = \Delta R_m(t)/L_m$ and the admissible interval for $\Delta R_m(t)$: if

$$\Delta R_m(t) \in [-\Delta R_{m*}(t), \Delta R_{m*}(t)],$$

the system is healthy, otherwise the fault has occurred.

Simulation results are presented in Figures 1, 2, and 3 with the initial state $x(0) = (0\ 0\ 0)$. In Figure 1, $d(t) = 0$, $t < 30\,s$ and $d(t) = 0.08$, $t \geq 30\,s$; behavior of the variable $x_2(t)$ is shown. Clearly, the fault affects the system behavior. In Figure 2, $d(t) = 0$, $t < 30\,s$ and $d(t) = 0.08$, $t \geq 30\,s$; because 0.08 is the admissible value of $d(t)$, then $0 \in [r_l(t), r_u(t)]$, and a decision that there are no faults should be made. In Figure 3, $d(t) = 0$, $t < 30\,s$ and $d(t) = 0.2$, $t \geq 30\,s$; since $0 \notin [r_l(t), r_u(t)]$, $t \geq 30\,s$, one concludes that the fault has occurred at $t = 30\,s$. Then, the value of fault can be estimated by methods suggested in (Edwards et al., 2000); finally, the estimate can be used in fault tolerant scheme.

5. CONCLUSION

The problem of interval diagnostic observer design for systems described by the linear non-stationary models under the external disturbances has been studied. To construct such observer, the reduced-order model of minimal dimension insensitive to the disturbance is used based on the Jordan canonical form of the model. Such a model is stationary but becomes nonlinear. The nonlinear interval observer has been used to solve the problem of fault detection. The theoretical results have been illustrated by practical example. The advantage of the suggested approach is that it does not use complex transformations which are characteristic for the methods developed for the non-stationary systems. A future research direction is the interval diagnostic parity relations design for nonlinear dynamic systems.

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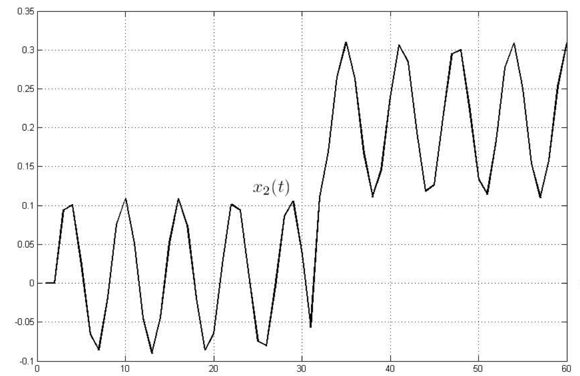

Figure 1. Behavior of the variable $x_2(t)$ for the healthy system and when the fault occurred.

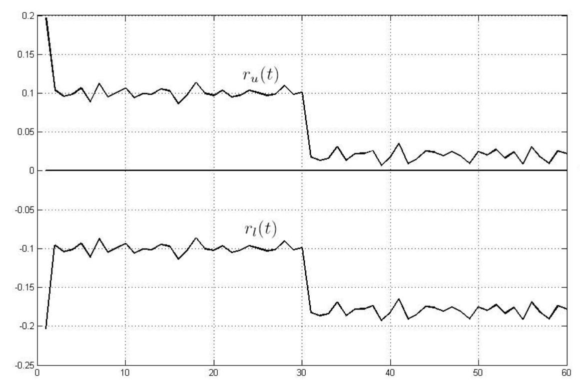

Figure 2. Behavior of the residuals $r_u(t)$ and $r_l(t)$ for the healthy system.

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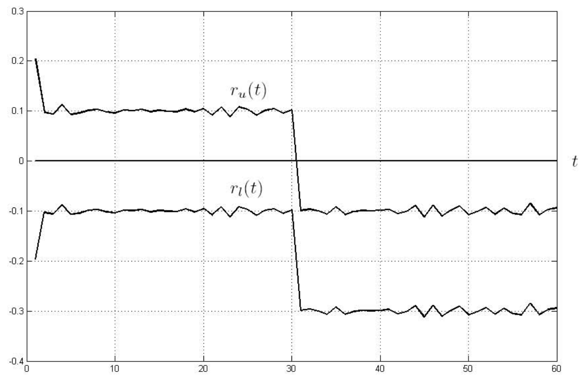

Figure 3. Behavior of the residuals $r_u(t)$ and $r_l(t)$ when the fault occurred.

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