A Composite Fault Feature Enhancement Approach for Rolling Bearings Grounded on ITD and Entropy-based Weight Method

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ABSTRACT

Aiming to precisely identify a compound fault of rolling bearing, the paper has contributed a fault characteristic enhancement method by combing entropy weight method (EWM) and intrinsic time scale decomposition (ITD). Firstly, to effectively segregate frequency components in vibration signals, proper rotation components (PRCs) were obtained by decomposing vibration signals based on ITD. Secondly, in view of the fact that amplitude, variance and correlation coefficient vary greatly in a bearing fault accompanied by impact components, parameter evaluation indexes were brought in to depict the fault characteristics of PRCs, including average, variance, correlation coefficient, margin factor, kurtosis, impulse factor, peak factor and so on. Thirdly, weight coefficient of each parameter index was calculated by entropy weight method and the characteristics of each PRC highlighted based on that. Finally, the signals were reconstructed according to the PRCs whose characteristics had been enhanced. Meanwhile reconstructed signals were denoised with singular differential spectrum (SDS) to reduce the influence of noise components, and then the type of compound fault was distinguished grounded on the frequency spectrum. To further prove the efficiency of proposed method, it is compared with other methods (SDS, ITD + entropy method). The result indicates that the proposed method can further highlight the characteristic information of compound faults of bearing and embody more exact identification and judgment on the type of faults.

1. INTRODUCTION

1.1. Background

As an important component of rotary machinery, rolling bearing is closely linked with the running state of equipment (Zhao Lei et al., 2018). If it fails to recognize a bearing fault in time, it will probably result in the damage of equipment causing serious economic loss and even personal injury. Therefore, it is crucial to correctly extract the fault information of bearing. Due to the influence of noise from environment and complexity of structure, when a fault happens in rolling bearing, its vibration signals are strongly linear and contains impulse components (Mingyue Yu and Xiang Pan, 2020), which largely increases the difficulty of extraction of fault characteristics. Therefore, the fault feature information contained in the signal is enhanced to further highlight the fault feature contained in the sensitive fault component signal, which can better extract the fault characteristics of rolling bearing and precisely identify a fault type (Zhou Yiwen et al., 2020).

1.2. Related Works

The study based on signal decomposition algorithm is an important direction in the research of fault diagnosis of rolling bearing. Common signal decomposition algorithms include wavelet transform (Guo Dazhi et al., 2021), empirical mode decomposition (EMD) (Debiao Meng et al., 2022), variational mode decomposition (VMD) (Deng Linfeng, Zhang Aihua et al., 2022), intrinsic time scale decomposition (ITD) (Ding Jakai et al., 2022) and so on. ITD algorithm manages to exactly extract the dynamic properties of non-stationary signal at higher speed and sees a rather wide application in fault diagnosis of bearing (Liu Feng et al., 2021). Fei Wang et al (2017) proposed a fault classification approach based on ITD and extreme learning
machine (ELM). Gao Yajuan et al (2019) proposed the combination of ITD and kernel principal component analysis to detect bearing faults. Pan Xiang et al (2021) brought forward the approach combining ITD and autocorrelation function to reconstruct rotation components obtained from ITD and calculate its autocorrelation function, and thereby extracted the characteristic information of bearing fault.

Signal parameter evaluation index is capable to measure and describe various characteristic information. Among them, kurtosis, impulse factor and peak factor can make better description of impact characteristics of signal; variance is designed to reflect the dispersion of signals; correlation coefficient exists as an index to determine the correlation among data; margin factor is often used to detect the wear degree of machine and its physical significance is similar to peak factor and impulse factor; average can represent the energy and intensity of signal. These index parameters all have certain applications in fault identification field of bearing. The original concept of entropy was about a single-flowing and irreversible energy transfer process (Zhang Yan, 2009) and gradually came into 3 thoughts, thermodynamic entropy, statistical entropy and information entropy. Entropy weight method (Li Hongxian et al., 2019) is a classical method to calculate the weight of multiple indexes. The method is dependent on entropy to evaluate the discrete degree of index in such a way that the greater discrete degree is, the more influence an index has on the evaluation of comprehensive data. Due to being less demanding to data and easy to manage, entropy weight method has been widely applied to multiple fields of engineering technology, such as comprehensive evaluation of engineering construction risks, evaluation of logistic development indexes, land use efficiency and so on. As for fault diagnosis, entropy weight method is mainly used for fault diagnosis of transformer, bearing and modular machine tool (Wang Hao et al., 2016). Yang Zhifei et al., (2017) fused the signals with entropy weight method and subjected them to variational modal decomposition and studied the optimal frequency band, which was screened out, and effectively extracted the fault characteristics of bearing. Entropy weight method was used in the paper to calculate the parameter evaluation index when a compound fault of rolling bearing occurred.

Singular value decomposition (SVD) (Li hua et al., 2020), a classical orthogonal decompose method (Li Hua et al., 2021), has been vastly applied to multiple fields of signal processing. Dong Shaolijian et al., (2022) precisely extracted the characteristic frequency of bearing fault by blending SVD algorithm, sliding window linear regression and multi-attention mechanism deep neural network. Zou Tiangang et al., (2021) proposed a fault diagnosis method for rolling bearing based on the combination of singular spectral decomposition, SVD and frequency weight energy operator. This method can effectively recognize a typical rolling bearing fault, highlight fault characteristics and improve fault diagnosis effect. Gougam Fawzi et al., (2020) brought forward the combination of 2 times domain features, SVD and fuzz logical system and made effective extraction of fault characteristic frequency of bearing. Singular value decomposition differential spectrum (SDS), a classical noise reduction method (Te Han et al., 2016), can eliminate the interference of strong noise and make extraction of fault information more precise (Jun Ma et al., 2018).

It can be seen that scholars from a pretty wide range of fields have made great efforts to implement precise identification of bearing faults.

1.3. Method

In an attempt to precisely identify a compound fault of rolling bearing, the paper combines ITD and entropy weight method, entropy weight method and typical signal evaluation indexes are used to obtain weight coefficients of component signals which are used as the basis to strengthen the characteristics of each PRC. To further reduce the influence of noise, the paper makes use of excellent denoising ability of SDS to reduce the noise of reconstructed signals after feature enhancement. The result has indicated that the proposed method is capable of precisely extracting the characteristic information of compound faults of rolling bearing.

2. ALGORITHM

2.1. Intrinsic time scale decomposition

Intrinsic time scale decomposition (ITD) can decompose non-stationary signals into the sum of multiple proper rotation components (PRCs) and a residual trend component. The process is shown as follow (Xueli An et al., 2012; Berlin, Lu Chao et al., 2015):

Step 1: Set input vibration signal is X and locate all the extreme points $X_k$ in the signal, and record the time slot of them and substitute into formula (1):

$$L_{k+1} = \alpha [X_k + \left(\frac{t_{k+1} - t_k}{t_{k+2} - t_k}\right) (X_{k+2} - X_k)] + (1 - \alpha) X_{k+1}$$

in this formula: $k = 1, 2, ..., M - 2$; $\alpha = 0.5$.

Step 2: The formula of piecewise linear extraction operator is as follow:

$$L = L_k + \left(\frac{t_{k+1} - t_k}{t_{k+2} - t_k}\right) (X_t - X_k)$$

Step 3: The formula of signal decomposition is as follow:

$$X_t = L_t + H_t = H_t + (H + L) L_t = H \sum_{k=1}^{P} L_k + L_t$$

In the formula $t \in (t_k, t_{k+1})$, $X_t$ represents input sampling signal, $L_t$ baseline component, $H_t$ intrinsic rotation component, $L$ piecewise linear extraction operator, $H$ intrinsic rotation component of highest frequency and $P$ number of iterations.
2.2. Entropy-based weight method

Information entropy can be used to determine the chaos and randomness of signal and information entropy of signal index can judge the extent of chaos of index (Wang Hao et al., 2016). When the extent of chaos is larger, the influence of index will have greater influence (weight) on comprehensive evaluation of system and related information entropy be smaller. It is being based on this principle that entropy weight method (EWM) can be used to evaluate a signal index. The detailed process of EWM is as follow (Wu Yu et al., 2020):

Step 1: Choose required parameter evaluation indexes. Set the total number of chosen parameter evaluation indexes is \( n \) and each vibration corresponds to \( n \) indexes. Given there are \( m \) groups of data, label the \( j \)-th index from \( i \)-th data group as \( x_{ij} (i = 1, \ldots, m; j = 1, \ldots, n) \) and all the indexes of data make up matrix \( A \):

\[
A = \begin{bmatrix}
x_{11} & x_{12} & \cdots & x_{1n} \\
x_{21} & x_{22} & \cdots & x_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
x_{m1} & x_{m2} & \cdots & x_{mn}
\end{bmatrix} \quad (i = 1, \ldots, m; j = 1, \ldots, n)
\]

(4)

Calculate \( P_{ij} \), the proportion of \( j \)-th index \( x_{ij} \) from \( i \)-th data group in the \( j \)-th indexes of all data;

\[
P_{ij} = \frac{x_{ij}}{\sum_{j=1}^{n} x_{ij}} \quad (j = 1, \ldots, n)
\]

(5)

All \( P_{ij} \) forms matrix \( P \):

\[
P = \begin{bmatrix}
P_{11} & P_{12} & \cdots & P_{1n} \\
P_{21} & P_{22} & \cdots & P_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
P_{m1} & P_{m2} & \cdots & P_{mn}
\end{bmatrix}
\]

(6)

Step 2: Calculate \( e_j \), the information entropy of \( j \)-th index:

\[
e_j = -k \sum_{i=1}^{m} p_{ij} \ln(p_{ij})
\]

(7)

In this formula : \( k = \frac{1}{\ln(m)} > 0 \).

Step 3: Calculate \( d_j \), the information entropy redundancy rate of \( j \)-th index:

\[
d_j = 1 - e_j
\]

(8)

Step 4: Calculate \( w_j \), the weight of \( j \)-th index:

\[
w_j = \frac{d_j}{\sum_{j=1}^{n} d_j}
\]

(9)

Step 5: Calculate \( s \), comprehensive evaluation value matrix of each index:

\[
S = \begin{bmatrix}
s_1 \\
s_2 \\
\vdots \\
s_m
\end{bmatrix} = \begin{bmatrix}
P_{11} & P_{12} & \cdots & P_{1n} \\
P_{21} & P_{22} & \cdots & P_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
P_{m1} & P_{m2} & \cdots & P_{mn}
\end{bmatrix} \begin{bmatrix}
w_1 \\
w_2 \\
\vdots \\
w_n
\end{bmatrix}
\]

(10)

Step 6: Strengthen the characteristics of component signal and heightened signal is \( Y_i \):

\[
Y_i = s_i \cdot PRC_i
\]

(11)

Step 7: Reconstruct signals according to characteristic-enhanced component signals and recon-structured signal is \( Y \):

\[
Y = \sum_{i=1}^{m} Y_i
\]

(12)

2.3. Bearing fault feature frequency

A bearing fault is often accompanied by the characteristic frequency related with different kinds of fault. For that, a fault type of bearing can be judged by calculating its characteristic frequency (Yuan Zhe et al., 2019). According to definition of paper, fault feature frequency in inner-ring, outer-ring, rolling-element and retainer is represented by \( f_i \), \( f_o \), \( f_r \) and \( f_c \), respectively; rotation frequency \( f_r \) and rotate speed \( f_n \). \( N \) represents the numbers of balls, \( D \) bearing diameter and \( d \) rolling ball diameter. Then, characteristic of rolling can be calculated by formula (13) – (17), which is shown as follow:

Rotate frequency:

\[
f_r = \frac{f_n}{60}
\]

(13)

Inner-ring fault feature frequency:

\[
f_i = \frac{1}{2} \cdot N \cdot (1 + \frac{d}{D}) \cdot f_r
\]

(14)

Outer-ring fault feature frequency:

\[
f_o = \frac{1}{2} \cdot N \cdot (1 - \frac{d}{D}) \cdot f_r
\]

(15)

Rolling-element fault feature frequency:

\[
f_b = \frac{D}{2 \cdot d} \cdot (1 - \frac{d}{D})^2 \cdot f_r
\]

(16)

Holder fault feature frequency:

\[
f_c = \frac{1}{2(1 - \frac{d}{D})} \cdot f_r
\]

(17)

3. COMPONENT SIGNAL FEATURE ENHANCEMENT BASED ON EWM– PROPOSED METHOD

The paper blends ITD and EWM to strengthen the compound fault characteristics of rolling bearing. In view of the characteristics of vibration signals when a bearing fault occurs, the paper selects average, variance, correlation coefficient, margin factor, kurtosis, impulse factor, and peak factor as evaluation indexes. The conceptual framework of proposed method as is shown in Fig 1. For the convenience of presentation, the symbols in Table 1 will prevail in the description of comprehensive weight of indexes. Concretely, \( W_K \) represents the comprehensive weight of kurtosis.
Detailed steps are as follow:

Step 1: Decompose original vibration acceleration signals into 4 layers (Ming Yue Yu and Xiang Pan, 2020; Pan Xiang et al., 2021) based on ITD and corresponding PRCs are obtained;

Step 2: Calculate the signal evaluation index of PRC in each layer, including average, variance, correlation coefficient, margin factor, kurtosis, impulse factor and peak factor;

Step 3: According to entropy weight method (eq. 4-10), calculate the comprehensive weight of each evaluation index;

Step 4: According to comprehensive weight calculated above, strengthen the characteristics of component signals and reconstruct the enhanced signals;

Step 5: Denoise reconstructed signals based on singular value difference spectrum. Meanwhile, extract characteristics of compound faults of bearing and identify the type according to the frequency spectrum of denoised signals.

### 4. Compound Fault Identification for Rolling Bearings

#### 4.1. Rolling bearing compound faults experiment

All the data of paper originates from the tester of rolling bearing of aero-engine shown in Fig 2. The tester mainly consists of drive motor, step-up gear box, lubricating system, rolling bearing and retainer. The tester can simulate different types of bearing faults, including single and compound. In the experiment, vibration acceleration sensors are fixed to the horizontal and vertical positions on the left-sided bearing seat. Electrical vortex sensor is applied for speed measurement and fixed to the right of tester. Specific sensor installation positions and corresponding channel number (CH1 – CH4) is shown in Fig 2. A bearing is cut by electric spark and the depth is 0.2mm. The number of rolling elements is represented by N, N = 7; diameter of rolling element, d = 9.6mm; diameter of bearing, D = 36mm. Fig 3 corresponds to the bearings in different compound faults: 3(a) compound faults of inner ring and rolling element; 3(b) outer and inner ring; 3(c) outer ring and rolling element; 3(d) outer ring, inner ring and rolling element.
4.2. Case 1

To verify the effectiveness of proposed method, randomly choose the data of compound faults of inner ring, outer ring and rolling element for analysis. This state is related with rotate speed 1875 r/min and rotate frequency $f_r$ is 31.3 Hz. The characteristic frequencies of the bearing are as follow: Inner-ring fault feature frequency $f_i$ is 138.5 Hz, outer-ring fault feature frequency $f_o$ is 80.2 Hz, rolling-element fault feature frequency $f_b$ is 54.4 Hz, holder fault feature frequency $f_c$ is 11.5 Hz.

4.2.1. Comparison method: SDS

Firstly, signals are denoised based on singular value difference spectrum and the result as is shown in Fig 4. Fig 4(a1) shows original vibration acceleration signal; Fig 4(a2) and Fig 4(a3) corresponds to the frequency spectrum and its local enlargement; Fig 4(b1) is the result of singular value difference spectrum denoising of acceleration signals; Fig 4(b2) and Fig 4(b3) is the frequency spectrum of Fig 4(b1) and its local enlargement.
Analysis of Fig 4(a3) and (b3) reveals that no matter the frequency spectrum of original signal or denoised signals by singular value difference spectrum, they have the following features:

- Noise components are evident;
- Most of frequency components correspond to the multiples of rotation frequency (for example, frequency components 287Hz and 1001Hz correspond to the 9x, and 32x of $f_r$), which is irrelevant with faults of the bearing;
- There is the frequency component 800.8Hz, equal to 10x of $f_o$;
- No evident frequency component corresponding to the type of compound fault has been found (this state is related with the compound fault of inner ring, outer ring and rolling element).

Namely, the method directly based on singular value difference spectrum, when a compound fault of inner race, outer race and rolling element occurs, the above method can only capture the fuzzy feature frequency of outer race and not the feature frequency of inner race and rolling element corresponding to bearing fault. The evaluation on compound faults of bearing is not precise.

### 4.2.2. Proposed method

To prove the efficiency of proposed method, the paper will analyze the data from section 4.2.1 (the compound faults of inner ring combined with outer ring and rolling element) based on the proposed method, the result as is shown in Figs 5 – 6. Fig 5(a) shows the PRCs of Fig 4(a1) after ITD. Fig 5(b) shows the function value of signal evaluation index related with each component signal. Fig 5(c) shows the comprehensive weight value of each evaluation index obtained by entropy weight method according to Fig 5(b) and Table 2. Table 2 shows the comprehensive weight value of each evaluation index obtained by entropy weight method. Table 3 provides the proportion value of each evaluation index for PRC obtained by formula (5). Table 4 and Fig 5(d) shows the component signals obtained after characteristic enhancement to each PRC according to formula (10) – (11). Fig 5(e) and (f) corresponds to reconstructed signal and its frequency spectrum obtained from Fig 5(d). Fig 5(f1) and (f2) is the local enlargement of frequency spectrum of Fig 5(f).
(b) The function value of signal evaluation index

(c) The weight value of each evaluation index

<table>
<thead>
<tr>
<th>P-value</th>
<th>Kurtosis</th>
<th>Correlation coefficient</th>
<th>Margin factor</th>
<th>Variance</th>
<th>Impulse factor</th>
<th>Peak factor</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRC1</td>
<td>0.2387</td>
<td>0.3217</td>
<td>0.2510</td>
<td>0.5483</td>
<td>0.2630</td>
<td>0.2748</td>
<td>0.4636</td>
</tr>
<tr>
<td>PRC2</td>
<td>0.2807</td>
<td>0.2868</td>
<td>0.2791</td>
<td>0.1973</td>
<td>0.2713</td>
<td>0.2614</td>
<td>0.0118</td>
</tr>
<tr>
<td>PRC3</td>
<td>0.2526</td>
<td>0.2452</td>
<td>0.2542</td>
<td>0.2219</td>
<td>0.2492</td>
<td>0.2448</td>
<td>0.4183</td>
</tr>
<tr>
<td>PRC4</td>
<td>0.2280</td>
<td>0.1464</td>
<td>0.2156</td>
<td>0.0324</td>
<td>0.2165</td>
<td>0.2189</td>
<td>0.1062</td>
</tr>
</tbody>
</table>

Table 2. The proportion of each evaluation index of the component signals - Eq. (5)

<table>
<thead>
<tr>
<th>W-value</th>
<th>$W_K$</th>
<th>$W_C$</th>
<th>$W_M$</th>
<th>$W_V$</th>
<th>$W_I$</th>
<th>$W_P$</th>
<th>$W_A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRC1-PRC4</td>
<td>0.0043</td>
<td>0.0520</td>
<td>0.0058</td>
<td>0.4057</td>
<td>0.0050</td>
<td>0.0049</td>
<td>0.5223</td>
</tr>
</tbody>
</table>

Table 3. The weight value of each evaluation index in this signal - Eq. (9)

- $Y_1 = (W_K * 0.2387 + W_C * 0.3217 + W_M * 0.2510 + W_V * 0.5483 + W_I * 0.2630 + W_P * 0.2748 + W_A * 0.4636) * PRC1$
- $Y_2 = (W_K * 0.2807 + W_C * 0.2868 + W_M * 0.2791 + W_V * 0.1973 + W_I * 0.2713 + W_P * 0.2614 + W_A * 0.0118) * PRC2$
- $Y_3 = (W_K * 0.2526 + W_C * 0.2452 + W_M * 0.2542 + W_V * 0.2219 + W_I * 0.2492 + W_P * 0.2448 + W_A * 0.4183) * PRC3$
- $Y_4 = (W_K * 0.2280 + W_C * 0.1464 + W_M * 0.2156 + W_V * 0.0324 + W_I * 0.2165 + W_P * 0.2189 + W_A * 0.1062) * PRC4$

Table 4. Component signals after characteristic enhancement of PRCs - $Y_1$
(d) Component signals after characteristic enhancement of (a) by proposed method

(e) Reconstruct signal according to (d)

(f) Spectrum of (e)

(f1) Local enlargement of (f)

(f2) Local enlargement of (f)

Fig 5. Bearing compound faults feature extraction - ITD+EWM

(a) Denoised signal of Fig 5(e) by SDS

(b) Spectrum of (a)

Fig 6. Time domain and frequency domain diagram of proposed method
On the frequency spectrum of signal obtained by the combination of ITD and entropy weight method (no denoising), an analysis of Fig 5(f1) and (f2) can reveal the following characteristics:

- Evident noise components exist in great number;
- There exist 286.9Hz and 966Hz frequency, which correspond to the 9x and 31x of \( f_r \);
- There is 197.8Hz frequency component \(((197.8+31.3-11.5)/4=54.4Hz)\), which corresponds to the 4x of \( f_b \);
- There is 800.8Hz frequency component \( (800.8Hz/10=80Hz)\), which matches with 10x of \( f_o \) (characteristic frequency of outer ring fault).

It is obvious that the combination of ITD and entropy weight method can obtain the characteristic frequency relative to the type of bearing fault and judge the type of fault. But, in the frequency spectrum of signal, noise components are very obvious, and some frequency components are already overwhelmed by noise, which is disadvantageous to identification of fault.

To further reduce the influence of noise, noise reduction is implemented based on singular value difference spectrum. The result as is shown in Fig 6. Fig 6(a) and (b) are the time domain and frequency spectrum of signal after singular value difference spectrum denoising of Fig 5(e).

From the analysis of Fig 6(b), the frequency spectrum of signal obtained by the proposed method of paper, it can be found that:

- Noise components are sharply decreased;
- There are two frequency components: 73.24Hz, equal to the sum of 1x of \( f_b \) and 1x of \( f_r \) subtracting 1x of \( f_e \)(73.24-31.3+11.5=53.4Hz) and 943.6Hz, the sum of 17x of \( f_b \) and 2x of \( f_e \) ((943.6-11.5*2)/17=54.2Hz). These frequencies correspond to the fault feature frequency of rolling element of bearing;
- 1556Hz, 11x of \( f_i \) adding 1x of \( f_r \) ((1556-31.3)/11=138.6Hz), which is consistent with fault feature frequency of inner ring;
- 1289Hz (1289Hz/16=80.6Hz), 16x of \( f_o \), which is consistent with fault feature frequency of outer ring.

These frequencies are helpful to judge the occurrence of a compound fault among inner ring, outer ring and rolling element, which is consistent with actual fault type of bearing. Namely, there exists a perfect matching between the above frequency components and the compound fault types of bearing. That means the proposed method of paper can greatly reduce the noise, precisely draw the characteristic frequency of bearing matching the type of compound fault and make a correct judgment.

5. Analysis the Effectiveness of Proposed Method under Other Composite Fault Types

To take a further step to verify the efficiency of proposed method, the signals from other 3 types of compound faults were chosen for analysis. These faults include: 1) inner ring and rolling element; 2) outer ring and inner ring; 3) outer ring and rolling element. The result is shown in Figs 7 - 9. The characteristic frequency of each fault mode is shown in Table 5 in the following.

<table>
<thead>
<tr>
<th>Fault mode</th>
<th>Rotate speed</th>
<th>Rotate frequency ( f_r )</th>
<th>Characteristic frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Holder ( f_c )</td>
<td>Rolling element ( f_b )</td>
<td>Inner ring ( f_i )</td>
</tr>
<tr>
<td>inner ring and rolling element</td>
<td>1812.7r/min</td>
<td>30.2Hz</td>
<td>11.1</td>
</tr>
<tr>
<td>outer ring and inner ring</td>
<td>1519.0r/min</td>
<td>25.3Hz</td>
<td>9.3</td>
</tr>
<tr>
<td>outer ring and rolling element</td>
<td>1812.7r/min</td>
<td>30.2Hz</td>
<td>11.1</td>
</tr>
</tbody>
</table>

Table 5. The characteristic frequency of each fault mode

5.1. Inner ring and rolling element faults analysis

Fig 7(a) and (b) are the time domain and frequency spectrum of signal from compound fault between inner ring and rolling element; Fig 8 shows the PRCs of Fig 7(a) after ITD. Fig 9 shows the function value of signal evaluation index related with each component signal. Table 6 provides the proportion value of each evaluation index for PRC obtained by formula (5). Fig 10 shows the comprehensive weight value of each evaluation index obtained by entropy weight method according to Fig 9 and Table 6, and the specific values as are shown in Table 7. Table 8 and Fig 11 shows the component signals obtained after characteristic enhancement to each PRC component signal according to formula (10) – (11). Fig 12(a) and (b) are the time domain and frequency spectrum of signal after singular value difference spectrum denoising of Fig 11.
(a) Horizontal acceleration signal

(b) Spectrum of (a)

Fig 7. Time domain and frequency domain diagram of original signal

Fig 8. PRCs of Fig 7(a) by ITD

Fig 9. The function value of signal evaluation index

Fig 10. The weight value of each evaluation index
Table 6. The proportion of each evaluation index of the component signals - Eq. (5)

<table>
<thead>
<tr>
<th>P-Value</th>
<th>Kurtosis</th>
<th>Correlation Coefficient</th>
<th>Margin Factor</th>
<th>Variance</th>
<th>Impulse Factor</th>
<th>Peak Factor</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRC1</td>
<td>0.2830</td>
<td>0.3862</td>
<td>0.2839</td>
<td>0.6692</td>
<td>0.2916</td>
<td>0.2950</td>
<td>0.2848</td>
</tr>
<tr>
<td>PRC2</td>
<td>0.2891</td>
<td>0.2583</td>
<td>0.2717</td>
<td>0.1216</td>
<td>0.2676</td>
<td>0.2601</td>
<td>0.0624</td>
</tr>
<tr>
<td>PRC3</td>
<td>0.1774</td>
<td>0.2117</td>
<td>0.1750</td>
<td>0.1787</td>
<td>0.1725</td>
<td>0.1750</td>
<td>0.5634</td>
</tr>
<tr>
<td>PRC4</td>
<td>0.2505</td>
<td>0.1438</td>
<td>0.2694</td>
<td>0.0305</td>
<td>0.2683</td>
<td>0.2699</td>
<td>0.0893</td>
</tr>
</tbody>
</table>

Table 7. The weight value of each evaluation index in this signal - Eq. (9)

<table>
<thead>
<tr>
<th>PRCs</th>
<th>W_K</th>
<th>W_C</th>
<th>W_M</th>
<th>W_V</th>
<th>W_I</th>
<th>W_P</th>
<th>W_A</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRC1-PRC4</td>
<td>0.0187</td>
<td>0.0690</td>
<td>0.0184</td>
<td>0.5003</td>
<td>0.0202</td>
<td>0.0194</td>
<td>0.3540</td>
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</tbody>
</table>

Table 8. Component signals after characteristic enhancement of PRCs - Y_i

Fig 11. Component signals after characteristic enhancement of Fig 8 by proposed method
5.2. Outer ring + inner ring faults analysis

Fig 13(a) and (b) corresponds to the time domain and frequency spectrum of signal from compound fault between outer ring and inner ring. Fig 14 shows the PRCs of Fig 13(a) after ITD. Fig 15 shows the function value of signal evaluation index related with each component signal. Table 9 provides the proportion value of each evaluation index for PRC obtained by formula (5). Fig 16 shows the comprehensive weight value of each evaluation index obtained by entropy weight method according to Fig 15 and Table 9, and the specific values as are shown in Table 10. Table 11 and Fig 17 shows the component signals obtained after characteristic enhancement to each PRC component signal according to formula (10) – (11). Fig 18(a) and (b) are the time domain and frequency spectrum of signal after singular value difference spectrum denoising of Fig 17. Fig 18(c) is the local enlargement of Fig 18(b).

Fig 12(b) shows the evident fault characteristic frequency extracted from the frequency spectrum treated by the proposed method of paper:

- There exists a frequency component 802Hz which corresponds to the 6x of $f_i$ (802/6 = 133.6 Hz);
- 1222Hz, the sum of 23x of $f_b$ and 1x of $f_c$ ($(1222-11.1)/23 = 52.6$ Hz);
- 1278Hz, the sum of 24x of $f_b$ and 1x of $f_r$ subtracting 1x of $f_c$ ($(1278-30.2+11.1)/24 = 52.5$ Hz);
- 1698Hz, the sum of 32x of $f_b$ and 1x of $f_r$ subtracting 2x of $f_c$ ($(1698-30.2+11.1*2)/32 = 52.8$ Hz).

It can be found from the above analysis that when a compound fault of inner race and rolling element takes place, the proposed method of paper can still effectively eliminate the influence of noise and successfully extract the fault feature frequency related to a specific type. Thus, the method can be used to correctly judge a type of compound bearing fault.
### Table 9

<table>
<thead>
<tr>
<th>P-value</th>
<th>Kurtosis</th>
<th>Correlation Coefficient</th>
<th>Margin Factor</th>
<th>Variance</th>
<th>Impulse Factor</th>
<th>Peak Factor</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRC1</td>
<td>0.2795</td>
<td>0.2783</td>
<td>0.2569</td>
<td>0.4360</td>
<td>0.2653</td>
<td>0.2694</td>
<td>0.3979</td>
</tr>
<tr>
<td>PRC2</td>
<td>0.3082</td>
<td>0.2791</td>
<td>0.2873</td>
<td>0.2045</td>
<td>0.2762</td>
<td>0.2598</td>
<td>0.2890</td>
</tr>
<tr>
<td>PRC3</td>
<td>0.1674</td>
<td>0.3260</td>
<td>0.1795</td>
<td>0.2963</td>
<td>0.1794</td>
<td>0.1867</td>
<td>0.0903</td>
</tr>
<tr>
<td>PRC4</td>
<td>0.2448</td>
<td>0.1166</td>
<td>0.2763</td>
<td>0.0632</td>
<td>0.2791</td>
<td>0.2841</td>
<td>0.2228</td>
</tr>
</tbody>
</table>

**Fig 14.** PRCs of Fig 13(a) by ITD

**Fig 15.** The function value of signal evaluation index

**Fig 16.** The weight value of each evaluation index

### Table 10

<table>
<thead>
<tr>
<th>W-value</th>
<th>( W_K )</th>
<th>( W_e )</th>
<th>( W_M )</th>
<th>( W_i )</th>
<th>( W_p )</th>
<th>( W_A )</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRC1-PRC4</td>
<td>0.0591</td>
<td>0.1465</td>
<td>0.0381</td>
<td>0.4149</td>
<td>0.0364</td>
<td>0.0300</td>
</tr>
</tbody>
</table>

**Table 9.** The proportion of each evaluation index of the component signals - Eq. (5)

**Table 10.** The weight value of each evaluation index in this signal - Eq. (9)
differs section 5.1 and 5.3’s, the method of paper can still
detect the fault of outer and inner race occurs and the rotate speed
can be known from above analysis that when a compound

\[
\begin{align*}
Y_1 &= (W_R \cdot 0.2795 + W_C \cdot 0.2783 + W_M \cdot 0.2569 + W_f \cdot 0.4360 + W_i \cdot 0.2653 + W_p \cdot 0.2694 + W_a \cdot 0.3979) \cdot \text{PRC1} \\
Y_2 &= (W_R \cdot 0.3082 + W_C \cdot 0.2791 + W_M \cdot 0.2873 + W_f \cdot 0.2045 + W_i \cdot 0.2762 + W_p \cdot 0.2598 + W_a \cdot 0.2890) \cdot \text{PRC2} \\
Y_3 &= (W_R \cdot 0.1674 + W_C \cdot 0.3260 + W_M \cdot 0.1795 + W_f \cdot 0.2963 + W_i \cdot 0.1794 + W_p \cdot 0.1867 + W_a \cdot 0.0903) \cdot \text{PRC3} \\
Y_4 &= (W_R \cdot 0.2448 + W_C \cdot 0.1166 + W_M \cdot 0.2763 + W_f \cdot 0.0632 + W_i \cdot 0.2791 + W_p \cdot 0.2841 + W_a \cdot 0.2228) \cdot \text{PRC4}
\end{align*}
\]

Table 11. Component signals after characteristic enhancement of PRCs - \( Y_i \)

Fig 17. Component signals after characteristic enhancement of Fig 14 by proposed method

It can be known from the analysis of Fig 18(c) that:

- At the spot of 1283Hz (1283Hz/2=64.2Hz), it meets the 20x of \( f_0 \);
- There exists a frequency component 64.7Hz which corresponds to the 1x of \( f_0 \);
- 72Hz, the sum of 1x of \( f_i \) and 1x of \( f_r \) subtracting the 2x of \( f_c \) ((72+25.3*2-9.3)/6=113.3Hz);
- 1217Hz, the 48x of \( f_r \).

It can be known from above analysis that when a compound fault of outer and inner race occurs and the rotate speed differs section 5.1 and 5.3’s, the method of paper can still successfully extract the frequency pertinent to the type of fault and effectively reduce the interference of noise.

5.3. Outer ring + rolling element faults analysis

Fig 19(a) and (b) corresponds to the time domain and frequency spectrum of signal from compound fault between outer ring and rolling element; Fig 20 shows the PRCs of Fig 9(a) after ITD. Fig 21 shows the function value of signal evaluation index related with each component signal. Table 12 provides the proportion value of each evaluation index for PRC obtained by formula (5). Fig 22 shows the comprehensive weight value of each evaluation index obtained by entropy weight method according to Fig 21 and Table 12, and the specific values as are shown in Table 13.
Table 14 and Fig 23 shows the component signals obtained after characteristic enhancement to each PRC component signal according to formula (10) – (11). Fig 24(a) and (b) are the time domain and frequency spectrum of signal after singular value difference spectrum denoising of Fig 23.

![Fig 19. Time domain and frequency domain diagram of original signal](image)

![Fig 20. PRCs of Fig 19(a) by ITD](image)

![Fig 21. The function value of signal evaluation index](image)

![Fig 22. The weight value of each evaluation index](image)
Table 12. The proportion of each evaluation index of the component signals - Eq. (5)

<table>
<thead>
<tr>
<th>P-value</th>
<th>W&lt;sub&gt;K&lt;/sub&gt;</th>
<th>W&lt;sub&gt;C&lt;/sub&gt;</th>
<th>W&lt;sub&gt;M&lt;/sub&gt;</th>
<th>W&lt;sub&gt;V&lt;/sub&gt;</th>
<th>W&lt;sub&gt;f&lt;/sub&gt;</th>
<th>W&lt;sub&gt;p&lt;/sub&gt;</th>
<th>W&lt;sub&gt;A&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRC1-PRC4</td>
<td>0.1159</td>
<td>0.0882</td>
<td>0.0555</td>
<td>0.2175</td>
<td>0.0526</td>
<td>0.0384</td>
<td>0.4320</td>
</tr>
</tbody>
</table>

Table 13. The weight value of each evaluation index in this signal - Eq. (9)

- \( Y_1 \) = \((W_K*0.4492+W_C*0.2785+W_M*0.3666+W_V*0.3478+W_f*0.3737+W_p*0.3595+W_A*0.0870) \) * PRC1
- \( Y_2 \) = \((W_K*0.2363+W_C*0.3294+W_M*0.2660+W_V*0.4001+W_f*0.2457+W_p*0.2289+W_A*0.5773) \) * PRC2
- \( Y_3 \) = \((W_K*0.1795+W_C*0.2983+W_M*0.2214+W_V*0.2159+W_f*0.2256+W_p*0.2386+W_A*0.3103) \) * PRC3
- \( Y_4 \) = \((W_K*0.1349+W_C*0.0938+W_M*0.1460+W_V*0.0362+W_f*0.1550+W_p*0.1730+W_A*0.0255) \) * PRC4

Table 14. Component signals after characteristic enhancement of PRCs - \( Y_i \)

Fig 23. Component signals after characteristic enhancement of Fig 20 by proposed method

Fig 24. Time domain and frequency domain diagram of proposed method - outer ring + rolling element faults
It can be known from Fig 24(b) that:

- There exists a frequency component 73.24Hz, equal to the sum of 1x of \( f_0 \) and 1x \( f_r \) subtracting 1x \( f_c \) (73.24-31.3+11.5=53.4Hz);
- 2427Hz, 46x of \( f_0 \) (2427/46=52.8Hz);
- 2573Hz (2573Hz/33=78.0Hz), consistent with the 33x of \( f_0 \).

It can be known from the analysis of Fig 9 that when a compound fault of outer race and rolling element occurred, the proposed method can not only effectively reduce the interference of noise, but also correctly extract the fault feature frequency of outer race and rolling element and recognize the type of compound fault.

It can be known that in different compound faults of bearing, the proposed method can equally reduce the influence of noise. According to the signal frequency spectrum, the method can extract the characteristic frequency consistent with the type of compound faults and then make precise judgment.

6. CONCLUSION

To solve the difficulty of single signal evaluation index to manifest characteristic information of compound faults of rolling bearing, this method has combined multiple indexes and proposed the fault characteristic enhancement method mixing ITD algorithm and entropy weight method. To further lessen the influence of noise, SDS is recruited to denoise the signals. The following conclusions can be drawn:

- By taking advantage of average and variance information of signal in a bearing fault and the impact components in fault signals, the paper has chosen 7 signal evaluation indexes to depict the compound fault characteristic information of bearing;
- By blending entropy weight method and ITD algorithm and calculating comprehensive weight of multiple signal evaluation indexes, the proposed method can embody the enhancement of fault characteristic information of component signals, effectively draw the characteristic frequency of compound faults and correctly determine on a fault type;
- The proposed method is capable to effectively restrain noise components and is sensitive to various compound faults of rolling bearing. This method can work on multiple compound fault types, including the faults among outer and inner ring and rolling element, inner ring and rolling element, outer ring and inner ring, outer ring and rolling element.

DECLARATION OF COMPETING INTEREST

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

ACKNOWLEDGEMENT

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NOMENCLATURE

- PRCS: proper rotation components
- EWM: entropy-based weight method
- ITD: intrinsic time scale decomposition
- SDS: singular differential spectrum
- SVD: singular value decomposition
- \( f_r \): rotate frequency
- \( f_i \): inner-ring fault feature frequency
- \( f_o \): outer-ring fault feature frequency
- \( f_b \): rolling-element fault feature frequency
- \( f_c \): holder fault feature frequency

REFERENCES


Gougam Fawzi et al. Bearing faults classification under various operation modes using time domain features, singular value decomposition, and fuzzy logic system[J].


